AN EVALUATION OF ADAPTIVE EQUALIZERS ON NON-LINEAR HF/IONOSPHERIC CHANNEL MODELS

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ABSTRACT

HF communications (3-30 MHz) have been a popular medium for emergency, military, and hobbyist applications because they generally do not require significant architecture or equipment. For long range HF communications, which are common, this involves having signals reflect off the Earth's ionosphere. However, the ionosphere is highly unstable, varying with respect to space, time, and frequency, which has prompted research into compensating the effects of the ionosphere in the receiver to maintain robust communications even when faced with harsh channel conditions. This instability in the ionosphere causes the channel to change between non-linear and linear models at different times. Thus, the objective of this paper is to utilize and evaluate the performance of two non-linear adaptive equalizers in recovering distorted signals transmitted through a non-linear HF channel. The performance of the equalizers will be characterized using mean squared error (MSE).

1. INTRODUCTION

The High Frequency (HF) band (3-30 MHz) is very popular among radio enthusiasts. Unlike mainstream communication architectures that require a significant amount of equipment to maintain (i.e. internet, satellites, fiber optic cables, etc.), transmissions in the HF band are made by only using radios to transmit signals by reflecting them off of the ionosphere, which enables long-range communications at a low cost. My work in [1], for example, showed that such communications were possible over the HF band, as our team (stationed in Tucson, Arizona) was able to hear communications from Tokyo, Japan. Because of this, in addition to being inexpensive, HF systems are used in emergency crises more than mainstream communication systems. One example of this is Hurricane Katrina, where HF systems were used to organize "rescue and recovery operations" as a result of damaged, regularly used communication infrastructures [2]. In addition, the HF band is used for various military applications, such as enabling troops to establish communication links in different locations [3]. However, while the ionosphere enables long-range/low-power communications over the HF band, it itself is not a stable medium. As [4] elaborates, the ionosphere varies significantly as a result of different atmospheric variations (i.e. solar radiation, sunspot cycles, seasonal changes), which can result in the transmitted signal suffering from effects like multipath and fading. This has been the motivation for the development of ionospheric models, with the most heralded of these developed by Clark Watterson. As will be elaborated on later, his model assumes that the channel is stationary in time and frequency making it only accurate for small bandwidths [5]. Thus, it is desirable to design a system capable of obtaining an adaptive, real-time model of the ionosphere that can be used to improve transmissions made over the HF frequency band.

Equalizers are an ideal signal processing tool for this application because they take the received signal from the channel as input and output an estimate of the transmitted signal by "creating an inverse model of the transmission channel" [6]. The error between this estimate and the transmitted signal is calculated and, in the case of adaptive equalization, is fed into a learning algorithm that utilizes the error to fine tune the equalizer's coefficients. Thus, the objective of this paper is to survey/compare the performance of different equalizers that have been implemented specifically for the HF channel. These equalizers are specifically the Least Mean Squares Decision Feedback Equalizer (i.e. LMS-DFE) and Constant Modulus Algorithm equalizer (i.e. CMA). The structure of the paper is as follows. Section 2 provides background on previous implementations of these equalizers for the HF channel. Section 3 provides an explanation on the functionality of a general Decision Feedback Equalizer (DFE). Section 4 provides an explanation on LMS-DFEs and CMA equalizers specifically. Section 5 elaborates on the Watterson and non-linear models used in this effort. Section 6 describes how the experiments were implemented and provides an analysis on the results obtained. Section 7 summarizes the work completed in this effort and provides next steps we will take in future projects.

2. BACKGROUND

Historically, different types of equalizers have been implemented in the context of HF channels. The authors of [7] showed that decision feedback equalizers (DFEs) were better suited for the HF channel than maximum-likelihood sequence estimation (MLSE) equalizers, due to having a similar performance but much simpler complexity. As such, different types of DFEs have been implemented for the HF channel. One type of equalizer that has been implemented frequently for HF environments is the Kalman Decision Feedback Equalizer (i.e. Kalman-DFE), where different variations of the standard Kalman state-tracking algorithm have been used to update the taps of the DFE. In [8], the recursive least squares (RLS) algorithm was utilized to update the taps because its usage in adaptive equalization is analogous to tracking states with standard Kalman filtering. In [9], a square root Kalman algorithm is utilized to update the weights of the DFE. In [10], a fast recursive least squares (FRLS) DFE is implemented and shown to have significantly better performance than a LMS-DFE and a similar performance to the DFE formed in [9].

In addition, different types of equalizers have been used in the HF channel. In [11], various blind equalization algorithms, including the Constant Modulus algorithm, were implemented over the air. While the above algorithms require sending a training sequence over the channel, to allow the equalizers to adapt the taps, the main attraction of blind equalization algorithms is that they don't require training sequences - allowing for more useful data to be sent at higher speeds [12].

3. DFE FUNCTIONALITY

The DFE consists of a feed-forward and feedback filter. Different adaptive algorithms are used to update the taps of the DFE, with the mean squared error (MSE) algorithm being one of the most common algorithms utilized. A standard DFE can be expressed as follows [13]:

$$\hat{I}_k = \sum_{j=-K_i}^{0} c_j v_{k-j} + \sum_{j=1}^{K_2} c_j I_{k-j}^{\check{}} \tag{1}$$

where the indices of j and c_j represent the taps of the DFE, v represents the sequences received from the channel, $\check{I}$ represents symbols decoded in previous iterations, and $\hat{I}_k$ represents the output of the equalizer. In equation 1, the first summation represents the feed-forward filter and the second represents the feedback filter. As [13] describes, if the mean squared error is being used to update the weights, the following cost function is minimized as shown in equation 2:

$$E(K_1, K_2) = |\hat{I}_k - \check{I}_k|^2 \tag{2}$$

where $(K_1 + 1)$ and K_2 represent the number of taps used in the feed-forward and feedback filter respectively. However, as will be shown in section 4, this criteria will vary based on the learning algorithm used.

4. LEAST MEANS SQUARES AND CONSTANT MODULUS ALGORITHMS

4.1. LMS

Similar to equation 2, [14] defines the error of the equalizer to be the following:

$$e(n) = d(n) - y(n) \tag{3}$$

where e represents the error, d represents the desired signal, and y represents the equalized output as given in equation 4:

$$y(n) = \hat{w}^H(n)u(n) \tag{4}$$

where $u(n)$ is the input sequence, $\hat{w}$ represents an estimate of the ideal tap vector (i.e. w), and H represents the Hermitian transposition. Contrary to equation 2, the LMS algorithm assumes that the function to be minimized is as shown in equation 5 [14]:

$$J(n) = E[|e(n)|^2] \approx e(n)e^*(n) \tag{5}$$

where $E[]$ denotes the expectation value and $e^*(n)$ is the complex conjugate of the error vector. As [14] elaborates this removal of the expectation is used to make the estimation more feasible to adapt to a varying environment. To find the subsequent tap vector that is most effective in optimizing the above cost function, equation 6 is used to update the taps at each iteration of the adaptation process:

$$\hat{w}(n+1) = \hat{w}(n) + \mu u(n)e^*(n) \tag{6}$$

where μ is a step-size parameter. Thus, the LMS algorithm would be an alternative to the MSE algorithm for updating the weights of the DFE. An important attribute to note about the LMS algorithm is that it does have knowledge about attributes of the signal (i.e. modulation) prior to equalization, unlike the CMA algorithm.

4.2. CMA

The Constant Modulus algorithm (CMA) is commonly used as a learning algorithm for blind equalizers. It is ideally used when managing signals with a constant amplitude. Similar to the LMS and MSE algorithms, it also has a cost function as shown in equation 7 [15]:

$$D^{(p)} = E(|z(n)|)^p - R_p)^2 \tag{7}$$

where D is referred to as the dispersion of order p, $z(n)$ represents the equalizer output, and the meaning of R_p will be explained later. As [11] summarizes, steepest descent can be used to optimize equation 7 resulting in the following update equation:

$$e_{n+1} = e_n - \mu \frac{\partial D^{(p)}}{\partial e}|_{e=e_n} \tag{8}$$

where e represents each of the taps of the equalizer, μ is a step-size parameter, and $\frac{\partial D^p}{\partial e}$ is given by the following expression:

$$\frac{\partial D^{(p)}}{\partial e}|_{e=e_n} = 2pE[y_n^* |z(n)|^{p-2}(|z(n)|^p - R_p)] \tag{9}$$

As [11] indicates, a specific form of R_p can be derived making the following two assumptions. The first is that the equalized signals, $z(n)$, are assumed to have the form $a(n) * e^{j*(\phi+2\pi\delta fnT)}$. The second is that $\frac{\partial D^{(p)}}{\partial e}$ is set to zero. Plugging this information into equation 9, it can be shown that R_p is given by the following expression:

$$R_p = \frac{E[|\mathbf{a(n)}|^{\mathbf{2p}}]}{E[|a(n)|^p]} \tag{10}$$

5. CHANNEL MODELS

5.1. Watterson Model

As explained in the introduction, Clark Watterson's model, referred to as the Watterson model, is the most frequently used ionospheric model in HF experiments. The model represents the ionosphere's effects on transmitted signals as a tapped delay line as shown in figure 1 [16]. Each tap modulates the transmitted signal in amplitude and phase to simulate Gaussian scattering effects using a complex gain function specific to each tap, represented in figure 1 as $G_i(t)$. Each function has a bi-variate Gaussian power spectrum, which enables the model to capture the effects of Rayleigh fading on the outputted signal [17].

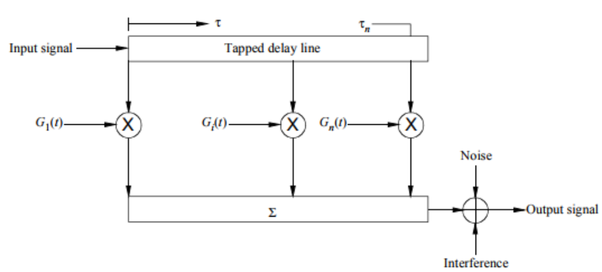

Figure 1: Watterson Model - Tapped Delay Line

Each main tap of the delay line shown in figure 1 is created using a series of taps generated using equation 11 [17]:

$$h_n(t) = ke^{-\pi^2 f_j^2 n^2 T_s^2}; -N < n < N \tag{11}$$

where f_j represents the Doppler spread, k is used to preserve a unity gain, n represents the tap index, N represents the total length of the filter, and T_s represents the sample period. For this effort, the above tap generation was implemented as shown so that it could accept the delay spread, Doppler spread, and number of taps as parameters. Complex Gaussian noise is filtered through these taps and sampled, with these final values used as the main taps of the Watterson model [17].

The International Telecommunications Union (ITU) [18] defines different HF channel conditions based on the delay spread and Doppler spread inputted into the Watterson Model, which are displayed in tables 1 and 2. As will be shown in later sections, these values were used in the simulations to simulate the ionosphere under different conditions.

Table 1: ITU Poor Channel Conditions

Channel Condition	Poor
Delay Spread (ms)	2
Doppler Spread (Hz)	1

Table 2: ITU Moderate Channel Conditions

Channel Condition	Moderate
Delay Spread (ms)	1
Doppler Spread (Hz)	0.5

5.2. Non-Linear Channel Model

While the Watterson model has been experimentally verified and used in multiple projects analyzing the HF band, as stated earlier, the assumptions Watterson made when constructing this model makes it valid only for small bandwidths [5]. In addition, because of the ionosphere's instability, it is possible for it to exhibit non-linear attributes at different instances, which the Watterson model does not neccesarily capture. However, Watterson's model did verify certain attributes of the ionosphere that are accurate; specifically, the channel having a Gaussian distribution and Rayleigh fading. Thus, in addition to using Watterson's original model, a non-linear model was also implemented to simulate such effects. To our knowledge, there is not an existing non-linear Ionospheric model that has been verified/heralded as the Watterson model. Thus, the following model, shown in equation 12 is our initial attempt to simulate non-linear effects in the HF channel:

$$y = x^2 + x + n \tag{12}$$

where y represents the signal received by the equalizer, x is the signal corrupted from the Watterson model, and n represents Gaussian noise. Thus, for this project, both Watterson's original model and the above non-linear model were used in the experiments for both equalizers.

6. RESULTS

6.1. System Architecture

The equalizers explained in section 4 were simulated using existing implementations in GNU Radio, an open source language used for rapid prototyping of communication systems/algorithms [19]. As explained earlier in section 4.1., the LMS equalizer does have knowledge about the incoming signal unlike the CMA equalizer. GNU Radio accounts for this by having one of the inputs to its LMS decision-directed (i.e. LMS-DD) equalizer block be the constellation of the incoming signal. In replacement of this parameter, the CMA equalizer block takes as an input the desired modulus (i.e. amplitude) that will drive the signal's equalization. Despite this distinction both blocks have the following parameters: number of taps, gain of the update loop, and samples per symbol rate of the incoming signal, which is used to apply proper down-conversion if the signal was interpolated at an point in the system. During the simulations, each equalizer had 4 taps, a gain of 0.01, and a sample per symbol rate of 2. A vector sink block was used to store the equalizer's output, and Python was used to calculate the subsequent MSEs.

8-PSK was the only modulation used in all experiments. Complex Gaussian noise was filtered through 101 taps used to generate a Watterson model with two main taps on its delay line, as explained in section 5.1. Both the Watterson and non-linear model were simulated using GNU Radio and Python.

6.2. MSE Results

Two different experiments were executed to gauge the performance of the two equalizers. The first involved observing the MSE of each equalizer as the SNR of the Watterson model was varied from 10 to 50 dB, with a step size of 5 dB. Each point was averaged over 5000 trials. Figures 2 and 3 show the results of these experiments for poor and moderate channels.

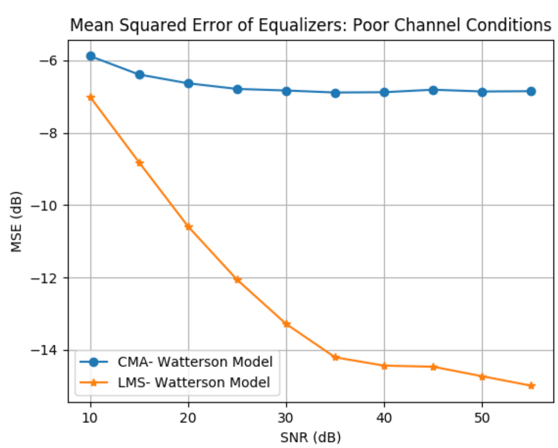

Figure 2: MSEs of LMS and CMA for Poor Channel Conditions as a Function of SNR

As the figures show, for both poor and moderate channel conditions, the LMS-DD equalizer has a much smaller MSE compared to the CMA equalizer. This may be due to the LMS-DD equalizer having "a-priori" knowledge of the signal's modulation, giving it an advantage over the CMA equalizer. However, for both equalizers, the MSE does decrease as the SNR of the channel is increased, as expected.

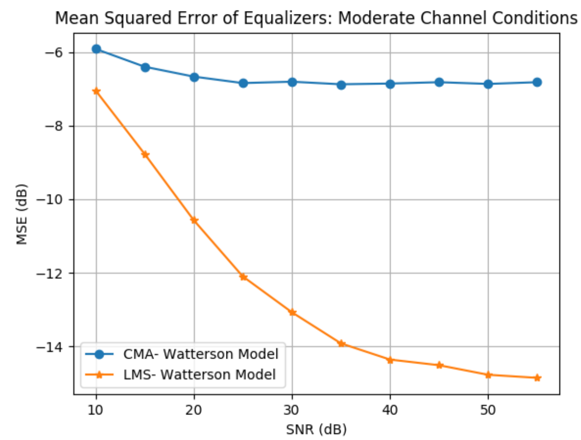

Figure 3: MSEs of LMS and CMA for Moderate Channel Conditions as a Function of SNR

As stated earlier, a vector sink block in GNU Radio was used to store the equalizer's output. A head block was then used to limit how many samples were stored in the vector. Subsequently, the second experiment involved observing the MSE of each equalizer as the number of samples outputted from the equalizers were varied. These trials were performed with the channel having an SNR of 10 dB, under poor and moderate conditions, with a range of 50-300 samples and a step size of 10 samples. Similar to the first experiment, each point was averaged over 5000 trials. The results of these experiments are shown in figures 4 and 5.

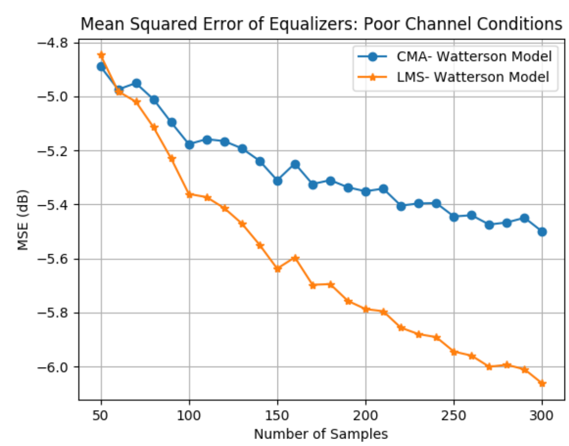

Figure 4: MSEs of LMS and CMA for Poor Channel Conditions as a Function of Number of Samples

Similar to the results from the first experiment, the LMS-DD equalizer has a smaller MSE than the CMA equalizer, under

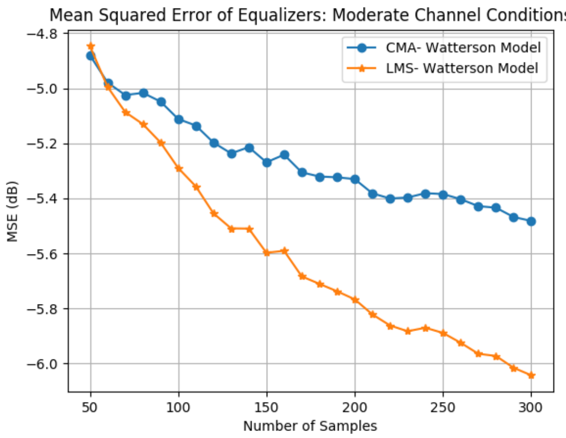

Figure 5: MSEs of LMS and CMA for Moderate Channel Conditions as a Function of Number of Samples

both poor and moderate conditions. As a sanity check, the MSE does decrease as the number of samples is increased again, as expected. However, it is noteworthy that at a low number of samples, the equalizers seem to have a similar performance under poor and moderate conditions. The above experiments were then repeated to obtain MSE results from usage of both the Watterson and non-linear channel models. Figures 6 and 7 show the MSE results when the Watterson and non-linear model are used as the SNR of the channel is varied. Figure 8 shows the MSE results in a similar experiment, with the number of samples sent to the equalizer being varied, but the channel set at an SNR of 10 dB under poor conditions.

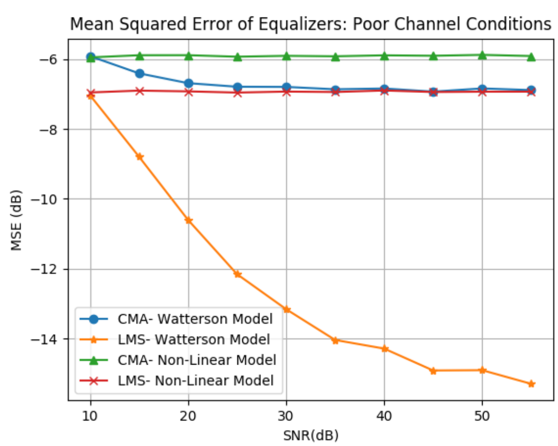

Figure 6: MSEs of Equalizers Under Poor Linear/Non-Linear Conditions as a function of SNR

Figures 6, 7, and 8 show that using the non-linear channel model produces a higher MSE than using the Watterson model. This is to be expected as the non-linear model provides more distortion to the signal prior to being processed by the equalizer. It's

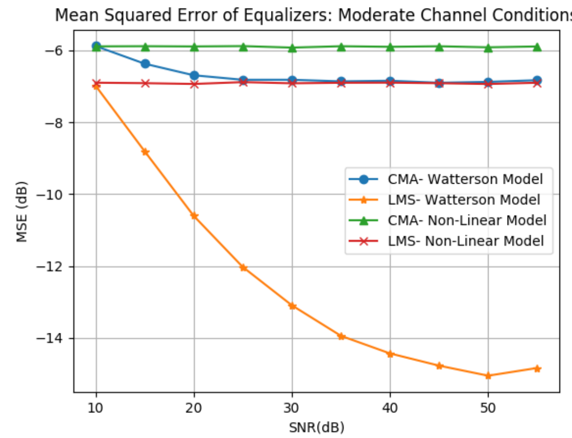

Figure 7: MSEs of Equalizers Under Moderate Linear/Non-Linear Conditions as a function of SNR

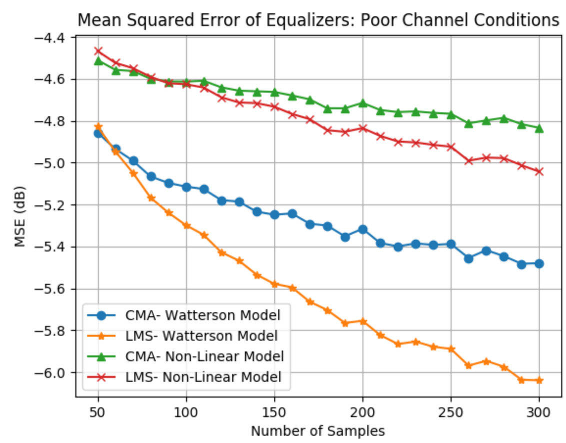

Figure 8: MSEs of Equalizers Under Poor Linear/Non-Linear Conditions as a Function of Number of Samples

interesting to observe, however, that despite this added complexity the LMS-DD equalizer still produces an MSE smaller than that of the CMA-equalizer under non-linear conditions. Figures 6 and 7 also indicate that the LMS equalizer under non-linear conditions has an MSE that is comparable to that of the CMA equalizer when using the Watterson model, which again affirms the LMS equalizer's superiority in these experiments. In addition, it seems that increasing the SNR is not effective towards reducing the MSE in this scenario. Figure 8 affirms these results but provides some additional insights as it appears the equalizers seem to again have a similar performance when the number of samples collected from the output of the equalizer is small. However, as the number of samples is increased, despite the LMS-DD equalizer having a better performance, figure 8 indicates there is not a significant reduction in the MSE when using the non-linear model. These results imply that further signal processing techniques will need to be implemented to compensate for non-linear effects.

7. CONCLUSION

The objective of this paper was to observe the performance of LMS and CMA equalizers in restoring signals transmitted through linear and non-linear Ionospheric models. It was shown in all experiments that the LMS-DD equalizer had a smaller MSE than the CMA equalizer, which could be due to the LMS knowing the constellation of the transmitted equalizer whereas the CMA did not, due to it being a blind equalizer. As expected, it was shown that usage of a non-linear Ionospheric model produced higher MSEs compared to using the Watterson model. However, it's noteworthy that despite these added complexities the LMS equalizer had a better performance than that of the CMA equalizer. These experiments also indicated that increasing the SNR, as well as the number of samples outputted from the equalizer, are not sufficient means for handling these non-linear effects. This implies that additional signal processing techniques are required to fully recover a signal corrupted due to non-linearities in the HF channel.

One of our next steps will be incorporating different adaptive equalizers/equalization techniques; such as, Minimum Mean Square Equalizers (MMSEs), Neural Networks and Volterra Equalizers, and observing their performance over similar experiments. As in this paper, the effectiveness of these equalizers will be classified based on their average MSE as well as additional metrics such as BER and convergence rate. We will research/investigate the ionosphere's unstable behavior to craft a more realistic non-linear model. In addition, we will work towards having analysis of the equalizers' performances when transmitting over the air, to capture the real-time impact of Ionospheric reflections on a signal transmitted through the HF band.

8. ACKNOWLEDGEMENTS

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9. REFERENCES

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