Quality of Service Assurance-based Auction for Spectrum Sharing Systems

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Abstract: Spectrum sharing has emerged as a promising solution to address the radio frequency (RF) spectrum bottleneck. The FCC has recently proposed a spectrum sharing framework that introduces a Spectrum Access System (SAS) as the governing entity that manages the desired coexistence among the primary and secondary users. An important aspect of dynamic spectrum management is the pricing of spectrum from the perspective of both the Primary and secondary users. Existing auction-based spectrum sharing models do not take into consideration an important aspect of successful secondary user operation: the duration of the available spectrum opportunity. In this paper we propose an auction-based spectrum sharing framework that accounts for the quality of the available spectrum opportunity. The proposed auctioning process allows both the PU and the SU to iteratively adjust their evaluation about the available spectrum opportunities over time and to achieve a price combination that maximizes their objectives.

I. INTRODUCTION

Wireless communities throughout the world have recognized the shortage of spectrum for commercial broadband uses and acknowledged the urgent need for a global effort to make additional spectrum available for broadband data services. Spectrum sharing has captured the center stage as the solution to the issue of spectrum scarcity [1]. Based on the recommendations by the PCAST report [2] and the experience gained with spectrum sharing in the television white spaces (TVWS) [3], the Federal Communications Commission (FCC) has taken a series of steps [1, 4, 5] towards sharing Federal spectrum, especially Department of Defense spectrums, with commercial broadband applications. The FCC proposed a spectrum sharing framework and introduced Spectrum Access System (SAS) as the governing entity of the framework. The SAS manages the shared spectrum and arbitrates the issues associated with the coexistence of the primary and secondary users. The research efforts directed towards maturing this concept mainly focus on efficient detection [5-7] and secondary allocation of the spectrum opportunities [8-10].

The dynamic spectrum sharing approach proposed by the FCC is a challenging problem because of the requirements of peaceful coexistence between the primary and secondary users. In recent years, game theory has been increasingly used to model and understand the difficulties of dynamically managing limited resources in a competitive environment [11, 12]. An important aspect of dynamic spectrum sharing is the pricing of spectrum from the perspective of both the primary and the secondary users [13-15]. A number of auction-based spectrum sharing models have been used to provide a framework for spectrum pricing and resource allocation problems [16-18].

The research efforts in the context of spectrum sharing have overlooked an important aspect of successful secondary user (SU) operation in the shared band which is the duration of available spectrum opportunity. Following the research trend of spectrum sharing aspects, the application of game theory and auction theory has been limited to power and spectrum allocation among the SUs and pricing structure for auction-based spectrum sharing. In order to achieve any desired level of QoS, the service providers need information about the QoS predictability of the channel. The QoS predictability of a shared spectrum is highly influenced by the duration of the available spectrum opportunity [19] and plays an important role in reducing interference to the PU. In its current form, the SU is responsible for predicting the achievable QoS in the shared band. The initiatives for QoS prediction are mainly focused on primary usage modeling. In a previous work [20], we proposed the Quality of Service (QoS) Assurance (QoSA) approach with minimum primary user (PU) involvement to estimate the achievable QoS for a spectrum opportunity.

In this paper, we propose an auction-based spectrum sharing framework that accounts for the quality of the available spectrum opportunity. The framework allows both the PU and SU to evaluate the spectrum opportunities over time and achieve a price combination that maximizes both parties' objectives. The rest of the paper if organized as follows. In section II, we briefly discuss the concept of the QoSA approach and present an expression for normalized effective spectrum duration. Section III presents the system model for the proposed auction-based spectrum sharing framework. In section IV, we discuss the problem formulation including utility, revenue, and payoff function of the SU and the objective function for the PU. We also propose an iterative algorithm which allows the PU and SU to adjust their evaluation of the spectrum opportunities to

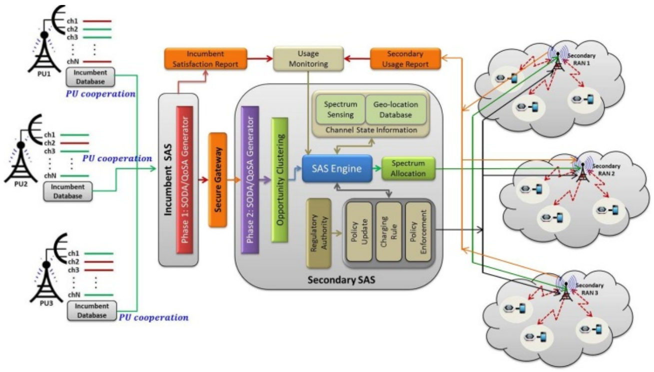

Figure 1: QoS assurance for spectrum sharing systems

achieve desired objective maximization. Section V discusses the simulation results and section VI derives the conclusions.

II. Background: Quality of Service Assurance

Our work in [20] proposed a QoS Assurance (QoSA) approach. The QoSA approach, as opposed to the QoS prediction approach, estimates the spectrum opportunity duration for operations with non-deterministic or unknown usage reservations or patterns. The SAS arbitrates among the PU and SU operations and bridges the interaction between two sides of the spectrum sharing system [21]. QoSA is a metric that measures the probabilistic assurance about idle PU channels stating that the channel under consideration will be available for secondary use for certain duration of time with probability QoSA. The QoSA approach requires minimal involvement from the PU and reduces the likelihood of exposing sensitive PU information using an "*opportunity clustering"* mechanism.

The SAS framework proposed to incorporate the QoSA approach has two major blocks: the incumbent SAS (I-SAS) and the secondary SAS (S-SAS) (Figure 1). I-SAS gathers the usage statistic and the tentative future usage plan from the incumbent database. The I-SAS and S-SAS perform two phases of the probabilistic assurance calculation for each of the idle channels. The S-SAS then generates clusters of similar quality opportunities based on the associated QoSA. Based on QoSA, the S-SAS helps the SU resource manager (SU-RM), serving multiple SU with different QoS demands, to perform an assignment problem that maximizes the overall spectrum efficiency for the shared bands.

In this paper we focus more on the effective duration of the spectrum opportunity based on the assurance from the PUs. The QoSA approach does not make any assumption on the PU traffic. We only assume the trust on the probabilistic assurance about the time of PU return within the assured opportunity is exponentially distributed. This assumption is justified as follows: The PU provides assurance for only those channels that are idle at the moment of reporting. Since this assurance is coming from the source, it is most likely that the assurance will be more reliable in near future and the reliability will decrease as time goes by. Based on the QoSA associated with the channels, the SAS generates cluster of similar quality channels (Figure 2).

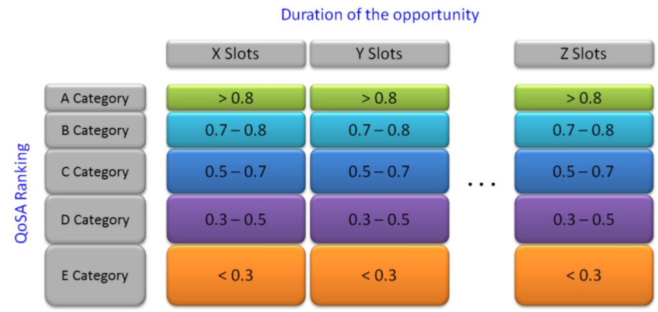

Figure 2: Opportunity clustering by the SAS

The normalized effective duration in terms of QoSA can be expressed as [20]

$$\alpha = \frac{QoSA-1}{\ln(QoSA)} \tag{1}$$

We will use this normalized effective duration of the spectrum opportunities as a measure of the quality of the channel. The players of the auction-based spectrum sharing framework will consider this measure of the channel quality to adjust the valuation of the resource.

III. System Model

This paper considers a simple spectrum sharing system that is governed by the SAS (Figure 3). There are a set of M PUs and N SU service providers (SU-AP). The PUs, with the help of I-SAS, determine the available spectrum opportunities, related QoSA values, and initial price of the opportunities and forward this information to the S-SAS. The S-SAS generates cluster of similar quality opportunities and determines the effective duration metric for each of these clusters. The S-SAS also receives requests for spectrum opportunities from the SU-APs along with their individual offered prices and makes a decision about the allocation of spectrum for secondary usage. The SU-AP, on its end, performs an estimation of the aggregate QoS demand of all of its end users. Based on the projected revenue earned from these users, the SU-AP determines the price of the bid for the current stage.

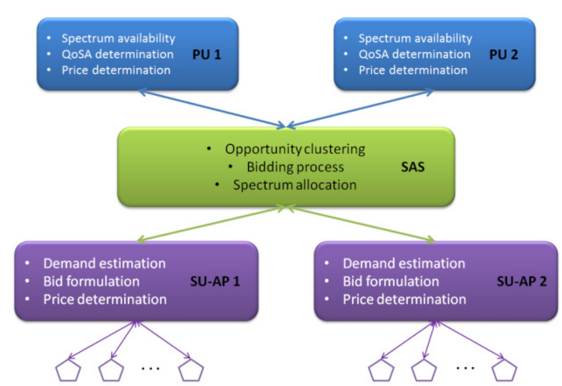

Figure 3: System model for the auction based spectrum sharing framework

We follow the system parameters proposed by the FCC for the Citizen Broadband Radio Service (CBRS) [21]. Each of the PU channels has a bandwidth of 10 MHz and the transmission power of each of the SUs is 24 dBm. Each PU determines the number of available 10 MHz spectrum opportunities and associated QoSA value. The SAS generates L clusters of opportunities and corresponding α_l values, where $l = 1, 2, \dots, L$. The PUs also inform the SAS about the asking price for its opportunities. For PU_i, the price of one 10 MHz channel is given by $x_i\theta_i$ where x_i is the initial reserve price and θ_i is the PU price adjustment factor. Each PU will adjust the price of its spectrum opportunities as a function of the demand on its channels in the previous auctioning stage by adjusting θ_i. This way the PU can make its channels more attractive for the SUs based on the SU demands in the previous stages.

For the purpose of this analysis, the SUs measure QoS in terms of bit error rate (BER). For the uncoded quadrature amplitude modulation (QAM) with square constellation (e.g., 4-QAM, 16-QAM) the BER is approximated as follows [22]:

$$BER = 0.2 \exp\left(\frac{-1.5\gamma}{2^k - 1}\right) \tag{2}$$

where γ is the Signal to Noise Ratio (SNR) at the receiver and k is the spectral efficiency of the modulation scheme used. We maintain BER at a target level $\left(BER_{Target}\right)$ so that the spectral efficiency of the transmission for SU-AP j (SU_j) can be described as:

$$k_j = log_2\left(1 + \frac{1.5\gamma}{log_e\frac{0.2}{BER_{Target}}}\right) \tag{3}$$

The SUs determine the bidding price depending on the price $(x_i\theta_i)$ and the channel quality factor (α_l) provided by the PU. In determining the bidding price, the SU also considers the revenue (r_j) earned from the end users. For any spectrum opportunity the base price $(x_i\theta_i)$ is given by the SAS. The SU may decide to bid at a higher price to improve its chance to win. The amount of increase in the bidding price depends on the channel quality factor (α_l). The SU decides on a price y_j for a channel and adjust the bidding price according to α_l. Thus, SU_j determines the price of each of its desired 10 MHz channels of PU_i that belong to the cluster l as follows:

$$\begin{aligned} &c_{i,j,l} = x_i\theta_i + y_j\alpha_l \\ &s.t. \quad c_{i,j,l} < r_j \\ &i = 1,2,\dots,M \\ &j = 1,2,\dots,N \\ &l = 1,2,\dots,L \end{aligned} \tag{4}$$

IV. Auction-based Spectrum Sharing: Problem Formulation

A. Utility Function of the Secondary Users

Each SU-AP determines the aggregate QoS demand, $\left(QoS_{agg_j}\right)$, of its end users. It also calculates the spectral efficiency $\left(k_{ij}\right)$ based on its operational parameters and determines the quality of each channel (l) using α_l and k_{ij}. First the SU-AP tries to determine its desired channels that are needed to satisfy the aggregate QoS demand.

$$\sum_{l=1}^{L}\sum_{i=1}^{M} k_{i,j}\alpha_l B\tau I_{j,desired_{l,i}} \geq Q_{agg_j} \tag{5}$$

Here B is the bandwidth of each channel (10 MHz), τ the duration for which the channel will be allocated, and $I_{j,desired_{l,i}}$ the channel indicator information about the desired channels. The SU-AP also determines the desired utility that it may achieve if its desired channels are allocated by the SAS. The utility function for SU-AP j can be expressed as

$$U_{j_{desired}} = \sum_{l=1}^{L}\sum_{i=1}^{M} k_{i,j}\alpha_l I_{j,desired_{l,i}} \tag{6}$$

B. Revenue and Payoff function of the Secondary Users

The SU-APs knows the revenue per unit $\left(r_j\right)$ that it earns from its end users. The revenue function of the SU_j can be expressed as

$$R_{j_{desired}} = \sum_{l=1}^{L}\sum_{i=1}^{M} r_j k_{i,j}\alpha_l B\tau I_{j,desired_{l,i}} \tag{7}$$

The SU-AP determines the channel indicator information $I_{j,desired_{l,i}}$ according to Eq. (5) and a bid price y_j to calculate the price $\left(c_{i,j,l}\right)$ it bids for each of its desired channels. The cost function of the SU_j can then be expressed as:

$$C_{j_{desired}} = \sum_{l=1}^{L}\sum_{i=1}^{M} c_{i,j,l} k_{i,j}\alpha_l B\tau I_{j,desired_{l,i}} \tag{8}$$

Using Eq. (7) and Eq. (8), the payoff function of the SU_j can be expressed as:

$$\begin{aligned} P_{j_{desired}} &= R_{j_{desired}} - C_{j_{desired}} \\ &= \sum_{l=1}^{L}\sum_{i=1}^{M} (r_j - c_{i,j,l}) k_{i,j}\alpha_l B\tau I_{j,desired_{l,i}} \end{aligned} \tag{9}$$

C. Problem Formulation and Solution Approach

The objective of each SU_j is to find the optimal bidding price per unit $\left(y_j\right)$ such that its payoff function $\left(P_{j_{desired}}\right)$ is maximized and the aggregated QoS demand is satisfied. So for an initial price of y_j the SU-AP tries to solve the following problem,

$$\max_{I} \quad P_j = \sum_{l=1}^{L}\sum_{i=1}^{M}(r_j - c_{i,j,l})k_{i,j}\alpha_l B\tau I_{j,desired_{l,i}}$$
$$s.t. \quad \sum_{l=1}^{L}\sum_{i=1}^{M} k_{i,j}\alpha_l B\tau I_{j,desired_{l,i}} \geq Q_{agg_j} \tag{10}$$

This Problem can be easily modified so that it can be treated as a special binary integer programming problem known as the Knapsack problem [23]. The SU-AP solves problem (10) to find the $I_{j,desired_{l,i}}$ and forwards this information along with the associated cost $(c_{i,j,l})$ as the bidding information to the SAS. Upon receiving the bidding information from all the SU-APs, the SAS performs the auction algorithm (Algorithm 1.1. and 1.2.). This completes one bidding cycle and the SU-APs enjoy the allocated channels for the duration (τ) for which the channels have been allocated. Then the same bidding process is repeated with updated channel prices from both the PU and SU sides.

Algorithm 1.1: Auction Algorithm

1. Spectrum opportunity clusters: $\boldsymbol{SO}$: $\{(x_{il}\theta_{il},\ \alpha_l),\ \forall i,l\}$; SU demand for each cluster: $\boldsymbol{SU_D}$: $\{I_{jl} \in (1,0), \forall j,l\}$; SU bidding price for each cluster: $\boldsymbol{SU_P}$: $\{y_{jl},\ \forall j,l\}$
2. Auction decision set: $\boldsymbol{I_{allocated}}$: $\{I_{ijl} \in (1,0), \forall i,j,l\}$
3. If any channel has a single claim: $\boldsymbol{SO}$: $\{(x_{il}\theta_{il},\ \alpha_l)\} \rightarrow \boldsymbol{I_{allocated}}$: $\{I_{ijl} \in (1,0)\}$
4. If any channel has multiple claims:
   $\boldsymbol{for}\ each\ l\ and\ \forall j, \qquad if\left(sum\left((\boldsymbol{SU_D}:\{I_{jl}\} == 1)\right) > 1\right)$
   $\boldsymbol{find}\ j^* = \arg\max_{\forall j \in \{I_{jl}=1\}} \boldsymbol{SU_P}:\{y_{jl}, \qquad \forall i,l\}$
   $\boldsymbol{SO}:\{(x_{il}\theta_{il},\alpha_l)\} \rightarrow \boldsymbol{I_{allocated}}:\{I_{ij^*l}\}$
   $\boldsymbol{0} \rightarrow \boldsymbol{I_{allocated}}:\{I_{ijl}\}, \forall j \in \{1,2,\ldots,L; j \neq j^*\}$
5. S-SAS forwards the allocation indicator information, $\boldsymbol{I_{j,allocated}}$: $\{I_{ijl} \in (1,0)\}$, to each of the participating SU-APs
6. Perform the allocation procedure for all the available spectrum opportunities.

Algorithm 1.2: Price Adjustment by PU and SU Operations

1. S-SAS generate list of unassigned channel and forwards $\boldsymbol{I_{unassigned}}$ to I-SAS:
   $\boldsymbol{for}\ \forall l,i, \qquad if(SO_{il} \notin \boldsymbol{I_{allocated}}): SO_{il} \rightarrow \boldsymbol{I_{unassigned}}$
2. I-SAS in consultation with the corresponding PU adjusts the discount factor θ_{il}
   $\boldsymbol{for}\ \forall l,i \in \boldsymbol{I_{unassigned}}, \qquad \theta_{il}^{new} = \theta_{il} - \Delta_{PU_{il}}:\ \theta_{il}^{new} \rightarrow \boldsymbol{SO}(x_{il}\theta_{il}^{new},\alpha_l)$
3. The SU-AP calculates the loss in revenue due to the current bidding price
   $$\boldsymbol{for}\ \forall l,i \in \boldsymbol{I_{j,desired}}, \quad P_{j_{achievable}} = \sum_{l=1}^{L}\sum_{i=1}^{M}(r_j - c_{i,j,l})k_{i,j}\alpha_l B\tau \boldsymbol{I_{j,desired}}$$
   $$\boldsymbol{for}\ \forall l,i \in \boldsymbol{I_{j,allocated}}, \quad P_{j_{achieved}} = \sum_{l=1}^{L}\sum_{i=1}^{M}(r_j - c_{i,j,l})k_{i,j}\alpha_l B\tau \boldsymbol{I_{j,allocated}}$$
   $(P_{j_{missed}} = P_{j_{achievable}} - P_{j_{achieved}})$: $\Delta_{SU_{jl}} = \theta_{adjust}P_{j_{missed}}$
   $y_{jl}^{new} = y_{jl} + \Delta_{SU_{jl}}$
4. The SU-AP that gets allocated its desired channels does not perform the price adjustment mentioned in Step2
   $\boldsymbol{for}\ \forall l,i, \qquad if(\boldsymbol{I_{desired}} = \boldsymbol{I_{allocated}})$: $\Delta_{SU_{jl}} = 0;\ y_{jl}^{new} = y_{jl}$
5. The asking and bidding prices for the spectrum opportunities are updated.

V. Performance Evaluation

This section presents the simulation results to evaluate the performance of the proposed auction-based spectrum sharing system. We are mainly interested in observing how the bidding prices, SU payoff, spectrum usage, and PU revenue evolve over consecutive bidding stages.

A. Parameter Setting

For the purpose of simulation, we assume a spectrum sharing scenario with two PUs $(M = 2)$ and two SUs $(N = 2)$. Each PU offers three channels, each of 10 MHz bandwidth and time duration of 1 unit $(\tau = 1)$, for sharing with the SUs. The SAS calculates the associated QoSA values for each of these spectrum opportunities. The SAS also employs the opportunity clustering mechanism and generate four categories $(L = 4)$ of spectrum opportunities as shown in Table 1.

Table 1: Opportunity Clusters with associated QoSA values

Category	QoSA Range	Cluster QoSA	α *value*
A	$QoSA > 0.8$	0.9	0.9491
B	$0.6 > QoSA \geq 0.8$	0.7	0.8411
C	$0.4 > QoSA \geq 0.6$	0.5	0.7213
D	$0.4 \geq QoSA$	0.3	0.5814

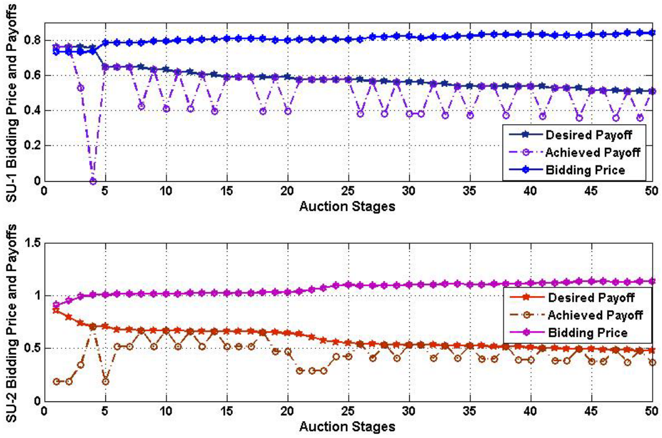

Figure 4: SU-AP bidding prices and corresponding payoffs: for SU-AP1 (upper) and SU-AP2 (lower)

Each of the SU-APs also calculates the spectral efficiency of each of the spectrum opportunities for a target BER of 10% using Equation (1). The asking price for each of channels at the beginning of the auction is 1.0 unit for PU-1 and 0.8 units for PU-2. The initial value for the PU price adjustment factor is 1. The SU-APs earn 2.0 units of revenue from each of the spectrum opportunities and starts the bidding with a price of 0.8 units for SU-AP1 and 0.6 units for SU-AP2. Using the above pricing and channel quality information, the SU-APs calculate the bidding price for the 1st auction stage according to Equation (4). After each iteration of the auction process the PUs re-evaluates their asking price based on the number of unassigned spectrum opportunities and missed revenue. The SU-APs also re-evaluate their bidding prices considering the difference in the desired and achieved payoffs as a result of the current bidding prices. In the following we present the simulation results and analyze the impact of the proposed iterative auction procedure on the bidding price and payoffs of the SU-APs, the revenue and asking price of the PUs, and the fraction of the available spectrum opportunity used by the spectrum sharing system.

B. SU Bidding Prices and Payoffs

The SU-APs use the α values associated with each of the spectrum opportunities to calculate the desired payoff. This helps the SU-APs to achieve payoffs close to the desired ones as a result of the auction results of each stage. Without the QoSA values, the SU-APs would have assumed that they would be able to access the spectrum opportunity for the whole duration of τ units and accordingly solve the optimization problem presented in Equation (10). But the uncertainty associated with the PU's return would affect the achievable payoffs out of the allocated opportunities. As can be seen from Figure 4, the iterative adjustments in the bidding prices (SU-AP1 in stage 4 and SU-AP2 in stage 21) also help the SU-APs to achieve payoffs close to the desired ones.

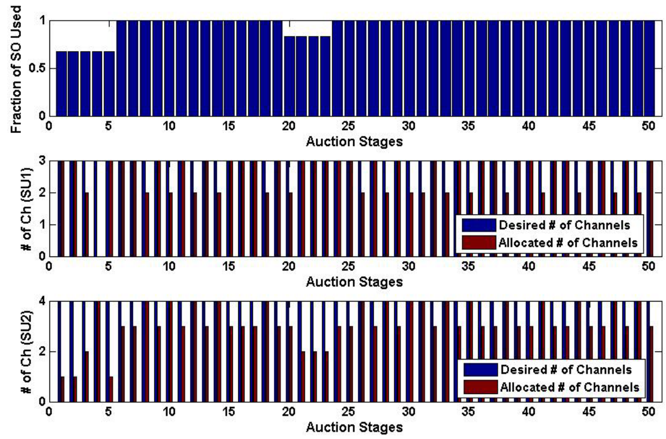

Figure 5: SU-AP channel allocation and overall Spectrum usage

C. Spectrum Usage

Figure 5 shows the desired and allocated channels for SU-AP1 and SU-AP2. The SU-APs determine the number of channels needed based on the aggregate QoS required by the end users served by the SU-APs. If the QoSA values of the spectrum opportunities are not known to the SU-APs, they will over-estimate the achievable QoS from the opportunities. This will result in degraded user experience for the SU-APs. Knowing the realistically achievable QoS considering the uncertainty due to PU return to the opportunity allows the SU-APs to more effectively determine the number of spectrum opportunities for which it will participate in the auction process. Figure 5 also presents the fraction of available spectrum opportunities used for sharing. The iterative price adjustment from both the PU and SU-APs helps the auction to be fair in terms of channel allocated to each of the SUs. This in turns increases the percentile use of the available spectrum opportunities. At stage 1 of the auction, the approximately 60% of the spectrum opportunities are allocated to the SU-APs. As a result of the iterative price adjustment at the later stages of the auction all the available spectrum opportunities are allocated to the SU-APs.

D. PU Asking Price and Revenue

The price adjustment done by the PUs help them to increase the overall revenue generated from the auction procedure. As can be seen from Figure 6, the revenue generated by PU1 is much lower compared to that of PU2 at the beginning of the auction procedure. PU1 identifies the problem and adjusts its asking price to make the spectrum opportunities more attractive for the bidding SUs. As a result of lower asking price, the revenue earned by PU1 improves significantly by the auction stage 6. The revenue generated by PU2 takes a hit at the auction stage 20 and that prompts PU2 to readjust its asking price. This readjustment based on the previous auction results and the α values of the opportunities improve the PU2 revenue and the auction procedure reaches an equilibrium state in terms of the revenue generated by both the PU operations.

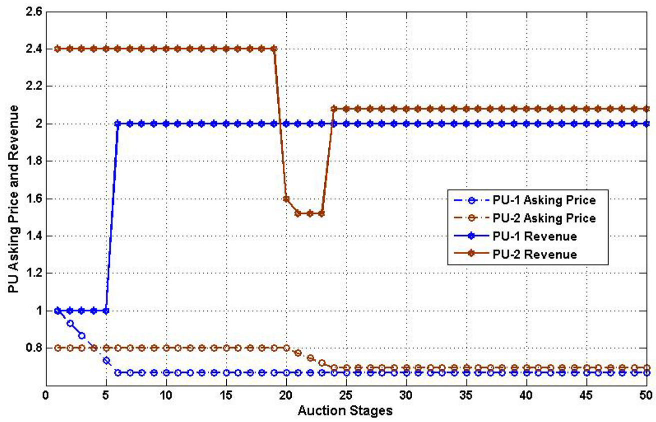

Figure 6: PU asking price and revenues

VI. Conclusion

Duration of spectrum opportunity is an important but often overlooked aspect of successful SU operation in a spectrum sharing system. In this paper we propose an auction-based spectrum sharing framework that includes the spectrum opportunity duration information in determining the price of the opportunities at each stage of the iterative auction procedure. In this multi-stage bidding process, both the PUs and SUs adjust their evaluation of spectrum opportunities based on the normalized effective duration of the spectrum opportunities and the auction result information from the previous stage of bidding.

In this paper we proposed an iterative auction algorithm where the PU and SU operations decide on the quality of the spectrum opportunities according to the QoSA values associated with each of the opportunities. The auction algorithm also allows flexibility in terms of adjusting the asking and bidding price of the opportunities based on the QoSA values and the auction results of each stage of the bidding. The simulation results presented in the paper show that the percentile of the available spectrum opportunities used by the SUs improves with the proposed algorithm. Simulation results also show that the achievable SU payoffs and PU revenues improve as the SUs and PUs adjust their evaluation about the spectrum opportunities after each stage in the auction procedure.

The work in this paper assumed that all the PUs and SUs use the QoSA information without any modification. In [20], we proposed interference avoidance algorithms that uses both the QoSA information and channel monitoring to access and determine the achievable QoS out of the spectrum opportunity. Considering this for auction decisions will allow the SUs to more effectively evaluate the bidding price of the opportunities. We also used a single price for all the channels, although the final price may be different for each of the channels. In future work, we plan to treat pricing of each channel individually, as a function of associated QoSA values, and analyze the impact on the performance metrics.

One important aspect of spectrum sharing auctioning is to meet the desired objectives of both the PU and SU applications. The auctioning mechanism presented in this paper is an initial analysis of a suggested approach. A coordinated effort from the government, industry, and academia to expedite the development of the dynamic SAS and auctioning procedure is imperative for successful SU operation in a spectrum sharing system. The successful implementation of the dynamic spectrum management approach will significantly improve the spectrum usage and influence the management approaches of other spectrum bands.

Acknowledgement

This work is supported by the National Science Foundation (NSF) under grant number 1343222.

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