An Analysis of Active Interference Cancellation for Wideband OFDM System from Multi-band OFDM System

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Abstract— **Recently, the shortage of assignable radio frequency spectrum became a serious issue because of the coexistence of many licensed wireless communications systems. Therefore, cognitive radio technology has gained much attention around the world, which may be aware of its environment and makes occupancy of radio spectrum more efficient. This paper examines numerical results of active interference cancellation (AIC) between wideband OFDM (WB-OFDM) system and multi-band OFDM (MB-OFDM). Since MB-OFDM systems share frequency spectrum with many WB-OFDM systems, and need to coexist with other ultra wideband (UWB) communications systems. The performance of avoidance the interference for WB-OFDM system in MB-OFDM system transmitter. Moreover, computer simulations have been performed to confirm these analytical results.**

I. Introduction

The usage of the radio spectrum and the regulation of radio emissions are coordinated by national regulatory bodies. As part of radio regulation, the radio spectrum is divided into frequency band, and licenses for the usage of frequency bands are provided to operators, typically for a long time such as one or two decades. With licensed frequency bands, operators have often the exclusive right to use the radio resources of the assigned bands for providing radio services. Depending on the type of radio service and on the efficiency of the radio systems, frequency bands may be used inefficiently. Therefore, many national regulatory and standards bodies such as the Federal Communications Commission (FCC) [1], IEEE 802.22 WG [2], and the Ministry of Internal Affairs and Communications in Japan have paid attention to the Dynamic Spectrum Access (DSA) technology. Using DSA technology, radio systems can dynamically use and release the radio spectrum wherever and whenever they are available. Moreover, DSA technology helps to minimize unused radio spectrum band [3]. This technology is also referred to as *cognitive radio technology*. Cognitive radio is defined as an intelligent wireless communication system, which may be aware of its environment and adapt to statistical variations in the input stimuli [4].

On the other hand, wireless communication systems such as Wireless Local Area Network (WLAN) and Bluetooth are becoming pervasive throughout the world. Especially, the main application of WLAN is wireless connection of PC's to a network but even includes such uses as wireless video transmission for indoor environment. Thus, WLAN has significantly impacted the world. Bluetooth is also a promising wireless interface solution in mobile ubiquitous environments and is expected to predominate among such applications soon. Meanwhile, some new technologies such as ultra wideband (UWB) radio systems have been proposed for short-range wireless applications [5]. They are expected to spread as a complement to developed technologies such as WLAN and Bluetooth or to be merged with such existing technologies.

UWB radio may inherently degrade the performance of the primary systems since the UWB radio band system overlaps that of primary systems such as Worldwide interoperability for Microwave Access (WiMAX), 4th generation mobile cellular systems (4G) and Field Pickup Unit (FPU). The technical conditions on the usage of UWB radio system were set up by the Ministry of Internal Affairs and Communications on March 2006, in Japan. In these conditions, it is essential for UWB radio to provide interference mitigation technique, *detect and avoid* (DAA) [6][7].

In the UWB environment, coexistence with heterogeneous wireless communications systems are enabled by using the concepts and techniques of the cognitive radio. Cognitive radio is a radio system that can sense the surrounding radio wave environment and use the radio resources efficiently by flexible reconfiguration of the system as a function of the changing environmen.

Although, UWB radio systems with DAA are allowed

TABLE I

MAJOR PARAMETERS OF MB-OFDM SYSTEM

Data sub-carrier	100
Pilot carrier	12
Guard carrier	10
Using sub-carrier	122
Sub-carrier interval	4.125 MHz (= 528MHz/128)
Effective symbol length	242.42 ns
Cyclic Prefix duration	60.61 ns
Guard interval duration	9.47 ns
Symbol interval	312.5 ns

to transmit with power level of -41.3 dB/MHz, those without DAA technique must limit their emission level at -70 dBm/MHz, which is lower than the noise level. Therefore, DAA is essential for UWB radio systems in order to allow them to transmit with the maximum allowed power level.

The question that may arise at this point is how to design cognitive radio systems such as UWB radio with DAA. Therefore, in this paper, this coexistence environment is analyzed and the design issue is discussed based on results. Moreover, we discuss the detection technique of primary system signals for UWB system with DAA and the effect of UWB system performance using DAA.

The rest of this paper is organized as follows. Section II presents the system models considered in this paper. The performance analysis of active interference cancellation technique for WB-OFDM system is detailed in Section III. Finally, conclusions are drawn in Section IV.

II. SYSTEM MODEL

A. Multiband OFDM System Model

The major parameters of MB-OFDM system are listed in Table I. MB-OFDM system proposed eight different high data rate wireless communication modes such as 53.3, 80, 110, 160, 200, 320, 400 Mega bit per second (Mbps)[8]-[11]. While these data rates, 53.3, 110, 200 Mbps are essential for MB-OFDM system. Moreover, MB-OFDM system uses OFDM system: transmission is using total number of 128 sub-carriers including the number of 122 multiplex modulated data. The coding method is forward error correction (EFC) and coding rate of 1/3, 11/32, 1/2, 5/8, 3/4. In addition, this MB-OFDM system also utilizes a time-frequency code (TFC) to interleave coded data over 3 frequency bands (called a band group). Four of such band groups are divided in 3 bands each and the fifth are 2 bands are defined.

TABLE II

MAJOR PARAMETERS OF THE WB-OFDM SYSTEMS

Bandwidth	101.5 MHz
Data rate objective	$>$ 100 Mbits/s
Center frequency	4635 MHz
Number of sub-carriers	768
Sub-Carrier spacing	131.8 kHz
OFDM symbol duration	7.5665 μs
Modulation	QPSK

B. WB-OFDM System Model

OFDM systems are considered amongst the most appropriate scheme for future high data rate communications systems due to their effective bandwidth utilization and the simplicity of the equalization strategies required to compensate the channel frequency selective fading. The OFDM technique has been adopted in several standards, e.g. digital audio broadcasting (DAB), digital video broadcasting (DVB), multimedia mobile access communications (MMAC), and WLAN. It has also been proposed for cable TV, broadband radio access networks, multi-user communications via satellite link, WiMAX, 4G and FPU.

In this paper, we focus on the coexistence environment of UWB radio systems and wideband systems based on OFDM such as 4G and WiMAX. The basic idea of OFDM is to divide the available spectrum into several sub-channels (sub-carriers). By making all sub-channels narrower than the coherence bandwidth of the radio channels, they experience almost flat fading, simplifying the equalization process (one tap equalizer). In order to obtain a high spectral efficiency, the sub-channels are overlapping, still keeping the orthogonality of their sub-carriers. This orthogonality is completely maintained, even when the signal experiences a time dispersive channel by introducing a cyclic prefix (the last part of the OFDM symbol is copied in front of the transmitted symbol). It introduces a signal-to-noise ratio (SNR) loss, the inter-symbol interference (ISI) and inter-carrier interference (ICI) are completely removed at the output of the channel [14].

The major parameters of primary systems are listed in Table II (c.f., [15]). In this paper, the bandwidth of the primary system such as 4G and WiMAX is about 100 MHz and 100 MHz bps data rate. The equivalent baseband model is used.

C. Interference System model

The effect of mutual interference between WB-OFDM system and MB-OFDM system is evaluated. Fig. 1 shows

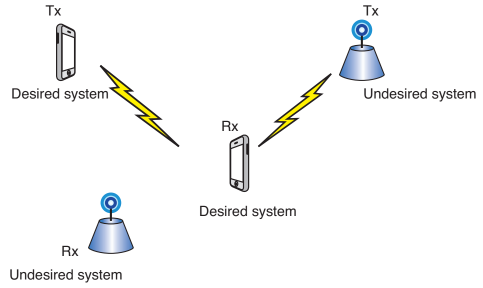

Fig. 1. The interference model which is considered in this paper

the interference model which is considered in this paper. Both MB-OFDM system and WB-OFDM system are operated as desired/undesired system,respectively.

The transmitted OFDM signals including interference are given by:

$$x(t)=\sum_v g(t-vT_s) \sum_{k=-K/2}^{K/2-1} X_{v,k} e^{j2\pi f_0 (t-vT_s)} e^{j2\pi f_i t} \quad (1)$$

where $X_{v,k}$ represents the v^{th} symbol and k^{th} sub-carrier transmitted signal, f_0 and f_i are sub-carrier spacing and center frequency, respectively. T_s is the symbol length of interference signals. $g(t)$ is described by:

$$g(t) = \begin{cases} 1 & (-T_g \le t \le T_e (= \frac{1}{f_0})) \\ 0 & (t < -T_g, t > T_e) \end{cases} \quad (2)$$

where T_g and T_e are guard interval and effective symbol length, respectively which satisfy $T_s = T_e + T_g$.

The received OFDM signals including interferencein the desired receiver are given as:

$$y(t) = \int_0^{\eta} h(\xi) x(t-\chi-\xi) d\xi \quad (3)$$

where $h(\xi)$ is the impulse response and $0 \le \xi \le \eta$. The χ represents the relative time difference between desired system and undesired system.

In the desired system case, for the u^{th} received signal, the FFT operates in terms of interval T. The l^{th} output is given as follows:

$$Y_{u,l}=\frac{1}{T} \int_{uT_d}^{uT_d+T} y(t) e^{-j\frac{2\pi l}{T}(t-uT_d)} e^{-j2\pi f_c t} dt$$

$$=\frac{1}{T} \int_{uT_d}^{uT_d+T} \int_0^{\eta} h(\xi) x(t-\chi-\xi) d\xi e^{-j\frac{2\pi l}{T}(t-uT_d)} e^{-j2\pi f_c t} dt$$

$$=\frac{1}{T} \int_{uT_d}^{uT_d+T} \int_0^{\eta} h(\xi) \sum_v g(t-\chi-\xi-vT_s) \sum_{k=K/2}^{K/2-1} X_{v,k}$$

$$e^{j2\pi f_0 (t-\chi-\xi-vT_s)} e^{j2\pi f_i (t-\chi-\xi)} d\xi e^{-j\frac{2\pi l}{T}} (t-uT_d) e^{-2\pi f_c t} dt$$

$$=\sum_v \sum_{k=-K/2}^{K/2-1} X_{v,k} e^{-j\frac{2\pi k}{T_e}(\chi+vT_s)} e^{-j2\pi f_i \chi} e^{j\frac{2\pi l}{T} uT_d}$$

$$\int_0^{\eta} h(\xi) e^{-j2\pi(\frac{k}{T_e}+f_i)\xi} \left\{ \frac{1}{T} \int_{uT_d}^{uT_d+T} g(t-\chi-\xi-vT_s) e^{j2\pi(\frac{k}{T_e}-\frac{l}{T}+f_i-f_c)t} dt \right\} d\xi$$

$$=\sum_v \sum_{k=-K/2}^{K/2-1} X_{v,k} e^{-j\frac{2\pi k}{T_e}(\chi+vT_s)} e^{-j2\pi f_i \chi} e^{j\frac{2\pi l}{T} uT_d} e^{j2\pi(\frac{k}{T_e}-\frac{l}{T}+f_i-f_c)uT_d}$$

$$\int_0^{\eta} h(\xi) e^{-j2\pi(\frac{k}{T_e}+f_i)\xi} \left\{ \frac{1}{T} \int_0^{T} g(t+uT_d-\chi-\xi-vT_s) e^{j2\pi(\frac{k}{T_e}-\frac{l}{T}+f_i-f_c)t} dt \right\} d\xi$$

$$=\sum_v \sum_{k=-K/2}^{K/2-1} X_{v,k} e^{-j\frac{2\pi k}{T_e}(\chi+vT_s)} e^{-j2\pi f_i \chi} e^{j\frac{2\pi l}{T} uT_d} e^{j2\pi(\frac{k}{T_e}-\frac{l}{T}+f_i-f_c)uT_d}$$

$$\int_0^{\eta} h(\xi) \mathrm{Z} \left\{ -(\frac{k}{T_e}-\frac{l}{T}+f_i-f_c); -(uT_d-\chi-\xi-vT_s) \right\} d\xi \quad (4)$$

where T_d and f_c denote the symbol length of the desired system including guard interval and center frequency of desired system, respectively.

III. The Performance Analysis of Active Interference Cancellation

A. *Detection Technique of WB-OFDM system*

In practical situations, the cognitive radio technologies cannot detect the primary systems ideally and this effect may degrade the performance of the WB-OFDM systems. Therefore, the performance of the interference mitigation technique for UWB radio communications is investigated under a practical scenario. Detection technique of WB-OFDM system for MB-OFDM system is discussed and also the avoidance technique of interference to WB-OFDM system for UWB system is presented [13]. Therefore, in this paper, we assume MB-OFDM system can detect the interference to WB-OFDM system by using arbitrary detection techniques. Hence, MB-OFDM system with interference mitigation technique can detect WB-OFDM systems, completely.

B. *Interference Avoidance Technique for WB-OFDM System*

MB-OFDM system has a lot of avoidance techniques to WB-OFDM system. MB-OFDM system is a kind of OFDM system, hence, the interference to WB-OFDM system can be avoided each sub-carrier intervals. Among the interference avoidance techniques to WB-OFDM systems, the Null data transmitting is the simplest one in the frequency domain. However, in case of transmitting null data, it cannot be avoided completely since the effect of side lobe exists. Hence, we propose the active interference cancellation (AIC) to mitigate of the effects of side lobe for WB-OFDM system signals. In this technique the MB-OFDM receiver detects the WB-OFDM system signals and the part of sub-carriers which overlap the frequency band of WB-OFDM system are not transmitted, therefore forming a notch. Moreover, this technique reduces the effect of side lobe from neighbor sub-carriers.

C. Numerical Results

First, we apply the condensed formula of Eq. (1), we obtain the equivalente baseband transmitted signals $x(t)$ given by;

$$x(t) = \sum_{k=-K/2}^{K/2-1} \mathrm{X}_k e^{j2\pi k f_0 t} \quad (-64 \leq k < 64) \tag{5}$$

where K represents the number of sub-carriers, X_k is the data of k^{th} sub-carrier, f_0 is sub-carriers interval. Generally, the transmission system operats DFT with $T_s = \frac{1}{Nf_0}$ sampling interval,

$$\begin{aligned} Y(f) &= \sum_{n=0}^{N-1} x(\frac{n}{Nf_0}) e^{-j2\pi f \frac{n}{Nf_0}} \\ &= \sum_{n=0}^{N-1} \sum_{k=-K/2}^{K/2-1} \mathrm{X}_k e^{j2\pi k f_0 \frac{n}{Nf_0}} e^{-j2\pi f \frac{n}{Nf_0}} \\ &= \sum_{n=0}^{N-1} \sum_{k=-K/2}^{K/2-1} \mathrm{X}_k e^{j\frac{2\pi n}{N} k - f\frac{1}{f_0}}. \end{aligned} \tag{6}$$

However, AIC is operated of 4 times the fundamental frequency ($f_0/4$) sampling, hence,

$$Y(\frac{f_0}{4} l) = \sum_{n=0}^{N-1} \sum_{k=-K/2}^{K/2-1} \mathrm{X}_k e^{j\frac{2\pi n}{N} k - f\frac{l}{4}} \tag{7}$$

where $P_{n,l,k} = \sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(k-\frac{l}{4})}$ is used. In terms of matrix $\mathbf{P}$ and the data vector $\underline{X}$, the output vector $\underline{Y}$ is given by:

$$\underline{Y} = \mathbf{P}\underline{X}. \tag{8}$$

$$\begin{bmatrix} Y_{-L/2} \\ \vdots \\ Y_l \\ \vdots \\ Y_{L/2-1} \end{bmatrix} = \begin{bmatrix} \sum_{n=0}^{N-1} P_{n,-L/2,0} & \cdots & \sum_{n=0}^{N-1} P_{n,-L/2,k} & \cdots & \sum_{n=0}^{N-1} P_{n,-L/2,K-1} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sum_{n=0}^{N-1} P_{n,l,0} & \cdots & \sum_{n=0}^{N-1} P_{n,l,k} & \cdots & \sum_{n=0}^{N-1} P_{n,l,K-1} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sum_{n=0}^{N-1} P_{n,L/2,0} & \cdots & \sum_{n=0}^{N-1} P_{n,L/2,k} & \cdots & \sum_{n=0}^{N-1} P_{n,L/2,K-1} \end{bmatrix} \begin{bmatrix} X_{-K/2} \\ \vdots \\ X_k \\ \vdots \\ X_{K/2-1} \end{bmatrix} \tag{9}$$

Here, the bandwidth of WB-OFDM system is assumed to be 10 MHz with the three sub-carriers of $k = 20, 21, 22$ (3×4.125 MHz = 12.375 MHz). Therefore, the position of these sub-carriers is crucial to make spectrum notch. Hence, the subcarrier of $k = 19, 23$ use AIC tones. Considering $\mathbf{P_1}$, part of $\mathbf{P}$, (Eq. (9)) and the null data vector $\underline{g}$ between $k = 19$ and $k = 23$, the output vector $\underline{d}$ is described by:

$$\underline{d} = \mathbf{P_1}\underline{g}. \tag{10}$$

$$\begin{bmatrix} d_{80} \\ \vdots \\ d_{l'} \\ \vdots \\ d_{88} \end{bmatrix} = \begin{bmatrix} \sum_{n=0}^{N-1} P_{n,80,0} & \cdots & \sum_{n=0}^{N-1} P_{n,80,k} & \cdots & \sum_{n=0}^{N-1} P_{n,80,K-1} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sum_{n=0}^{N-1} P_{n,l',0} & \cdots & \sum_{n=0}^{N-1} P_{n,l',k} & \cdots & \sum_{n=0}^{N-1} P_{n,l',K-1} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sum_{n=0}^{N-1} P_{n,88,0} & \cdots & \sum_{n=0}^{N-1} P_{n,88,k} & \cdots & \sum_{n=0}^{N-1} P_{n,88,K-1} \end{bmatrix} \begin{bmatrix} g_{-K/2} \\ \vdots \\ g_k \\ \vdots \\ g_{K/2-1} \end{bmatrix}$$

$$. \tag{11}$$

The output vector $\underline{a}$ which is part of making a notch spectrum, is given by:

$$\underline{a} = \mathbf{P_2}\underline{h}. \tag{12}$$

$$\begin{bmatrix} a_{80} \\ \vdots \\ a_{l'} \\ \vdots \\ a_{88} \end{bmatrix} = \begin{bmatrix} \sum_{n=0}^{N-1} P_{n,80,19} & \cdots & \sum_{n=0}^{N-1} P_{n,80,k'} & \cdots & \sum_{n=0}^{N-1} P_{n,80,23} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sum_{n=0}^{N-1} P_{n,l',19} & \cdots & \sum_{n=0}^{N-1} P_{n,l',k'} & \cdots & \sum_{n=0}^{N-1} P_{n,l',23} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \sum_{n=0}^{N-1} P_{n,88,19} & \cdots & \sum_{n=0}^{N-1} P_{n,88,k'} & \cdots & \sum_{n=0}^{N-1} P_{n,88,23} \end{bmatrix} \begin{bmatrix} g_{19} \\ \vdots \\ g_{k'} \\ \vdots \\ g_{23} \end{bmatrix} \tag{13}$$

where $\underline{h}$ represents the data vector of $k = 19, 20, 21, 22, 23$, the matrix $\mathbf{P_2}$ is considered only for $k = 19, 20, 21, 22, 23$. Hence, the smallest error $\underline{e^2}$ is:

$$\underline{e^2} = \|\mathbf{P_2}\underline{h} + \underline{d}\|^2 \tag{14}$$

However, inverse matrix cannot be obtained since $\mathbf{P_2}$ of Eq (14) is not nonsingular matrix. Hence, due to Moore Penrose equivalent inverse matrix, the solution is obtained. Moreover, the vector $\underline{h}$ values are chosen for depth of the spectrum notch.

$$\underline{h} = -\mathbf{P_2}^H\mathbf{P_2}(\mathbf{P_2}^H\mathbf{P_2}\mathbf{P_2}^H\mathbf{P_2})^{-1}\mathbf{P_2}^H\underline{d}. \tag{15}$$

Fig. 2 shows the null spectrum without AIC, Fig.3 depicts the null spectrum with AIC. From Fig.2 and 3, the spectrum notch is made 10dB and 40dB, respectively. Therefore, by using AIC, the depth spectrum notch becomes deeper.

Moreover, the avoidance with AIC for WB-OFDM system is analyzed. The major parameter of the simulation are listed in Table III. Fig. 4 and 5 show the effect of interference from MB-OFDM system to WB-OFDM system with AIC, where SIR is10dB and15dB,respectively.

From Fig. 4 and 5, the AIC shows the effectiveness of avoidance technique. However, in spite of Fig. 3 with interference 40dB, a 30dB gain is obtained since the relationship of sampling frequency between MB-OFDM system and WB-OFDM system is not integral multiple. Hence, it cannot make the spectrum notch deeper with AIC.

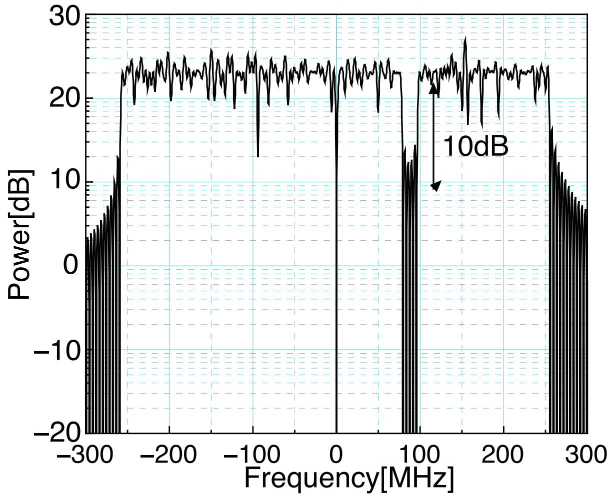

Fig. 2. The null spectrum without AIC

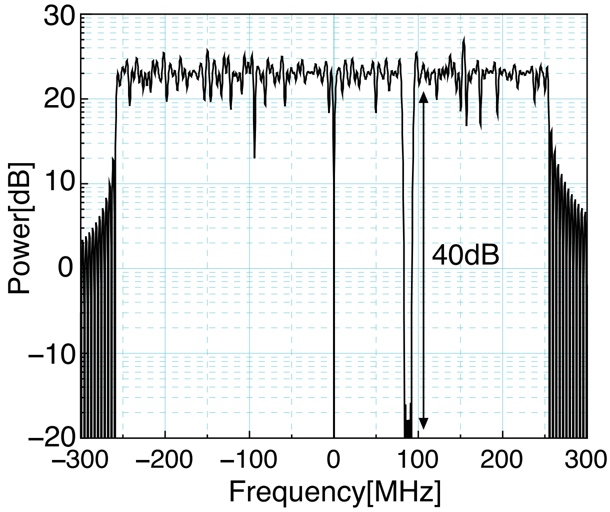

Fig. 3. The null spectrum with AIC

IV. Conclusion

We showed the performance of UWB radio system with DAA in the coexistence environment between UWB systems and WB-OFDM system. We showed the AIC is an effective interference mitigation technique. However, DAA technique should be chosen in consideration to the required performance quality of UWB applications. Moreover, the avoidance technique is related to the detection methods. The realtime applications such as speech and high-quality video in the wireless communication require high data rate. Therefore, the unused frequency band by the WB-OFDM systems needs to be allocated. In this case, UWB system needs a high detection probability. Therefore the miss-detection of the WB-OFDM system may not affect the performance of the UWB systems. Thus, DAA technology is the effective interference mitigation technique for high data rate UWB system.

TABLE III
Major parameters of the simulation

Bandwidth	10 MHz
FFT point	> 1024
Center frequency	3.4 GHz
Modulation	16QAM
undesired system	MB-OFDM system
interference system modulation	QPSK μs
Channel	AWGN

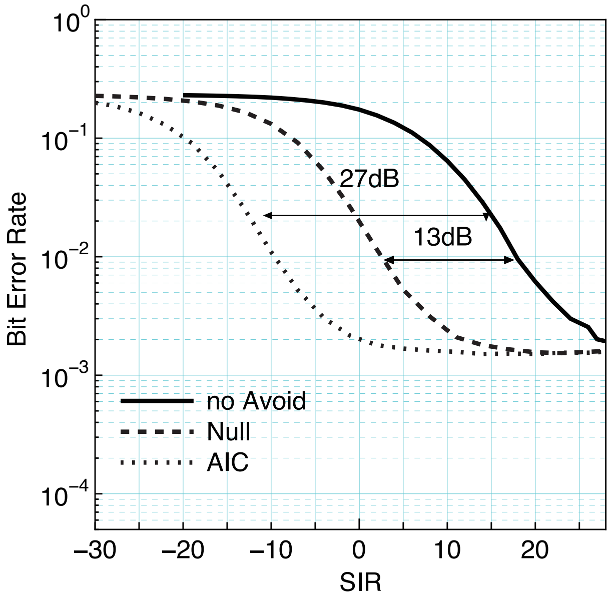

Fig. 4. The BER of WB-OFDM system vs SIR=10dB with AIC

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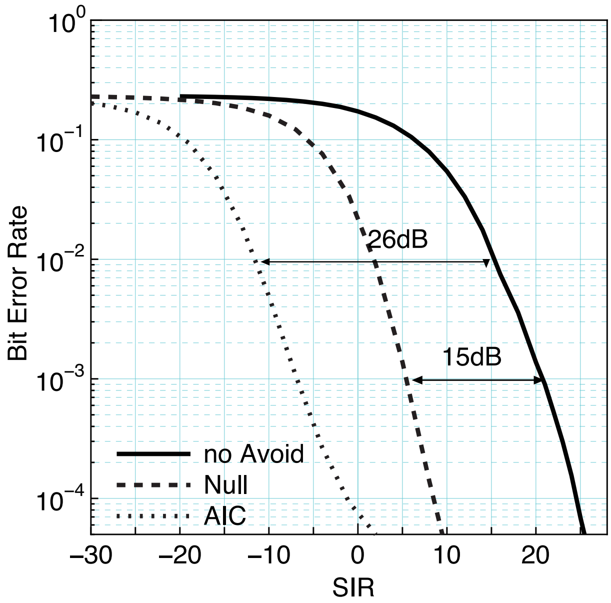

Fig. 5. The BER of WB-OFDM system vs SIR=15dB with AIC

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