Co-existence of MIMO Radar and Communication Systems: Real-Time Framework for Interference Control

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***Abstract*—Controlling interference in a time-varying wireless channel is essential to spectrum sharing and co-existence solutions. Some of radar bands are subject to a prospective spectrum sharing with communication services, and in this scenario radar is expected to receive and cause interference to other sharing devices. In this work, we take one direction of the problem and study whether a careful design for MIMO radar would result in a reduced interference at communication receivers. We derive a steepest-descent pre-coder for radar transmitter, and explain how it can be used in the co-existence of a MIMO radar and communication system.**

I. Introduction

Sharing the spectrum between dissimilar systems has already been proposed to be the solution for the shortfall of the free radio spectrum [1]. Spectrum sharing has many challenges, and one of these challenges pertain to the control of interference within a time-varying wireless channel [2]. Dissimilar systems can co-exist as long their transmissions do not harm other systems. Communication system are able to tolerate a certain level of interference, we study whether radar would harness this tolerated interference as an opportunity for successful operation. One challenge to this problem is that channels between the coexistent systems are sbject to small scale impairments, such as real-time shadowing and fading. Path-loss models can be used to estimate the received interference [3], but these models cannot capture the real time dynamics of the wireless channel. The development of real-time oriented tools that control the interference is essential for the success of spectrum-sharing and co-existence solutions. In this paper, we propose a framework that addresses these real-time changes within the co-existence of MIMO radar and communication systems.

Radar uses large portions of the radio spectrum. The measurements carried out by the National Telecommunciation Information and Administration (NTIA) show that spectrum bands allocated for radar systems are under-utilized, and that secondary spectrum access to these bands is feasible [4], and similar observations have been noticed around the globe. In this paper, we envision a model where a radar band will be released to communication services, and study whether radar can still function as an opportunistic user in this band[1]. We redesign radar's pre-coder so that its transmission is confined within the interference range tolerated by a cellular base station.

There are many approaches to design radar pre-coder for intereference management purposes, and one of these approaches is based on subspace methods. Interference can be controled through a subspace expansion or reduction. Authors in [5] proposed a null-space based projection to cancel *completely* the radar's generated interference, and they have analyzed how this nulling affects radar performance. They have concluded that this interference-nulling scheme would severely degrades radar performance. Authors in [6], [7] suggested a subspace expansion that alleviates this degradation. Their method allows radar to transmit interference up to limited number of levels that the communication receivers can tolerate. Unfortunately, the number of these levels is limited by the number of antennas that are used by radar and communication systems. Authors in [8] proposed a polynomial-defined subspace method that overcomes this limitation, they have introduced a matrix polynomial formula that enhances the resolution of the subspace-based interference control. The previously mentioned subspace methods are computationally heavy, and a more less complex solutions are required. In this work, we propose low complexity pre-coder design that manages the interference between radar and communication systems, and we present some preliminary results pertain to its computational performance. The proposed pre-coder is based on a steepest descent design.

A. Paper Organization

This paper is organized as follows. In Section II, we review the system model. In Section III, we review the theory behind steepest-descent pre-coder design, and we derive related solutions. In Section IV, we present and discuss the related numerical results.

[1]Some countries around the world might find releasing radar bands for communication services less expensive than the building and management of data-base oriented spectrum sharing framework

B. Notations

The $||\cdot||_F$ is the Frobenius norm, ∇ is the gradient (i.e, the first derivative), $()^H$ is the matrix Hermitian, $\mathbb{C}$ is the set of complex numbers, $P_R^{(i)}$ indicates the value of P_R at time slot-i, the $tr(A)$ indicates the trace of the matrix A,

II. System Model

We start with MIMO radar sharing a common spectrum with a communication system, as shown in Fig.1. The communication system consists of a MIMO base station equipped with N_T and N_R transmit-and-receive antennas, respectively. There are L users connected with the base station. The base station receiver has a post-processor module $F_B \in \mathbb{C}^{L \times N_R}$. The channel between the base station and the L users is denoted as $H_{BL} \in \mathbb{C}^{N_R \times L}$. Radar is equipped with M_T and M_R transmit-and-receive antennas, respectively. Radar's transmitter has a pre-coder module $P_R \in \mathbb{C}^{M_T \times M_T}$. The channel between the radar and the base station is denoted as $H_{BR} \in \mathbb{C}^{N_R \times M_T}$, and H_{BL} and H_{BR} are modeled as Complex-Symmetric-Gaussian-Channels (CSGC). Interference generated by the radar and received by the base station is quantified as $y_{BR} = ||F_B H_{BR} P_R||_F$. Our goal is to design the radar's pre-coder so that the received interference by the base station's receiver is confined to the tolerated threshold, d_{th}, i.e, $d_{th} = ||F_B H_{BR} P_R||_F$. In previous literature, the P_R matrix solution was obtained using the subspace methods [5], [6], [7]. In this work, we seek a less complex design,and we investigate a Steepest-Descent (SD) approach. First, we review how the SD solution is obtained, and then we introduce it into the co-existence model.

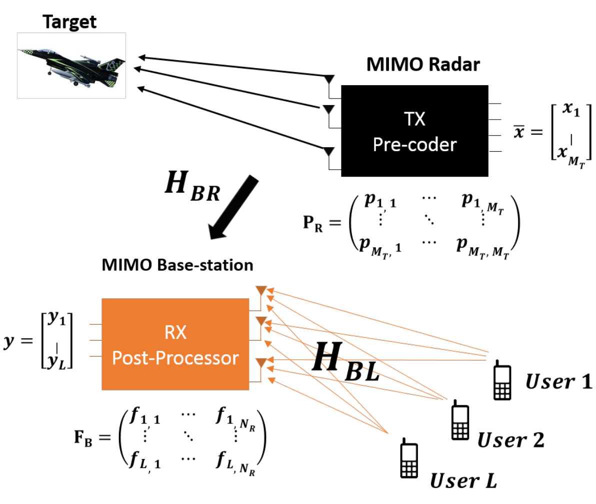


Fig. 1: Co-existence system model

III. Steepest Descent Review and Design

SD is based on minimizing the error between the actual and its desired outputs, as shown in Fig.2, where the error is minimized in an iterative manner. We can inlay this general process into our problem, and design the radar's pre-coder with the purpose of interference control. The actual output is the interference received by the base station, i.e, $y_{BR} = ||F_B H_{BR} P_R||_F$, and the desired output (or called the reference) is the tolerated interference threshold, i.e, d_{th}. The error is the absolute difference defined as $\epsilon = \Big| d_{th} - ||F_B H_{BR} P_R||_F \Big|$. Next, we show how this can be approached mathematically. We would like to mention that the inputs $\{F_B, H_{BR}\}$ are obtained through a channel training as we will explain later.

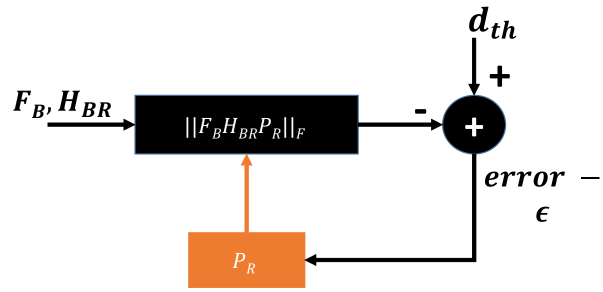


Fig. 2: SD block diagram

Define a Minimum Square Error (MSE) cost-function:

$$MSE = \epsilon^2 = \Big[d_{th} - ||F_B H_{BR} P_R||_F \Big]^2 \tag{1}$$

The pre-coder solution, i.e, $\widehat{P}_R$, is the one that minimizes the cost-function as follows:

$$\widehat{P}_R = arg \min_{P_R} \quad \Big[d_{th} - ||F_B H_{BR} P_R||_F \Big]^2 \tag{2}$$

The Frobenius norm makes the problem in (2) a non-linear problem with M_T^2 unknowns. Recall that the size of the pre-coder matrix P_R is M_T by M_T. The pre-coder solution is the minimum of this non-linear cost-function. There are numerical techniques that can be used to find this minimum, such as the steepest-descent and the normalized steepest-descent. Next we discuss them in more detail.

A. Pre-coder Design Based on Steepest Descent

The steepest descent solution is defined as [9]:

$$P_R[k+1] = P_R[k] - \mu \nabla[k] \tag{3}$$

where, $P_R[k]$ is the pre-coder matrix at the time step index k, $\nabla[k]$ is the gradient of the MSE and is supposed to be a matrix with the same size of $P_R[k]$, and μ is a scalar known as the *convergence factor* or *step size*. A typical range for the μ-factor is $0 < \mu \ll 1$.

The MSE is a scalar quantity and its gradient can be defined as the derivative of MSE with respect to the matrix P_R.

$$\nabla = \frac{\partial \text{MSE}}{\partial P_R} = \begin{bmatrix} \frac{\partial \text{MSE}}{\partial P_{R_{11}}} & \cdots & \frac{\partial \text{MSE}}{\partial P_{R_{1M_T}}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \text{MSE}}{\partial P_{R_{M_T 1}}} & \cdots & \frac{\partial \text{MSE}}{\partial P_{R_{M_T M_T}}} \end{bmatrix} \tag{4}$$

Notice that MSE is a non-linear function with respect to the P_R. We approximate the gradient as follows

$$\begin{aligned}\nabla = \frac{\partial \texttt{MSE}}{\partial P_R} &= \frac{\partial \epsilon^2}{\partial P_R} = 2\epsilon \frac{\partial \epsilon}{\partial P_R} \\ &= 2\left[d_{th} - ||F_B H_{BR} P_R||_F\right] \cdot \\ &\qquad\qquad \frac{\partial}{\partial P_R}\left[d_{th} - ||F_B H_{BR} P_R||_F\right] \\ &\approx 2\left[d_{th} - ||F_B H_{BR} P_R||_F\right]\left[-||F_B H_{BR}||_F \; P_R\right] \\ &= -2\,\epsilon \cdot \; ||F_B H_{BR}||_F \; \cdot P_R \end{aligned} \tag{5}$$

the previous approximation was obtained based on numerical tests. By substituting (5) in (3) the pre-coder solution becomes

$$\begin{aligned}P_R[k+1] &= P_R[k] + 2\mu\,\epsilon[k] \cdot \; \left|\left|F_B H_{BR}\right|\right|_F \; \cdot P_R[k] \\ &= P_R[k] + 2\mu\left[d_{th} - \left|\left|F_B H_{BR} P_R[k]\right|\right|_F\right] \\ &\qquad \cdot \left|\left|F_B H_{BR}\right|\right|_F \; P_R[k] \cdot \end{aligned} \tag{6}$$

This solution requires updating the radar pre-coder sequentially until the error tends to zero.

a) Special Case: For interference considered at the base station's receiver antennas, for example, before the post-processor, as in Fig.(1), and following same analysis we did as in (4) to (6) the pre-coder solution becomes

$$\begin{aligned}P_R[k+1] &= P_R[k] + 2\mu\,\epsilon[k] \cdot \; \left|\left|H_{BR}\right|\right|_F \; \cdot P_R[k] \\ &= P_R[k] + 2\mu\left[d_{th} - \left|\left|H_{BR} P_R[k]\right|\right|_F\right] \\ &\qquad \cdot \left|\left|H_{BR}\right|\right|_F \; \cdot P_R[k] \cdot \end{aligned} \tag{7}$$

The steepest descent method has two shortcomings: it has a large convergence time and an unstable behavior [9]. The unstable behavior occurs when the cost-function surface has a narrow valley shape around the minimum. The NSD method provides better performance, and we derive its solution next.

B. Pre-coder Design Based on Normalized Steepest Descent

The NSD solution to the problem in (2) can be stated as [10]:

$$P_R[k+1] = P_R[k] - \tilde{\mu}[k]\nabla[k] \tag{8}$$

The only difference between (8) and (3) is that the step size changes over time. This ensures a better convergence when compared with the fixed step size, μ. We define the time variable step size, $\tilde{\mu}[k]$, as follows:

$$\bar{\mu}[k] = \frac{\mu}{\left|\left|P_R[k]P_R^H[k]\right|\right|_F^2} = \frac{\mu}{tr\Big(P_R[k]P_R^H[k]\Big)} \tag{9}$$

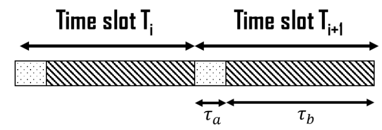


Fig. 3: Time slot allocation

where, $0 \leq \mu \leq 1$, $tr(.)$ is the trace. Substituting (9) and (5) into (8) results in the NSD solution

$$\begin{aligned}P_R[k+1] &= P_R[k] + 2\mu\,\epsilon[k] \cdot \; ||F_B H_{BR}||_F \; \cdot P_R[k] \\ &= P_R[k] + \frac{2\mu}{tr\Big(P_R[k]P_R^H[k]\Big)} \cdot \\ &\qquad \Big[d_{th} - ||F_B H_{BR} P_R[k]||_F\Big] \; ||F_B H_{BR}||_F \; P_R[k] \cdot \end{aligned} \tag{10}$$

b) Special Case: For interference considered at the base station's receive antennas, for example, before the post-processor, as in Fig.(1), the solution in (10) becomes

$$\begin{aligned}P_R[k+1] &= P_R[k] + \frac{2\mu}{tr\Big(P_R[k]P_R^H[k]\Big)}\,\epsilon[k] \; ||H_{BR}||_F \; P_R[k] \\ &= P_R[k] + \frac{2\mu}{tr\Big(P_R[k]P_R^H[k]\Big)} \cdot \\ &\qquad \Big[d_{th} - ||H_{BR} P_R[k]||_F\Big] \; ||H_{BR}||_F \; P_R[k] \cdot \end{aligned} \tag{11}$$

C. Time Framework

We presume that radar should be synchronized to the operation of the communication system. Current communication systems, such as LTE, transmit Reference Signals (RS) for channel estimation purposes, and the transmission of these RS signals is periodic overtime. Radar tunes to these transmitted RS signals, in a time slotted fashion as in Fig.3, to estimate the product $F_B H_{BR}$, which are the inputs required to initiate the pre-coder design. We assume the channel, H_{BR}, remains static within each time slot. Radar configures its pre-coder at the start of every time slot, at the training interval τ_a, following the steps shown in Algorithm 1. In the other interval, τ_b, radar operates normally, while the base station's received interference is confined to the tolerated interference threshold, d_{th}. The time that radar spends to configure its pre-coder is crucial for efficient spectrum utilization, and should be as short as possible. In the next section, we investigate the length of this time for various system parameters and conditions. We examine the convergence speed of the pre-coder solutions derived in (6) and (10).

IV. Numerical Results and Discussion

We are interested to examine the convergence speed of the solutions obtained in (6), (7), (10), and (11). First, we compare the convergence behavior of SD and NSD. Next, we examine

Algorithm 1 SD-based interference control

procedure DESIGN RADAR'S PRE-CODER P_R
Start new time slot T_i**:**
Inputs: $d_{th}^{(i)}$, $F_B^{(i)}$, and $H_{BR}^{(i)}$
State the error tolerance $\delta^{(i)}$ and the convergence factor $\mu^{(i)}$:
    $\delta^{(i)} = 0.01$, $0 \leq \mu^{(i)} \ll 1$
State the initial pre-coder settings, $P_R^{(i)}[k=0]$:
    $P_R^{(i)}[k=0] = randn(M_T) + j\, randn(M_T)$, or
    $P_R^{(i)}[k=0] = P_R^{(i-1)}$
Keep updating the radar' pre-coder using (6) or (10) and stop when $\left| d_{th}^{(i)} - ||F_B^{(i)} H_{BR}^{(i)} P_R^{(i)}[k]||_F \right| \leq \delta^{(i)}$ is satisfied
Outputs:
    $P_R^{(i)}$
Channel changes
    loop to: start a new time slot, T_{i+1}

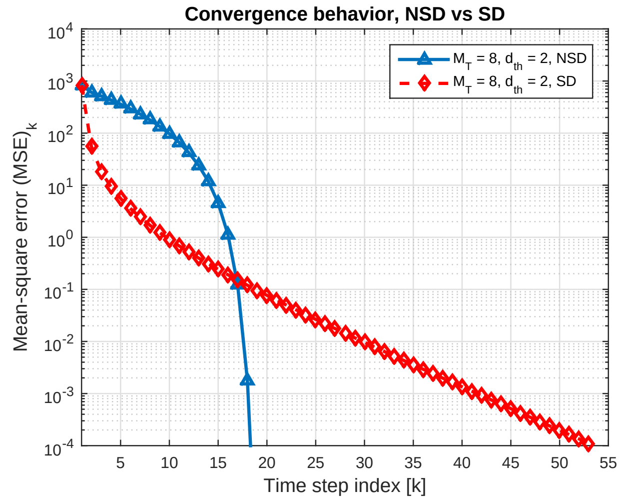
the influence of the initial settings on the convergence time, and then we probe the impact of increasing the number of antennas on the convergence time.

A. SD vs NSD

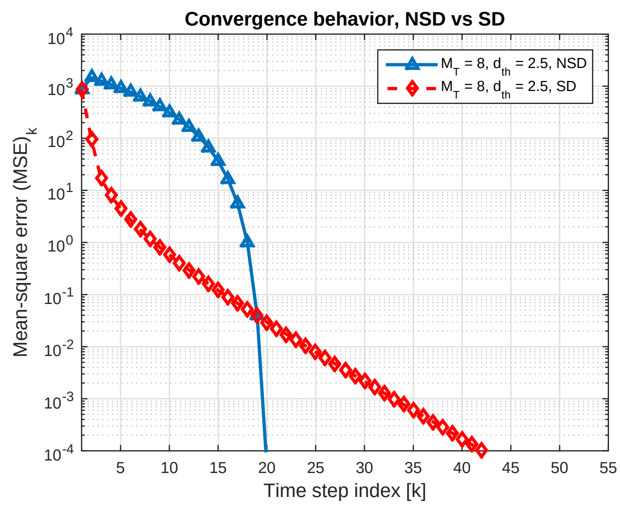
NSD solution has better convergence behavior when compared with its SD counterpart. This is due to the fact that the convergence factor, $\mu[k]$, of the NSD solution is defined to be time variant, as in (9). The step size is adjusted with every pre-coder's update in (8), this results in a faster and more stable convergence. We have compared SD and NSD convergence behaviors for the same operating scenario. Fig.4(a) shows the SD and NSD convergence behavior for $d_{th} = 2$ and in Fig. 4(b) we plot the convergence behavior for $d_{th} = 2.5$. The y-axis is the MSE error associated with the iterative updates, i.e, $MSE[k] = \epsilon^2[k] = \left| d_{th} - \left|\left| F_B H_{BR} P_R[k] \right|\right|_F \right|^2$. The x-axis is the time step index [k]. These plots represent the simulation of one time slot realization. The initial settings were assigned randomly, $P_R^{(i)}[k=0] = randn(M_T) + j\, randn(M_T)$. Radar and the base station are equipped with $M_T = M_R = N_T = N_R = 8$ antennas. Note that NSD converges with faster time when compared to the SD one. For an example see the blue lines in Fig.4(a) and Fig.4(b).

B. Initial Setting $P_R[k=0]$

Selecting the proper initial setting $P_{R[k=0]}$ is crucial for the convergence and stability of the pre-coder solutions in (6), (7), (10), and (11). There are two options to select the initial pre-coder setting. One option is to assign $P_{R[k=0]}$ randomly, while in the other option we use the pre-coder design computed in the previous time slot as an initial setting to the current time slot, $P_R^{(i)}[k=0] = P_R^{(i-1)}$. In Fig.(5(a)) we compare NSD convergence behavior for random initial settings with the "previous time slot" settings option. The y-axis is the MSE error associated with iterative updates, $MSE[k] = \epsilon^2[k] = \left| d_{th} - \left|\left| F_B H_{BR} P_R[k] \right|\right|_F \right|^2$. The x-axis is the time



(a) $d_{th} = 2$



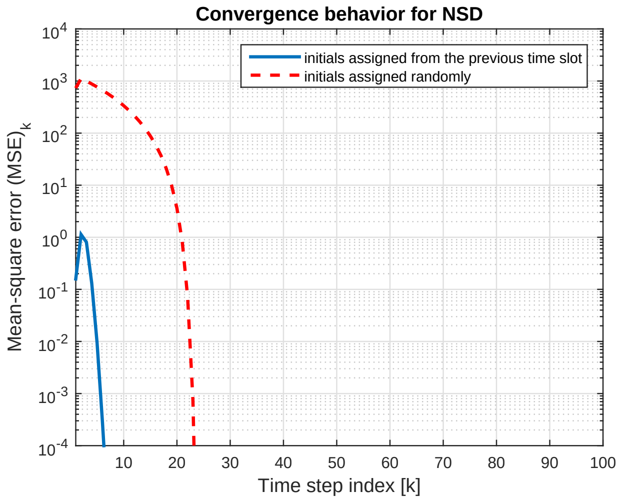
(b) $d_{th} = 2.5$

Fig. 4: Comparison between SD and NSD convergence behavior

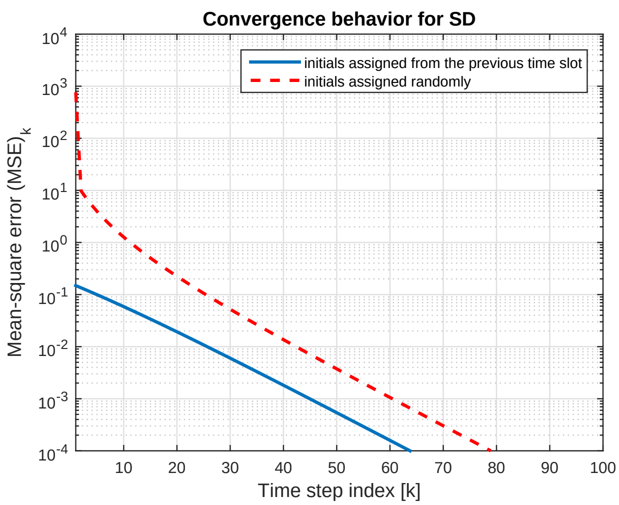
step index [k]. These plots represent the simulations for one time slot realization. Radar and base station have the following number of antennas $M_T = M_R = N_T = N_R = 8$. Tolerated interference level is $d_{th} = 2$. The NSD solution converges slower for random initial settings, the pre-coder solution with the random initial setting converges for $k = 23$ iterations, as in Fig. (5(a)), when compared with around $k = 5$ iterations for the previous time slot assignment. Similar observation is obtained for the SD solution, the random initial pre-coder assignment converges for $k = 79$, while the "previous time-slot" assignment converges faster, $k = 64$, as shown in Fig. (5(b)).

C. Number of Antennas

The increase in the number of MIMO radar antennas improves radar capability in estimating target's angle of arrival



(a)



(b)

Fig. 5: Convergence behavior and initial settings effects

and enhances the resolution of radar beam-pattern. However, this increase in the number of antennas increases the size of the pre-coder matrix, P_R, which in its turn will make the convergence slower. In Fig.6, we compare the convergence behavior for a different number of antenna. The initial pre-coder settings were assigned randomly. The radar and the base station have the same number of antennas, $M_T = M_R = N_T = N_R$. The relative interference threshold is $d_{th} = 2$. Increasing the number of antennas slows the convergence, for a typical example see the solid blue line with $M_T = 8$ antennas converges around $k = 19$, while the red dashed line with $M_T = 10$ antennas converges around $k = 26$, and the green dotted line with $M_T = 12$ antennas converges around $k = 34$.

V. Conclusion

We have presented a framework for controlling the interference generated by an opportunistic MIMO radar. In the proposed framework we redesign the radar's pre-coder such that

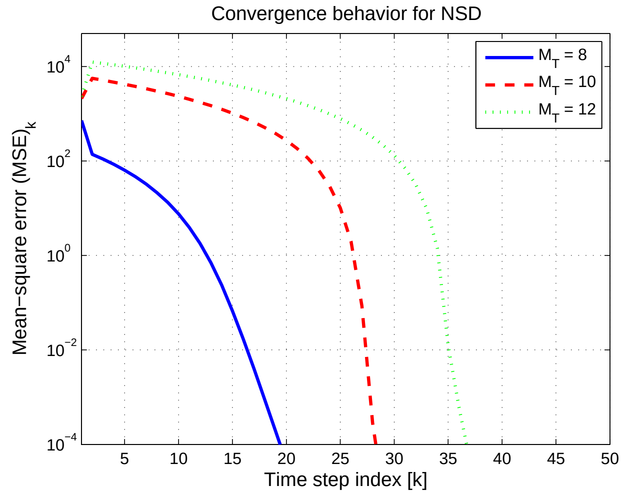


Fig. 6: Convergence behavior for NSD pre-coder with various number antennas

the interference received by a co-existent communication base-station is constrained up to an acceptable tolerance level. The radar's pre-coder is configured using a steepest-descent based approach. We have derived two solutions for the problem: the steepest descent (SD) and Normalized Steepest-Decent(NSD). Our result matches the expectations, NSD performs better in terms of its enhanced convergence rate when compared with its SD counterpart. Our future work will include an assessment on how radar performance will be affected.

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