New Directions in Channelized Receivers and Transmitters

fred harris

2-December 2011
Motivation For Using Multirate Filters
Processing Task: Obtain Digital Samples of Complex Envelope Residing at Frequency $f_C$

Multi-Channel FDM Input Signal

Receiver

Analog

Input Spectrum

Digital

Single-Channel Base banded Output Signal

Selected Narrow Band Signal

$\text{Input Spectrum}$

$\text{Receiver}$

$\text{Analog}$

$\text{Digital}$

$\text{Multi-Channel FDM Input Signal}$

$\text{Selected Narrow Band Signal}$

$\text{Input Spectrum}$

$f$

$f_C$
See!
First Generation DSP Receiver

Analog Signal Processing
Signal Conditioning for DSP Receiver

1. Input signal
2. Filtering
3. Process

Diagram showing the signal conditioning process with low pass filters and DSP processing.

- Low Pass Filter
- DSP Process

fs/2 and -fs/2 frequencies are indicated.
Replicate Analog Processing in DSP

Ignoring Good Advice!
• Fundamental Operations
  • Select Frequency
  • Limit Bandwidth
  • Select Sample Rate

Digital Down Converter (DDC)
Spectral Description of Fundamental Operations

- **Input Analog Filter Response**
- **Channel of Interest**
- **Translated Spectrum**
- **Output Digital Filter Response**
- **Filtered Spectrum**
- **Spectral Replicates at Down-Sampled Rate**
A Shell Game: Rearrange the Players! Keep Your Eye on the Pea!
Signal and Filter are at Different Frequencies
Which One to Move?

- **First Option**
- **Second Option**
Down Sample Complex Digital IF

1. Low Pass Filter
2. $f_s$
3. $M$
4. $-M$
5. $-M$
6. $+M$

No Spectral Image

- $f_s/2$
- $f_s/2$
- $f_s/2$
- $f_s/2$

1. $e^{j\theta_n}$
2. $e^{-j\theta_n}$
Fundamental Operation with Rearrange Operators

Up-Convert Filter, Filter Signal at IF, Down Convert Output of Filter
Equivalency Theorem

Down-Convert Signal at Input to Low-Pass Filter

\[ r(n) = s(n)e^{-j\theta_0 n} * h(k) \]

\[ = \sum_{k} s(n - k)e^{-j\theta_0 (n-k)} h(k) \]

\[ = e^{-j\theta_0 n} \sum_{k} s(n - k)h(k)e^{j\theta_0 k} \]

\[ = e^{-j\theta_0 n} \{ s(n) * h(n)e^{j\theta_0 n} \} \]

Down-Convert Signal at Output of Band-Pass Filter

Up-Convert Low Pass Filter To Become Complex Band-Pass Filter
Signal Flow Description of Equivalency Theorem

Not Finished: Moving Down Converter from Input to Output
Replaces 2-Multipliers (Complex Scalar) with 4-Multipliers (Complex Product)
Interchange Down Converter and Resampler

Only Down Convert the Samples we Intend to Keep!
Let the Resampler Alias the Center Frequency to Baseband
SPECTRAL DESCRIPTION

REORDERED FUNDAMENTAL OPERATION

INPUT ANALOG FILTER RESPONSE

CHANNEL OF INTEREST

FILTERED SPECTRUM

TRANSLATED FILTER

ALIASED REPLICATES AT DOWN-SAMPLED RATE

TRANSLATED REPLICATES AT DOWN-SAMPLED RATE
Successive Transformations Turn Sampled Data Version of Edwin Armstrong’s Heterodyne Receiver to Tuned Radio Frequency (TRF) Receiver and then to Aliased TRF Receiver.

\[
e^{-j\theta_k n}
\]

**Digital Band-Pass**

**Armstrong**

\[
H(Z)
\]

**Nyquist**

\[
H(Z e^{-j\theta_k})
\]

\[
M \cdot \theta_k = k \cdot 2\pi
\]

or

\[
\theta_k = k \cdot \frac{2\pi}{M}
\]

Any Multiple of Output Sample Rate Aliases to Baseband
Let’s Keep Rearranging the Players!
Linear Systems
Commute and are Associative

\[ R(f) = H(f)G(f) \]

\[ R(f) = G(f)H(f) \]
Linear systems Are Associative

\[ L_1 \rightarrow L_2 \rightarrow L_3 = L_1 L_2 \]

\[ X \rightarrow Y \rightarrow W \]

Filter \hspace{2cm} Resample

\[ X \rightarrow Y \rightarrow W \]

Resampled Filter
In Case you Couldn’t Wait to See the Proof

\[ y(n) = \sum_{k_1} x(n - k_1) h(k_1) \]

\[ dy(n) = \sum_{k_2} y(n - k_2) g(k_2) \]

\[ = \sum_{k_2} \sum_{k_1} x(n - k_1 - k_2) h(k_1) g(k_2) \]

\[ = \sum_{k_2} \sum_{k_3} x(n - k_3) h(k_3 - k_2) g(k_2) \]

\[ = \sum_{k_3} x(n - k_3) \sum_{k_2} h(k_3 - k_2) g(k_2) \]

\[ = \sum_{k_3} x(n - k_3) f(k_3) \]

where \( f(n) = \sum_{k_2} h(n - k_2) g(k_2) \)
Filter and Output Resampler can Commute to Input Resampler and Resampled Filter

\[ X(Z) \rightarrow H(Z) \rightarrow Y(Z) \rightarrow y(nM) \]

\[ y(n) = \sum_0^{M-1} y(nM + r) \]

\[ X(Z) \rightarrow \sum_0^{M-1} Z^r H_r(z^M) \rightarrow Y(Z^M) \rightarrow y(nM) \]

\[ y(nM) = \sum_0^{M-1} y(nM + r) \]

\[ Z^r X(Z^M) \rightarrow \sum_0^{M-1} Z^r H_r(z^M) \rightarrow Y(Z^M) \]
Coefficient Assignment of Low-Pass Polyphase Partition

For M-to-1 resample start at Index r and Increment by M
For 3-to-1 resample start at index r and increment by 3

This mapping from 1-D to 2-D is used by Cooley-Tukey FFT. Polyphase Filters and CT-FFT are kissing cousins!
Polyphase Partition of Low Pass Filter

1-Path to M-Path Transformation

\[ H(Z) = \sum_{n=0}^{N-1} h(n)Z^{-n} \]

\[ H(Z) = \sum_{r=0}^{M-1} \sum_{n=0}^{N-1} h(r + nM)Z^{-(r+nM)} \]

\[ H(Z) = \sum_{r=0}^{M-1} Z^{-r} \sum_{n=0}^{N-1} h(r + nM)Z^{-nM} \]

M-Path Partition Supports M-to-1 Down Sample
Also Supports Rational Ratio M-to-Q and M-to-Q/P Down Sample!
**Noble Identity**: Commute M-units of Delay followed by M-to-1 Down Sample

M-Units of Delay at Input Rate Same as 1-Unit of Delay at Output Rate
Interchange Filters and Resampler: Place Resampler at Input Rather Than at Output of Filter
Replace Delays with Commutator
Perform Path Operations Sequentially

\[ x(n) \rightarrow H_0(Z) \rightarrow H_1(Z) \rightarrow H_2(Z) \rightarrow \ldots \rightarrow H_{M-2}(Z) \rightarrow H_{M-1}(Z) \rightarrow y(nM) \]

Select

Coefficient Bank

8-tap
Transmitter Process, Up-Sample and Up-Convert
Receiver Process, Down Convert and Down-Sample

Modulator Raises Sample Rate & Applies Heterodyne at High Output Sample Rate!

De-Modulator Applies Heterodynes at High Input Rate & then Reduces...
Conventional and Ubiquitous DDC

DDS

DIGITAL LOW-PASS

M:1

DIGITAL LOW-PASS

M:1

DDS

CIC Correction

CIC

M:1

Half Band

2:1

Half Band

2:1

Half Band

2:1

Integrators

M:1

Derivative Filters

z^{-1} z^{-1} z^{-1} z^{-1} z^{-1} z^{-1} z^{-1} z^{-1} z^{-1}
Convert Two Parallel Paths into $M$ Sequential Paths for each Path
Replace CIC with Cascade 2-to-1 Half Band FIR Filters

<table>
<thead>
<tr>
<th>Filter Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Taps</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>11</td>
<td>19</td>
<td>70</td>
</tr>
<tr>
<td>Operations Per Filter</td>
<td>2-A 2-Shifts</td>
<td>2-A 2-Shifts</td>
<td>2-A 2-Shifts</td>
<td>4-A 2-Mult</td>
<td>4-A 2-Mult</td>
<td>4-A 2-Mult</td>
<td>4-A 2-Mult</td>
<td>6-A 3-Mult</td>
<td>10-A 5-Mult</td>
<td>___</td>
<td></td>
</tr>
<tr>
<td>Adds Ref to Input</td>
<td>2</td>
<td>2/2</td>
<td>2/4</td>
<td>2/8</td>
<td>4/16</td>
<td>4/32</td>
<td>4/64</td>
<td>4/128</td>
<td>6/256</td>
<td>10/512</td>
<td>4.26</td>
</tr>
<tr>
<td>Mult Ref to Input</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2/16</td>
<td>2/32</td>
<td>2/64</td>
<td>2/128</td>
<td>3/256</td>
<td>5/512</td>
<td>0.27</td>
<td></td>
</tr>
</tbody>
</table>
Impulse and Frequency Response of Last Stage Referred to Earlier Stages
Replace CIC with Cascade 2-to-1 Half Band Linear Phase IIR Filters

<table>
<thead>
<tr>
<th>Filter Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Taps</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>23</td>
</tr>
</tbody>
</table>
Impulse and Frequency Response of Last Stage Referred to Earlier Stages
Impulse, Frequency, & Group Delay Response of 2-Path Linear Phase, Recursive Half-Band Filter

**Impulse Response**

**Frequency Response**

**Group Delay**
2-to-1 Resampling 2-Path Polyphase Filter and Digital Down-Converter

Half Band Filter: $h(n)$

$h(2n)$

$h(2n+1)$

2-Point DFT

$f$
Resampling 4-Path Down-Sample Polyphase Filter and 4-Point IFFT Extracts Signal Component From One-of-Four Selected Nyquist Zones

Half Band Filters Centered on Cardinal Directions Each Reduces BW 2-to-1 and Reduces Sample Rate 2-to-1
4-Path, 2-to-1 Down-Sample with 4 Possible Trivial Phase Shifters

2-to-1 Down-Sample

4-Path Polyphase Filter
Path-0 Not Used
2.5-Multiplies per Input

4-Phase Rotators
\( f_s \cdot k/4: \{ c_0 \ c_1 \ c_2 \ c_3 \} \)
\( f_s \cdot 0/4: \{ 1 \ 1 \ 1 \ 1 \} \)
\( f_s \cdot 1/4: \{ 1 \ j \ -1 \ -j \} \)
\( f_s \cdot 2/4: \{ 1 \ -1 \ 1 \ -1 \} \)
\( f_s \cdot 3/4: \{ 1 \ -i \ -1 \ i \} \)
Four Bands Centered on the Cardinal Directions

Bands Centered on 0° and 180 ° (DC and f_s/2) Alias To DC When Down-Sampled 2-to-1

Bands Centered on +90° and -90 ° (+f_s/4 and –f_s/4) Alias To fs/2 When Down-Sampled 2-to-1
Spectra: Four Half Band Filters on Unit Circle Showing Alias Free Pass, Transition, and Aliased Bands

Any Narrowband Signal Must Reside in One of the 4 Alias Free Band Intervals. The Alias Free Band Intervals Overlap!
Pole-Zero Diagrams of Four Nyquist Zone Filters
Frequency Responses of Four Nyquist Zone Filters
Spectra of Signal Aliased to Different Sampled Data Frequencies in Successive 2-to-1 Sample Rate Reductions.
Most Efficient Multistage Half-Band Digital Down-Converter

\[ N = 2.5 \cdot [1 + \frac{2}{2} + \frac{2}{4} + \frac{2}{8} + \cdots] \]

\[ = 2.5 + 2.5 \cdot [1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots] \]

\[ < 2.5 \cdot 3 = 7.5 \]
Spectrum of Input Signal and Zoom to Spectral Segment

Received Signal Spectrum

Frequency / MHz

Zoom-in
Spectra: Last Four Stages Processing Chain.
Dotted Line Indicates Center Frequency of Desired Spectral Component
Sampled Data Frequency Locations on Successive Aliases
Spectrum at Input and Output of Final Heterodyne and Filter Stage

Post Processing--Input Signal Spectrum (ch # 600)

Post Processing--Down Convert (ch # 600)

Post Processing--Filtering (ch # 600)
A 375-to-1 down-sample:
90 MHz to 240 kHz with a 30 kHz output BW 80 dB dynamic range.
Require 6 CIC stages. The gain of each stage is 375: Gain of 6 stages becomes \((375)^6\) or \(2.8 \cdot 10^{15}\) or 52 bits growth in the CIC integrators.
With 16-bit input data integrator bit width is 16+52 or 68. Six integrators in both I & Q paths would be circulating 816 bits per input sample which if converted to the 16-bit width required of the arithmetic in the half-band filters proves to be same number of bits to manipulate 48 arithmetic operations per input sample.

Number of operations for the I-Q half band filter chain is on the order of 8-multiply and 16 adds per input sample which represents a workload 1/6 of the CIC chain. The efficient cascade CIC filter chain can be replaced with an even more efficient cascade four-path half band filter chain.
Linear Phase IIR Filter

Group Delay: Two Path, 4-Coefficient, Linear Phase 2-Path Filter

Frequency response
Most Efficient Multistage Half-Band Digital Down-Converter

Impulse Response, Two-Path, 4-Coefficient, Linear Phase IIR

\[ N = 2.0 \cdot \left[ 1 + \frac{2}{2} + \frac{2}{4} + \frac{2}{8} + \cdots \right] \]

\[ = 2.0 + 2.0 \cdot \left[ 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \right] \]

\[ < 2.0 \cdot 3 = 6.0 \]
Polyphase Partition of Band Pass Filter
1-Path to M-Path Transformation

Modulation Theorem of Z-Transform

\[ G(Z) = \sum_{n=0}^{N-1} h(n) e^{j\theta_k n} Z^{-n} = \sum_{n=0}^{N-1} h(n) (e^{-j\theta_k} Z)^{-n} = H(e^{-j\theta_k} Z) \]

\[ G(Z) = \sum_{r=0}^{M-1} \sum_{n=0}^{N-1} h(r + nM) e^{j\theta_k (r+nM)} Z^{-(r+nM)} \]

\[ G(Z) = \sum_{r=0}^{M-1} e^{j\theta_k} Z^{-r} \sum_{n=0}^{N-1} h(r + nM) e^{j\theta_k nM} Z^{-nM} \]

\[ \theta_k = k \cdot 2\pi \]

\[ \theta_k = k \cdot \frac{2\pi}{M} \]

\[ G(Z) = \sum_{r=0}^{M-1} e^{j\frac{2\pi}{M} k} Z^{-r} \sum_{n=0}^{N-1} h(r + nM) Z^{-nM} \]
Polyphase Band Pass Filter and M-to-1 Resampler

\[ H(z^M) \]

\[ x(n) \rightarrow H_0(z^M) \rightarrow y(n) \]

\[ y(nM) \]

\[ Z^{-1} \]

\[ Z^{-2} \]

\[ Z^{-(M-2)} \]

\[ Z^{-(M-1)} \]

\[ e^{j\frac{2\pi k}{M}} \]

\[ e^{j\frac{2\pi k}{M}} \]

\[ e^{j\frac{2\pi k}{M}} \]

\[ e^{j\frac{2\pi k}{M}} \]
Apply Noble Identity to Polyphase Partition

We Reduce Sample Rate
M-to-1 Prior to Reducing Bandwidth
(Nyquist is Raising His Eyebrows!)

We Intentionally Alias the Spectrum.
(Were you Paying Attention
in school when they discussed the
importance of anti-aliasing filters?)

M-fold Aliasing!
M-Unknowns!
M-Paths supply M-Equations
We can the separate Aliases!
Move Phase Spinners to Output of Polyphase Filter Paths

\[ x(n) \rightarrow H_0(Z) \rightarrow H_1(Z) \rightarrow H_2(Z) \rightarrow \ldots \rightarrow H_{M-2}(Z) \rightarrow H_{M-1}(Z) \rightarrow y(nM,k) \]
Polyphase Partition with Commutator
Replacing the “r” Delays in the “r-th” Path

Note: We don’t assign Phase Spinners to Select Desired Center Frequency Till after Down Sampling And Path Processing
This Means that The Processing for every Channel is the same till the Phase Spinner

No longer LTI, Filter now has M-Different Impulse Responses! Now LTV or PTV Filter.
Rather than selecting center frequency at input and reduce sample rate at output, we reverse the order, reduce sample rate at input and select center frequency at output. We perform arithmetic operations at low output rate rather than at high input rate!
Down Sample 6-to-1
Polyphase Partition
1-D filter becomes
2-D M-Path Filter
Reorder Filter and Resample

... this is very stuff....

\[ s(t) \]

\[ s(n) \]

\[ r(nM) \]

\[ r(nM,k) \]

\[ h(0+ nM) \]

\[ h(1+ nM) \]

\[ h(r+ nM) \]

\[ h(M-1+ nM) \]

\[ c(0,k) \]

\[ c(1,k) \]

\[ c(r,k) \]

\[ c(M-1,k) \]

\[ r(nM,k) \]

\[ j^{\frac{2\pi}{M} r k} \]

LOWPASS FILTER
POLYPHASE PARTITION

PHASE ROTATORS
ALIASED HETERODYNE

BANDPASS FILTER
POLYPHASE PARTITION
Phase and Gain Response

(3-Versions of Filter)

Prototype Filter,

Polyphase Filter Prior to Resampling,

Polyphase Filter after Resampling
Impulse Response and Frequency Response of Prototype Low Pass FIR Filter
Impulse Response of 6-Path Polyphase Partition Prior to 6-to-1 Resampling
Frequency Response of 6-Path Polyphase Partition Prior to 6-to-1 Resampling
Phase Response of 6-Path Polyphase Partition Prior to 6-to-1 Resampling
Overlay Phase Response of 6-Path Polyphase Partition Prior to 6-to-1 Resampling
De-Trended Overlay Phase Response:
6-Path Partition Prior to 6-to-1 Resampling
3-D Paddle-Wheel Phase Profiles, 6-Path Partition Prior to 6-to-1 Resampling
Overlay 3-D Paddle-Wheel Phase Profiles, 6-Path Partition Prior to 6-to-1 Resampling
Overlay 3-D Paddle-Wheel Phase Profiles, Showing Phase Shifts in +1 Nyquist Zone
Overlay 3-D Paddle-Wheel Phase Profiles, Phase Shifted to Align Phases in +1 Nyquist Zone
Single Channel Armstrong and Multirate Aliased Polyphase Receiver
ide Input Down Conversion to Output of Filter Where it
anishes Due to Down Sampling. Rotators in Filter Factor
Out and are Applied to Path Outputs Rather than to
Coefficients.
Advantage: Real sequence is made complex at output of
Filter Rather than at Input to Filter
Bad Mismatch: Sample Rate Large Compared to Transition Bandwidth

Nyquist Rate for Filter is

200 kHz + 200 kHz = 400 kHz or $fs/50$
Polyphase Partition of Low-Pass Filter

- 400 Taps
- 8 Taps
- 400 kHz
- 20 kHz
- 20 MHz
- 50-to-1
- \( \phi_0 \)
- \( \phi_1 \)
- \( \phi_2 \)
- \( \phi_{48} \)
- \( \phi_{49} \)
Cascade Polyphase Filter
Down-Sampling and Up-Sampling
Efficient Polyphase Filter Implementation

20 MHz Input Sample Rate → 400 Tap FIR Filter → 20 MHz Output Sample Rate

20 MHz → 8-tap → 400 kHz → 8-tap → 20 MHz

Select Coefficient Bank → 8-tap → Select Coefficient Bank
Two Processing in Boxes:
How can you tell which is which from outside box?

(The Wet Finger Test)

20MHz
60-Ops/Input

20MHz
16-Ops/Input

400-Tap Lowpass Filter

8-Tap Filter
Coefficient Bank
State Machine
Select

20MHz
White Box

400 kHz

20MHz
White Box

8-Tap Filter
Coefficient Bank
Select

OUCH

AHH
Polyphase Partition of Low-Pass Filter
Polyphase Partition of Band Pass Filter

\[ H_0(Z) \quad H_1(Z) \quad H_2(Z) \quad \ldots \quad H_{M-1}(Z) \]

\[ x(n) \quad e^{j0\frac{2\pi}{M}k} \quad e^{j1\frac{2\pi}{M}k} \quad e^{j2\frac{2\pi}{M}k} \quad \ldots \quad e^{j(M-1)\frac{2\pi}{M}k} \]

\[ y(nM,k) \quad y(n,k) \]
Polyphase Partition of Two Band Pass Filters
Workload for Multiple M-Path Filters

- **1-Channel M-to-1 Down Sample**
  - 1-Filter and M Complex Phase Rotators

- **2-Channels M-to-1 Down Sample**
  - 1-Filter and 2M Complex Phase Rotators

- **K-Channels M-to-1 Down Sample**
  - 1-Filter and kM Complex Phase Rotators

- **M-channels M-to-1 Down Sample (use FFT)**
  - 1-Filter and \([\log_2(M)/2]\)M Complex Phase Rotators

If \(k > \log_2(M)/2\), build all channels and discard the channels you don’t need!

- \(M = 16\), \(\log_2(16)/2 = 2\): thus if you want 2 or more, build them all!
- \(M = 128\), \(\log_2(128)/2 = 3.5\): thus if you want 4 or more, build them all!
- \(M = 1024\), \(\log_2(1024)/2 = 5\): thus if you want 5 or more, build them all!
M-Channel Channelizer: Resampled M-Path Narrowband Filter Channels Alias to Baseband: Phase Aligned Sums Separate Aliases: Work Performed at Low Output Rate Rather Than at High Input Rate.

One Input Filter Services M-Output Channels

\[ h(n) = h(r + nM) \]
Dual Channel Armstrong and Multirate Aliased Polyphase Receiver

![Diagram of Dual Channel Armstrong and Multirate Aliased Polyphase Receiver](image-url)
Up-sampling by
Zero Packing and Filtering
Spectra Of Input, of Zero-Packed, and of Low-Pass Filtered Zero-Packed Signal
Spectra Of Input, of Zero-Packed, and of Band Pass Filtered Zero-Pack Signal
Polyphase Partition of Resampling Filter

\[ H(Z) = \sum_{n=0}^{N-1} h(n)Z^{-n} \]

1-to-5

\[ H(Z) = \sum_{r=0}^{M-1} \sum_{n=0}^{M-1} h(r + nM)Z^{-(r+nM)} \]
Factor Delays and Rearrange

\[ H(Z) = \sum_{r=0}^{M-1} Z^{-r} \sum_{n=0}^{N-1} h(r + nM)Z^{-nM} \]
Noble Identity:
Interchange M-Delays with M-to-1 Resample
Interchange Filter and Resampler

Replace Up Samplers, Delays, and Summer with M-Port Output Commutator
Low-Pass Replaced by Band-Pass

\[ G(Z) = \sum_{n=0}^{N-1} h(n) e^{\frac{2\pi}{M} kn} Z^{-n} \]

\[ G(Z) = \sum_{r=0}^{M-1} \sum_{n=0}^{N-1} h(r + nM) e^{\frac{2\pi}{M} (r + nM) k} Z^{-(r + nM)} \]

\[ G(Z) = \sum_{r=0}^{M-1} Z^{-r} e^{\frac{2\pi}{M} r k} \sum_{n=0}^{N-1} h(r + nM) e^{\frac{2\pi}{M} nM} Z^{-nM} \]

\[ G(Z) = \sum_{r=0}^{M-1} Z^{-r} e^{\frac{2\pi}{M} r k} \sum_{n=0}^{N-1} h(r + nM) Z^{-nM} \]

Spin The Delays,
Don’t Touch the M-Path Partitioned Weights
Low-Pass to Band-Pass
1-to-M Up-Sampling Filter
M-Path, M-Channel Channelizer: Spinners are in IFFT

\[ h_i(n) = h(r + nM) \]
M-Point IFFT Supplies Phase Spinners to Form Up Converters to all Multiples of Input Sample Rate

All Output Channels Centered on Multiples of Input Sample Rate

Example: Multiples of 6-MHz
Heterodyne Input Signal a Small Frequency Offset from DC: Channelizer Aliases DC to Channel Center and offset signal from DC is Offset from Channel Center
Two Filter Spectra
M-Channel Polyphase Channelizer:
M-path Filter and M-point FFT

\[ h(n) = h(r + nM) \]
Various Filter-Channelizer Configurations

- **Critically Sampled**
  - $f_s = f_C$

- **Nyquist Sampling**
  - $f_s = 2f_C$

- **RT Nyquist Filter**
  - $f_s = 2f_C$

- **QRT Nyquist Filter**
  - $f_s = 4f_C$

- **Nyquist Sampling**
  - $f_s = 4f_C$
Filter Sampled at Rate to Avoid Band Edge Aliasing
Prototype Low-Pass Filter for 120 Channel Channelizer

Impulse Response
Prototype Filter for 120-Path Channelizer

Time Index

Frequency Response
M-Path Polyphase Filter and M/2-to-1 Down Sampling
Use Noble Identity to Pull M/2-to-1 Resampler Through Path Filter

Path Filters: Polynomials in $Z^M$
Converted to Polynomials in $Z^2$
Use Noble Identity to Pull $\frac{M}{2}$-to-1 Resampler Through Delays in Lower Half of Paths
Replace Delays and M/2-to-1 Resamplers with Dual Input M/2 Path Commutator
Fold Unit Delays in lower half Filter Paths Into Filter Polynomials in $\mathbb{Z}^2$
M-to-1 Down Sample Aliases Multiples of Output Sample Rate to DC
M/2-to-1 Down Sample Aliases Odd Multiples of Output Sample Rate to Half sample Rate
Circular Buffer Between Polyphase Filter and IFFT Aligns Shifting Input Origin with IFFT’s Origin
M/2-to-1 Analysis Channelizer
1-to-M/2 Synthesis Channelizer
Frequency Domain Filtering With Cascade M/2-to-1 Analysis and 1-to-M/2 Synthesis Channelizers
Impulse response and Frequency Response
25-Enabled Ports: 2.4 MHz Bandwidth
Impulse Response and Frequency Response
40-Enabled Ports: 3.9 MHz Bandwidth
Mixed, Arbitrary Bandwidth Channelizers
Mixed Bandwidth Signals presented to Channel Synthesizer

- **Signal 1**:
  - **Signal 1A**
  - **Signal 1B**
  - **Signal 1C**

- **Signal 2**:
  - **Signal 2A**
  - **Signal 2B**
  - **Signal 2C**

- **Signal 3**:
  - **Signal 3A**
  - **Signal 3B**

**Axes:**
- **Magnitude (dB)**
- **Frequency (MHz)**
Compose Broadband Signals Using Short Analysis Filters and Present Components to Synthesizer
20-MHz Input Signal Partitioned into Five 10-MHz Sub Channels: $f_S = 20 \text{ MHz}$
Multiple Partitioned Input Bands Presented to Synthesizer
Assembled Multiple BW Channels in Single Synthesis Channelizer

Up-Converted Signal Spectrum

Frequency (MHz)
Reassemble Decomposed Broadband Signals Using Short Synthesis Filters formed by Multiple Channel Analysis Channelizer
partitioned spectral Components from single Multi-Channel Analyzer
Reassembled Wide band Channels from Short Synthesis Channelizers
Signal Fidelity Preserved under Multiple Sub-Channel Disassembly and Reassembly
BLAM!

CAPTAIN MARVEL. I SALUTE YOU. HENCEFORTH IT SHALL BE YOUR SACRED DUTY TO DEFEND THE POOR AND HELPLESS, RIGHT WRONGS AND CRUSH EVIL EVERYWHERE.

YES, SIRE.

AS BILLY SPEAKS THE MAGIC WORD HE BECOMES CAPTAIN MARVEL!
Cascade M/2-to-1 Analysis and 1-to-M/2 Synthesis Channelizers
Frequency Domain Filtering and Spectral Shuffle
Here pointed hair boss. This report explains how a small frequency offset at the input sample rate is converted to the same frequency offset from the channel center frequency at the high output sample rate.
Suspicitions Confirmed!

BLAH, BLAH, BLAH...

I HAVE NO IDEA WHAT HE'S TALKING ABOUT.
Dilbert, is it true that DSP makes the world go around but multirate signal processing supplies the music for the ride?
Is There any Doubt???