Multipath Interference Characterization in Wireless Communication Systems

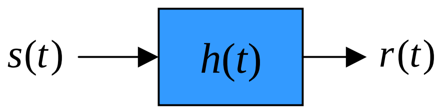
Michael Rice

BYU Wireless Communications Lab

Multipath Propagation

- Multiple paths between transmitter and receiver
- Constructive/destructive interference
- Dramatic changes in received signal amplitude and phase as a result of small changes ($\lambda/2$) in the spatial separation between a receiver and transmitter.
- For Mobile radio (cellular, PCS, etc) the channel is time-variant because motion between the transmitter and receiver results in propagation path changes.
- Terms: Rayleigh Fading, Rice Fading, Flat Fading, Frequency Selective Fading, Slow Fading, Fast Fading ….
- What do all these mean?

LTI System Model



$$h(t) = \sum_{k=0}^{N-1} a_k e^{j\theta_k} \delta(t - \tau_k)$$

$$= a_0 \delta(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} \delta(t - \tau_k)$$

line-of-sight propagation — multipath propagation

$$r(t) = a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t - \tau_k)$$

line-of-sight component — multipath component

Some Important Special Cases

All the delays are so small and we approximate $\tau_k \approx 0$ for all k

$$
\begin{aligned}
r(t) &= a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t-\tau_k) \\
&\approx a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t) \\
&= \underbrace{\left[a_0 + \sum_{k=1}^{N-1} a_k e^{j\theta_k} \right]}_{\alpha} s(t)
\end{aligned}
$$

- sum of complex random numbers (random amplitudes and phases)
- if N is large enough, this sum is well approximated by complex Gaussian pdf

$$
\begin{aligned}
\alpha &= \alpha_R + j\alpha_I \\
\alpha_R &\sim N(m_a, \sigma_a^2) \\
\alpha_I &\sim N(m_a, \sigma_a^2)
\end{aligned}
$$

$$
\begin{aligned}
m_a = E\{\alpha\} &= E\left\{ \sum_{k=1}^{N-1} a_k e^{j\theta_k} \right\} \\
&= \sum_{k=1}^{N-1} E\{a_k\} E\{e^{j\theta_k}\} \\
&= 0 \text{ when } \theta_k \sim U[-\pi, \pi]
\end{aligned}
$$

Some Important Special Cases

All the delays are so small and we approximate $\tau_k \approx 0$ for all k

$$r(t) = a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t-\tau_k)$$
$$\approx a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t)$$
$$= \left[a_0 + \sum_{k=1}^{N-1} a_k e^{j\theta_k} \right] s(t)$$
$$= [a_0 + \alpha] s(t)$$
$$= [a_0 + \alpha_R + j\alpha_I] s(t)$$
$$= \sqrt{(a_0 + \alpha_R)^2 + (\alpha_I)^2}\, e^{j\phi} s(t)$$

$$r(t) = \sqrt{(a_0 + \alpha_R)^2 + (\alpha_I)^2}\, e^{j\phi} s(t)$$
$$= \sqrt{X_1^2 + X_2^2}\, e^{j\phi} s(t)$$

$X_1 \sim N(a_0, \sigma_a^2)$ $\quad$ $X_2 \sim N(0, \sigma_a^2)$

Important PDF's

$$X_1 \sim N\left(a_0, \sigma_a^2\right)$$
$$X_2 \sim N\left(0, \sigma_a^2\right)$$

$$W = X_1^2 + X_2^2$$

$$p_W(w) = \frac{1}{2\sigma_a^2} e^{-\frac{a_0^2 + w}{2\sigma_a^2}} I_0\left(\sqrt{w}\frac{a_0}{\sigma_a^2}\right)$$

Non-central Chi-square pdf

$$U = \sqrt{X_1^2 + X_2^2}$$

$$p_U(u) = \frac{u}{\sigma_a^2} e^{-\frac{a_0^2 + u^2}{2\sigma_a^2}} I_0\left(\frac{ua_0}{\sigma_a^2}\right)$$

Rice pdf

$$X_1 \sim N\left(0, \sigma_a^2\right)$$
$$X_2 \sim N\left(0, \sigma_a^2\right)$$

$$W = X_1^2 + X_2^2$$

$$p_W(w) = \frac{1}{2\sigma_a^2} e^{-\frac{w}{2\sigma_a^2}}$$

Chi-square pdf

$$U = \sqrt{X_1^2 + X_2^2}$$

$$p_U(u) = \frac{u}{\sigma_a^2} e^{-\frac{u^2}{2\sigma_a^2}}$$

Rayleigh pdf

Back to Some Important Special Cases

All the delays are so small and we approximate $\tau_k \approx 0$ for all k

$|a_0| > 0$

$$r(t) = a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t-\tau_k)$$
$$\approx a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t)$$
$$= \underbrace{\sqrt{(a_0+\alpha_R)^2 + (\alpha_I)^2}}_{\text{Rice pdf}} e^{j\phi} s(t)$$

"Ricean fading"

$a_0 = 0$

$$r(t) = \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t-\tau_k)$$
$$\approx \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t)$$
$$= \underbrace{\sqrt{(\alpha_R)^2 + (\alpha_I)^2}}_{\text{Rayleigh pdf}} e^{j\phi} s(t)$$

"Rayleigh fading"

Some Important Special Cases

All the delays are small and we approximate $\tau_k \approx \bar{\tau}$ for all k

$|a_0| > 0$

$$r(t) = a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t-\tau_k)$$
$$\approx a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t-\bar{\tau})$$
$$= a_0 s(t) + \underbrace{\sqrt{(\alpha_R)^2 + (\alpha_I)^2}}_{\text{Rayleigh pdf}} e^{j\phi} s(t-\bar{\tau})$$

"Line-of-sight with Rayleigh Fading"

$a_0 = 0$

$$r(t) = \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t-\tau_k)$$
$$\approx \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t-\bar{\tau})$$
$$= \underbrace{\sqrt{(\alpha_R)^2 + (\alpha_I)^2}}_{\text{Rayleigh pdf}} e^{j\phi} s(t-\bar{\tau})$$

"Rayleigh fading"

Multiplicative Fading

In the past two examples, the received signal was of the form

$$r(t) = Fe^{j\phi}s(t)$$

The fading takes the form of a random attenuation: the transmitted signal is *multiplied* by a random value whose envelope is described by the Rice or Rayleigh pdf.

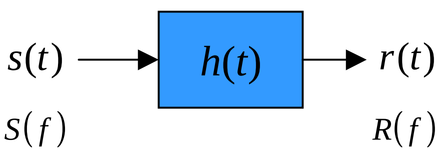
This is sometimes called multiplicative fading for the obvious reason. It is also called *flat fading* since all spectral components in $s(t)$ are attenuated by the same value.

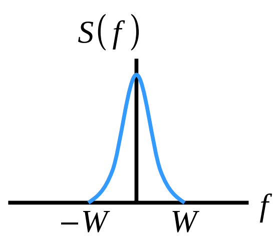
An Example

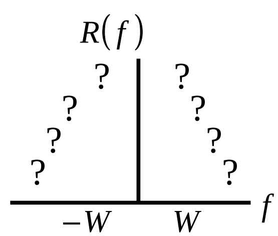
$$h(t) = \delta(t) + ae^{j\theta}\delta(t-\tau)$$

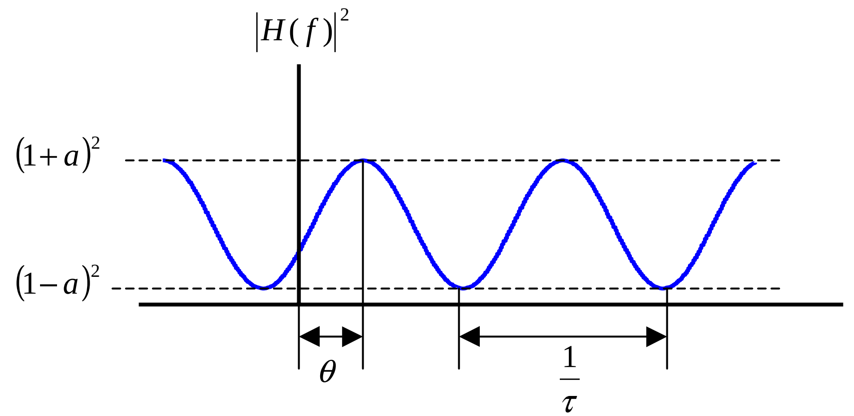
$$H(f) = 1 + ae^{j(\theta - 2\pi f\tau)}$$

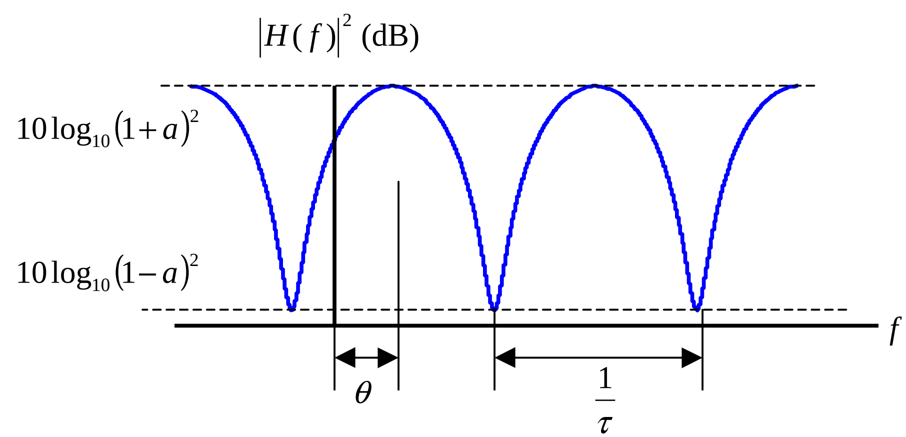
$$|H(f)|^2 = 1 + a^2 + 2a\cos(2\pi f\tau - \theta)$$



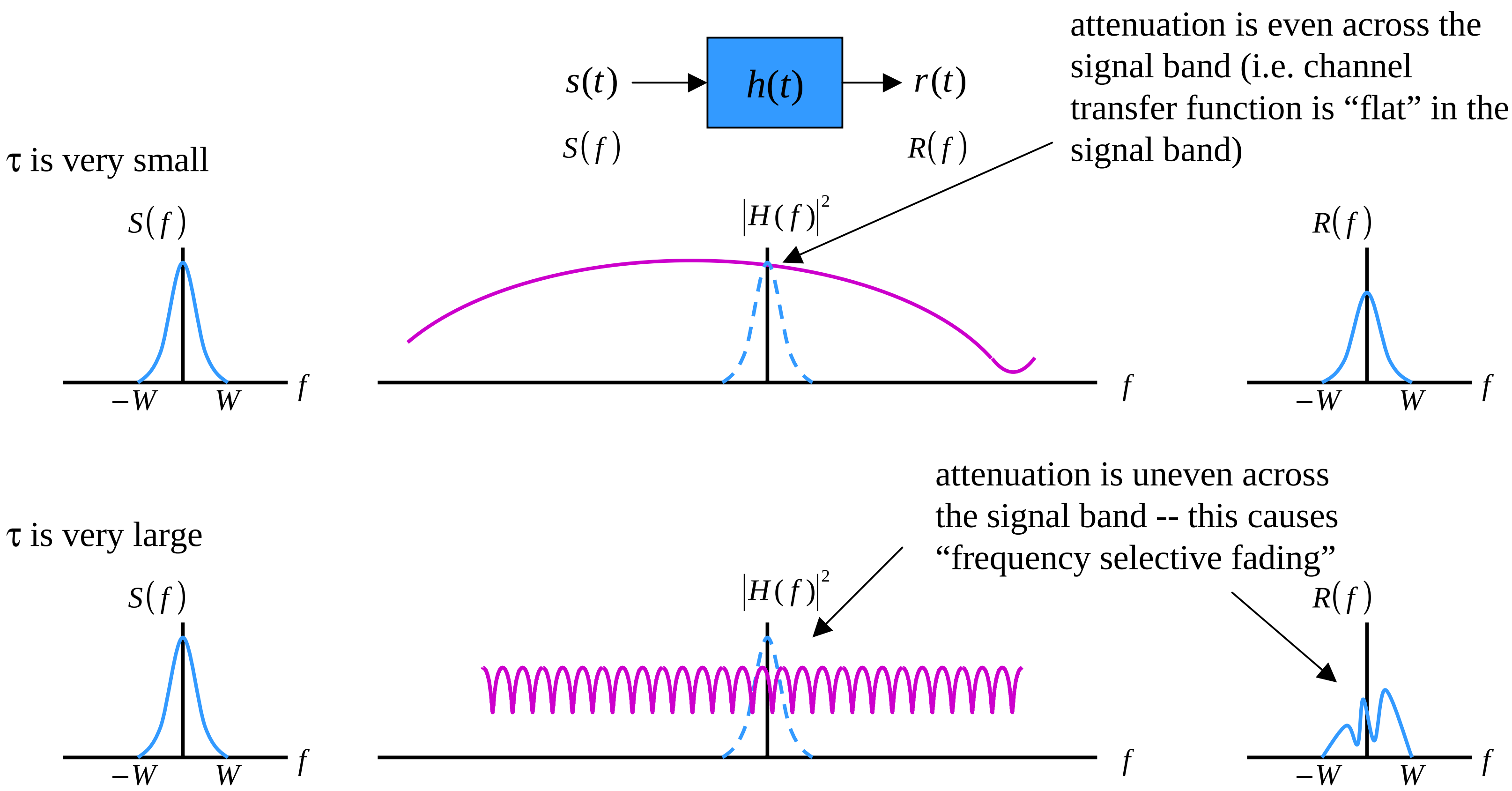








Example (continued)



Another important special case

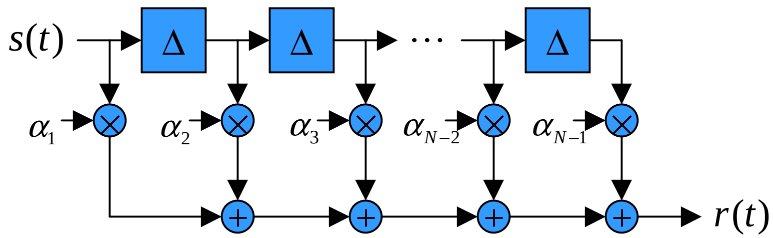
The delays are all different: $\tau_1 < \tau_2 < \cdots < \tau_{N-1}$

$$r(t) = a_0 s(t) + \underbrace{\sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t - \tau_k)}_{\text{intersymbol interference}}$$

if the delays are "long enough", the multipath reflections are *resolvable*.

Two common models for non-multiplicative fading

Taped delay-line with random weights



Additive complex Gaussian random process

central limit theorem: approximately a Gaussian RP

$$r(t) = a_0 s(t) + \overbrace{\sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t-\tau_k)}$$
$$\approx a_0 s(t) + \xi(t)$$

Multipath Intensity Profile

The characterization of multipath fading as either flat (multiplicative) or frequency selective (non-multiplicative) is governed by the delays:

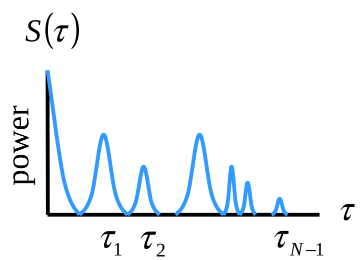
small delays $\Rightarrow$ flat fading (multiplicative fading)
large delays $\Rightarrow$ frequency selective fading (non-multiplicative fading)

The values of the delay are quantified by the multipath intensity profile $S(\tau)$

$$R_{hh}(\tau_1,\tau_2) = E\{h^*(\tau_1)h(\tau_2)\}$$
$$= S(\tau_1)\delta(\tau_1-\tau_2)$$

uncorrelated scattering (US) assumption

$$S(\tau) = E\{|h(\tau)|^2\}$$



1. "maximum excess delay" or "multipath spread"

$$T_m = \tau_{N-1}$$

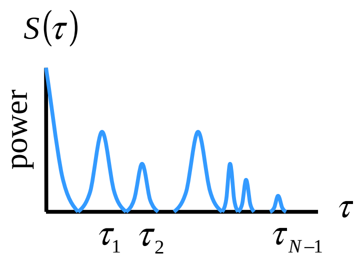
2. average delay

$$\bar{\tau} = \frac{1}{N-1}\sum_{k=1}^{N-1}\tau_k \quad \text{or} \quad \bar{\tau} = \frac{\sum_{k=1}^{N-1}|a_k|\tau_k}{\sum_{k=1}^{N-1}|a_k|}$$

3. delay spread

$$\sigma_\tau = \sqrt{\frac{1}{N-1}\sum_{k=1}^{N-1}\tau_k^2 - \bar{\tau}^2} \quad \text{or} \quad \sigma_\tau = \sqrt{\frac{\sum_{k=1}^{N-1}|a_k|^2\tau_k^2}{\sum_{k=1}^{N-1}|a_k|^2} - \bar{\tau}^2}$$

Characterization using the multipath intensity profile



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3. delay spread

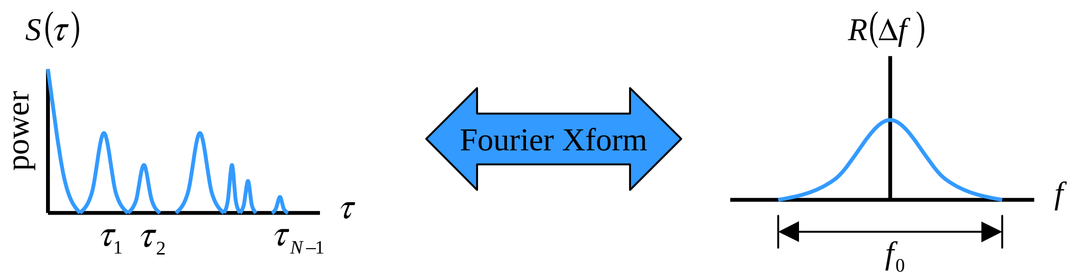
$$\sigma_\tau = \sqrt{\frac{1}{N-1}\sum_{k=1}^{N-1}\tau_k^2 - \bar{\tau}^2} \quad \text{or} \quad \sigma_\tau = \sqrt{\frac{\sum_{k=1}^{N-1}|a_k|^2\tau_k^2}{\sum_{k=1}^{N-1}|a_k|^2} - \bar{\tau}^2}$$

Compare multipath spread T_m with symbol time T_s:

$T_m < T_s \Rightarrow$ flat fading (frequency non-selective fading)

$T_m > T_s \Rightarrow$ frequency selective fading

Spaced Frequency Correlation Function



Compare coherence bandwidth f_0 with transmitted signal bandwidth W:

$f_0 > W \Rightarrow$ flat fading (frequency non-selective fading)

$f_0 < W \Rightarrow$ frequency selective fading

$R(\Delta f)$ is the "correlation between the channel response to two signals as a function of the frequency difference between the two signals."

"What is the correlation between received signals that are spaced in frequency $\Delta f = f_1 - f_2$?"

Coherence bandwidth f_0 = a statistical measure of the range of frequencies over which the channel passes all spectral components with approximately equal gain and linear phase.

equations (8) - (13) are commonly used relationships between delay spread and coherence bandwidth

Time Variations

Important Assumption

Multipath interference is spatial phenomenon. Spatial geometry is assumed fixed. All scatterers making up the channel are stationary -- whenever motion ceases, the amplitude and phase of the receive signal remains constant (the channel appears to be time-invariant). Changes in multipath propagation occur due to changes in the spatial location *x* of the transmitter and/or receiver. The faster the transmitter and/or receiver change spatial location, the faster the time variations in the multipath propagation properties.

$$h(t;x) = \sum_{k=0}^{N-1} a_k(x) e^{j\theta_k(x)} \delta(t - \tau_k(x))$$

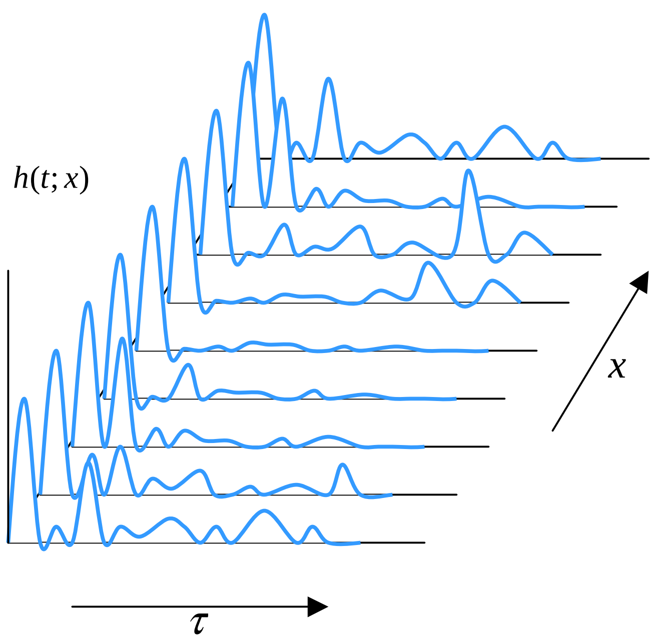
$$= a_0(x)\delta(t) + \sum_{k=1}^{N-1} a_k(x) e^{j\theta_k(x)} \delta(t - \tau_k(x))$$

complex gains and phase shifts are a function of spatial location *x*.

line-of-sight propagation

multipath propagation

Spatially Varying Channel Impulse Response



- channel impulse response changes with spatial location *x*
- generalize impulse response to include spatial information

$$h(t) \rightarrow h(t;x)$$

- Transmitter/receiver motion cause change in spatial location *x*
- The larger $\dot{x}$, the faster the rate of change in the channel.
- Assuming a constant velocity *v*, the position axis *x* could be changed to a *time* axis *t* using $t = x/v$.

Generalize the Multipath Intensity Profile

From before…

$$R_{hh}(\tau_1,\tau_2) = E\{h^*(\tau_1)h(\tau_2)\}$$
$$= S(\tau_1)\delta(\tau_1-\tau_2)$$

US assumption

$$S(\tau) = E\{|h(\tau)|^2\}$$

The generalization …

$$R_{hh}(\tau_1,\tau_2;x_1,x_2) = E\{h^*(\tau_1;x_1)h(\tau_2;x_2)\}$$
$$= S(\tau_1;x_1,x_2)\delta(\tau_1-\tau_2)$$

US assumption

$$S(\tau;x_1,x_2) = E\{h^*(\tau;x_1)h(\tau;x_2)\}$$

$t = x/v$

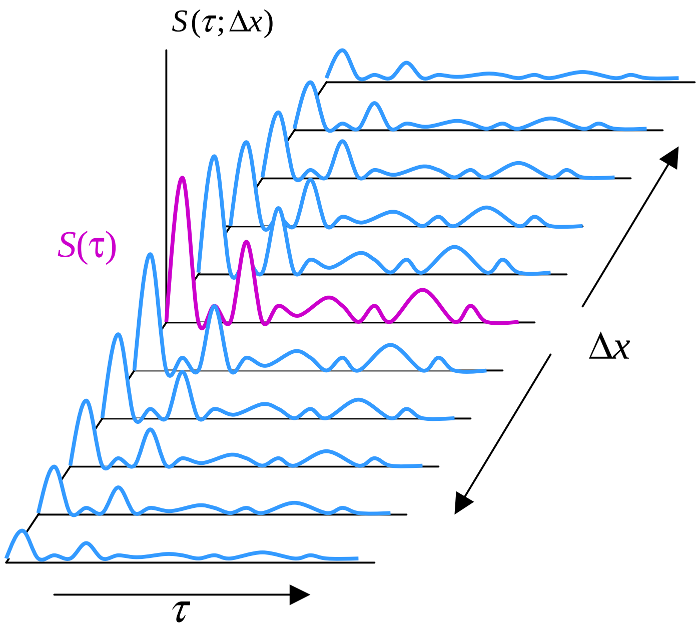
$$S(\tau;t_1,t_2) = E\{h^*(\tau;t_1)h(\tau;t_2)\}$$

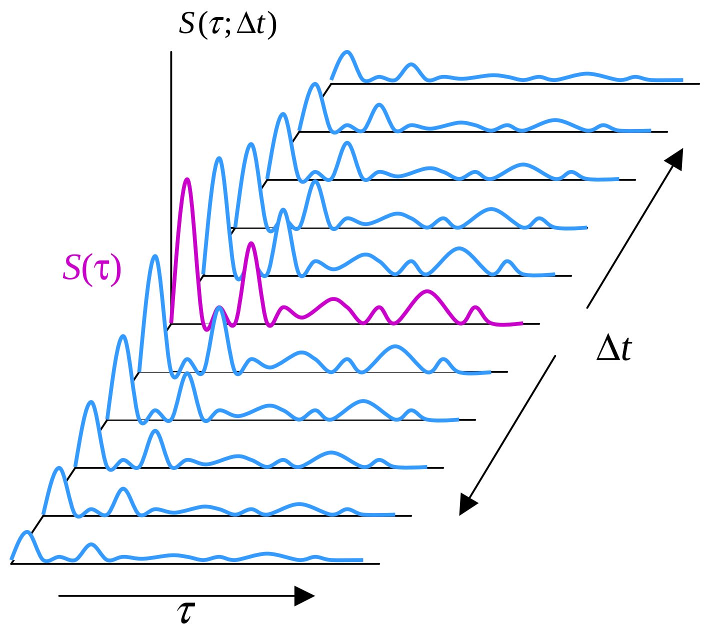
WSS assumption

$$S(\tau;\Delta t) = E\{h^*(\tau;t)h(\tau;t+\Delta t)\}$$

this function is the key to the WSSUS channel

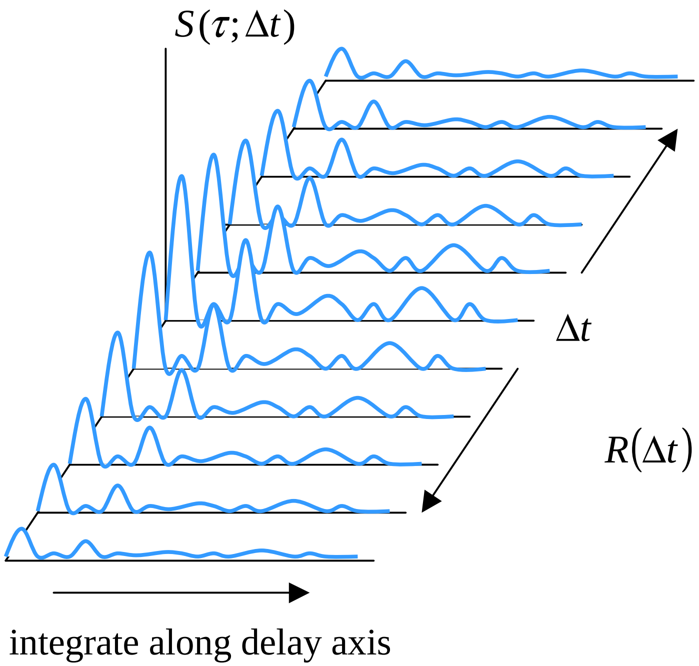
A look at $S(\tau;\Delta t)$



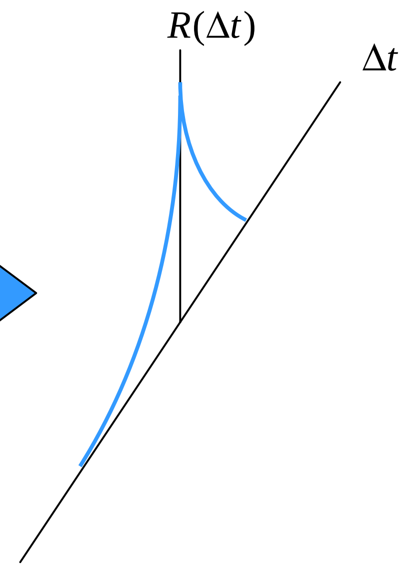


$$S(\tau) = S(\tau;0)$$

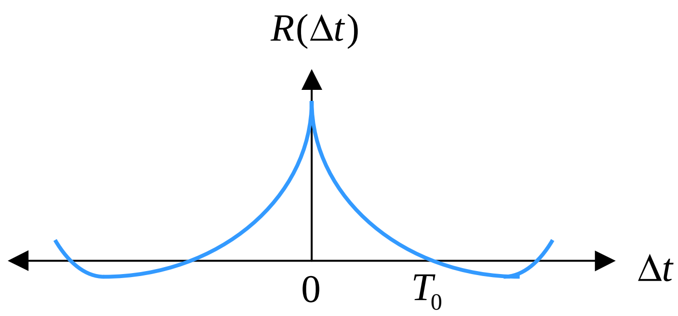
Time Variations of the Channel:
The Spaced-Time Correlation Function



$$R(\Delta t) = \int S(\tau;\Delta t)\,d\tau$$



Time Variations of the Channel: The Spaced-Time Correlation Function



$R(\Delta t)$ specifies the extent to which there is correlation between the channel response to a sinusoid sent at time t and the response to a similar sinusoid at time $t+\Delta t$.

Coherence Time T_0 is a measure of the expected time duration over which the channel response is essentially invariant. Slowly varying channels have a large T_0 and rapidly varying channels have a small T_0.

Re-examination of special cases

From before…

$$r(t) = a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t - \tau_k)$$

$$\approx a_0 s(t) + \sum_{k=1}^{N-1} a_k e^{j\theta_k} s(t)$$

$$= \left[a_0 + \sum_{k=1}^{N-1} a_k e^{j\theta_k} \right] s(t)$$

$$= [a_0 + \alpha] s(t)$$

complex Gaussian RV

The generalization …

$$r(t) = a_0 s(t) + \sum_{k=1}^{N-1} a_k(x) e^{j\theta_k(x)} s(t - \tau_k(x))$$

$$\approx a_0 s(t) + \sum_{k=1}^{N-1} a_k(x) e^{j\theta_k(x)} s(t)$$

$$= \left[a_0 + \sum_{k=1}^{N-1} a_k(x) e^{j\theta_k(x)} \right] s(t)$$

$$= [a_0 + \alpha(x)] s(t)$$

$t = x / v$

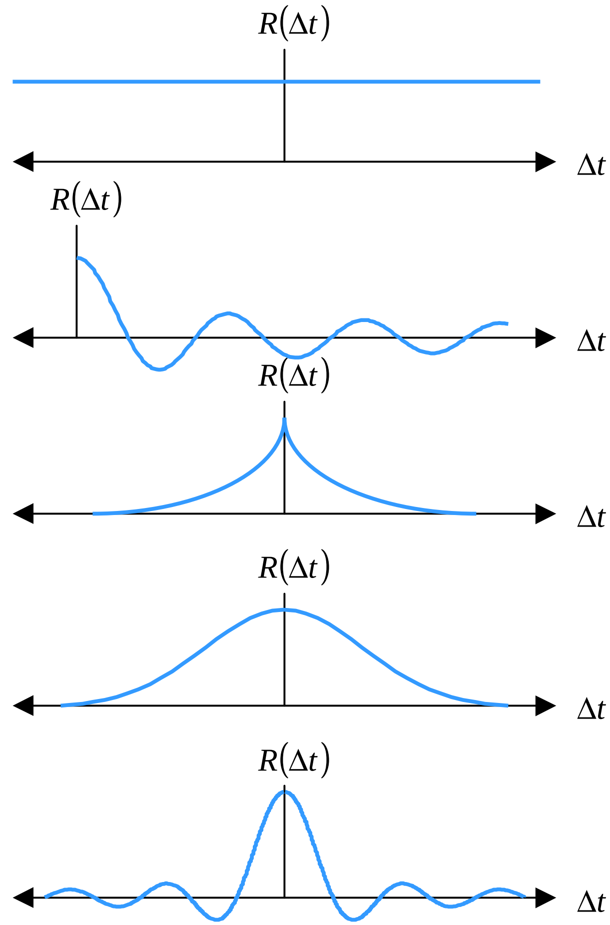
$$= [a_0 + \alpha(t)] s(t)$$

complex Gaussian Random *Process* with autocorrelation

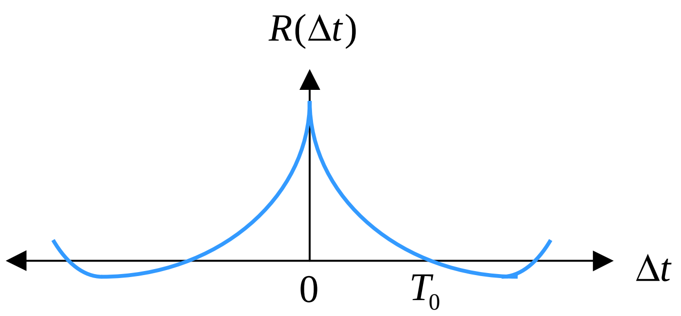
$$R_\alpha(\Delta t) = E\{\alpha^*(t)\alpha(t + \Delta t)\}$$
$$= R(\Delta t)$$

Commonly Used Spaced-Time Correlation Functions

Time Invariant	$R(\Delta t) = 2\sigma_a^2 \quad -\infty < \Delta t < \infty$
Land Mobile (Jakes)	$R(\Delta t) = 2\sigma_a^2 J_0\left(2\pi \frac{v}{\lambda} \Delta t\right)$
Exponential	$R(\Delta t) = 2\sigma_a^2 e^{-2\pi \frac{v}{\lambda}\lvert\Delta t\rvert}$
Gaussian	$R(\Delta t) = 2\sigma_a^2 e^{-\left(\pi \frac{v}{\lambda}\Delta t\right)^2}$
"Rectangular"	$R(\Delta t) = 2\sigma_a^2 \frac{\sin\left(2\pi \frac{v}{\lambda}\Delta t\right)}{2\pi \frac{v}{\lambda}\Delta t}$

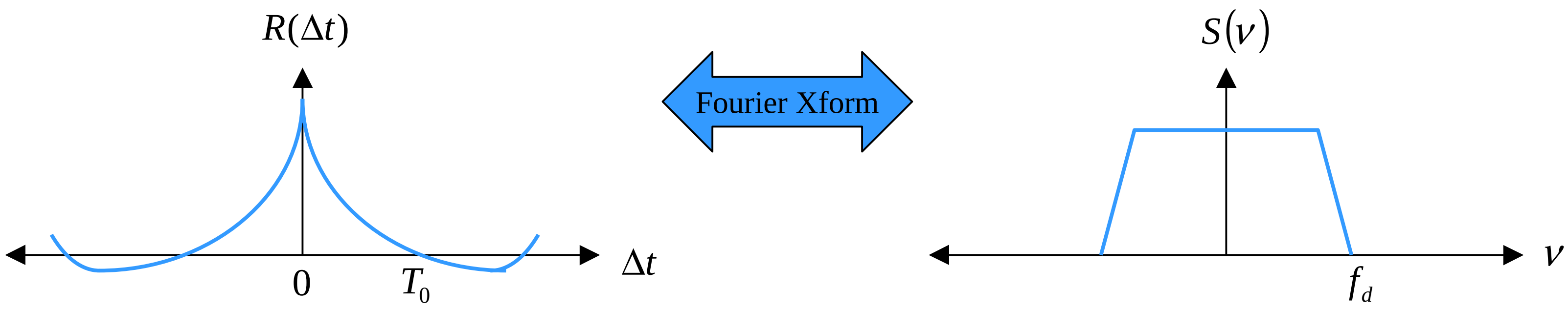


Characterization of time variations using the spaced-time correlation function



- Fast Fading
  - $T_0 < T_s$
  - correlated channel behavior lasts less than a symbol ⇒ fading characteristics change multiple times during a symbol ⇒ pulse shape distortion
- Slow Fading
  - $T_0 > T_s$
  - correlated channel behavior lasts more than a symbol ⇒ fading characteristics constant during a symbol ⇒ no pulse shape distortion ⇒ error bursts…

Doppler Power Spectrum
Frequency Domain View of Time-Variations

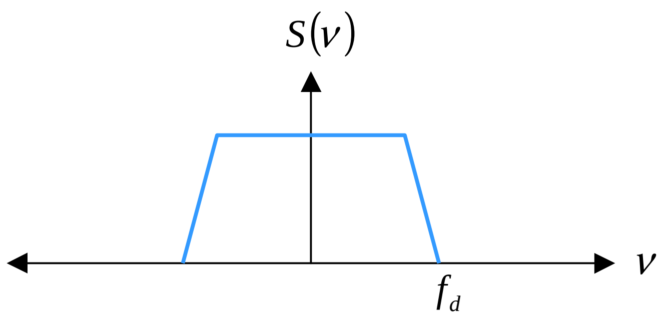


Time variations on the channel are evidenced as a Doppler broadening and perhaps, in addition as a Doppler shift of a spectral line.

Doppler power spectrum $S(\nu)$ yields knowledge about the spectral spreading of a sinusoid (impulse in frequency) in the Doppler shift domain. It also allows us to glean how much spectral broadening is imposed on the transmitted signal as a function of the rate of change in the channel state.

Doppler Spread of the channel f_d is the range of values of ν over which the Doppler power spectrum is essentially non zero.

Doppler Power Spectrum and Doppler Spread



Compare Doppler Spread f_d with transmitted signal bandwidth W:

$f_d > W \Rightarrow$ fast fading

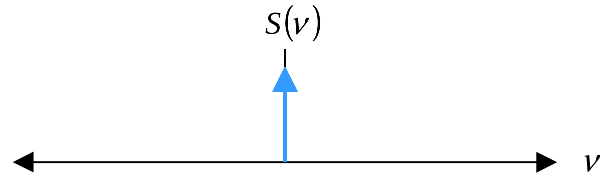
$f_d < W \Rightarrow$ slow fading

equations (18) - (21) are commonly used relationships between Doppler spread and coherence time

Common Doppler Power Spectra

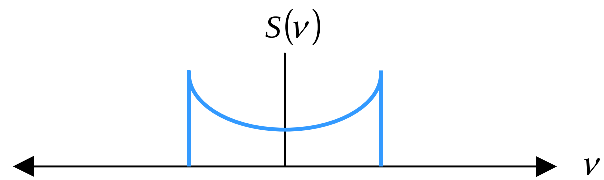
Time Invariant

$$S(\nu) = 2\sigma_a^2 \delta(\nu)$$



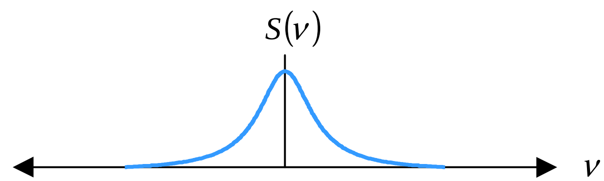
Land Mobile
(Jakes)

$$S(\nu) = \frac{2\sigma_a^2}{\sqrt{\nu^2 - (v/\lambda)^2}}$$



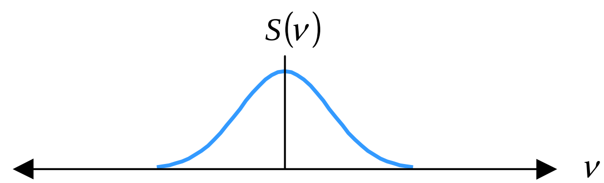
Exponential
(1st order Butterworth)

$$S(\nu) = \frac{2\sigma_a^2 v/\lambda}{\pi\left(\nu^2 + (v/\lambda)^2\right)}$$



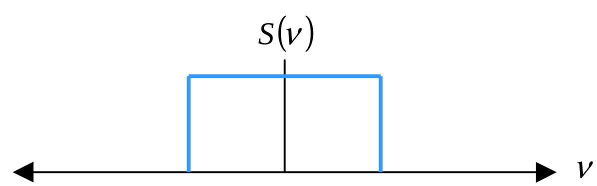
Gaussian

$$S(\nu) = \frac{2\sigma_a^2}{\sqrt{\pi (v/\lambda)^2}} e^{-\frac{\nu^2}{(v/\lambda)^2}}$$

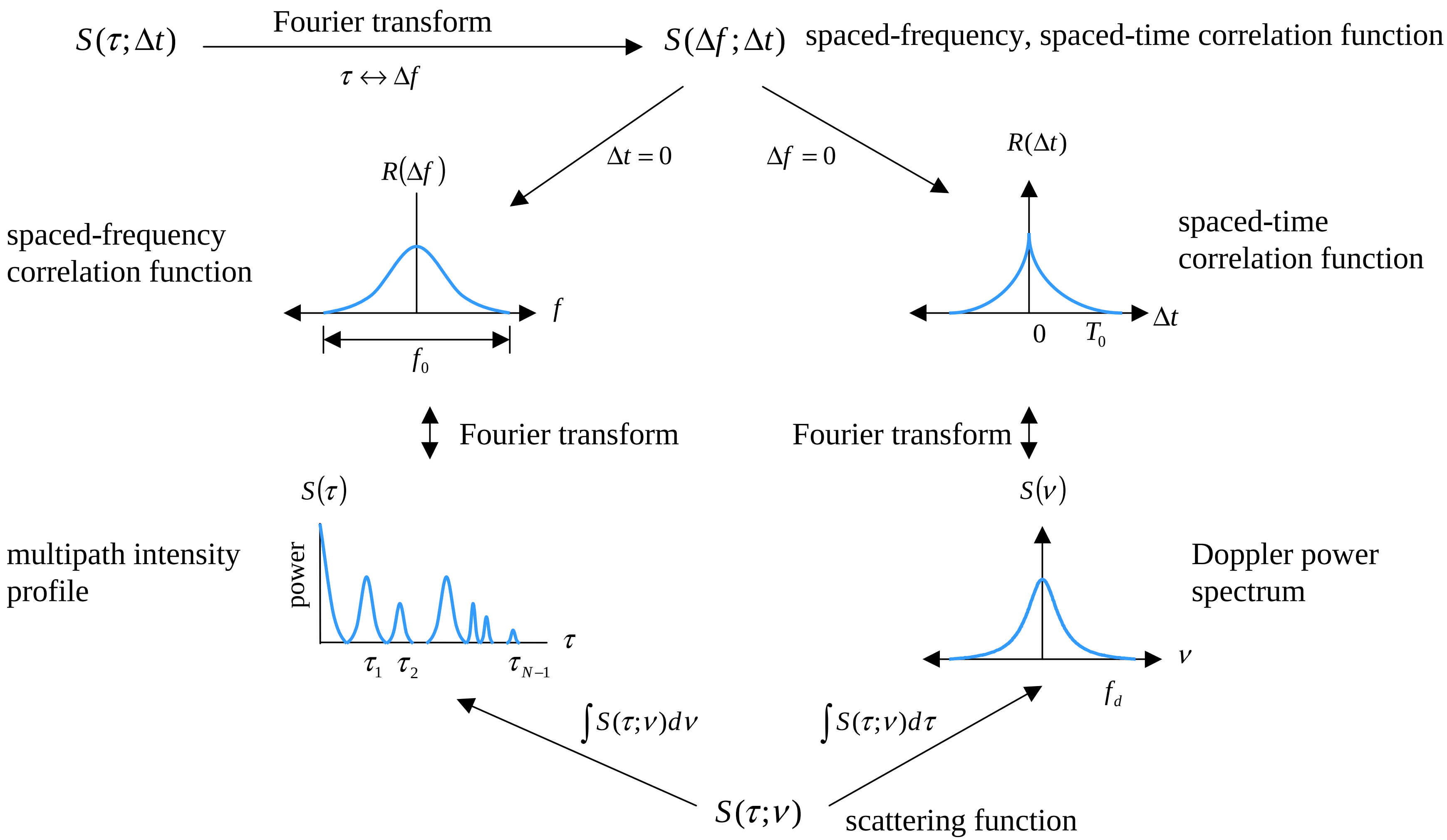


“Rectangular”

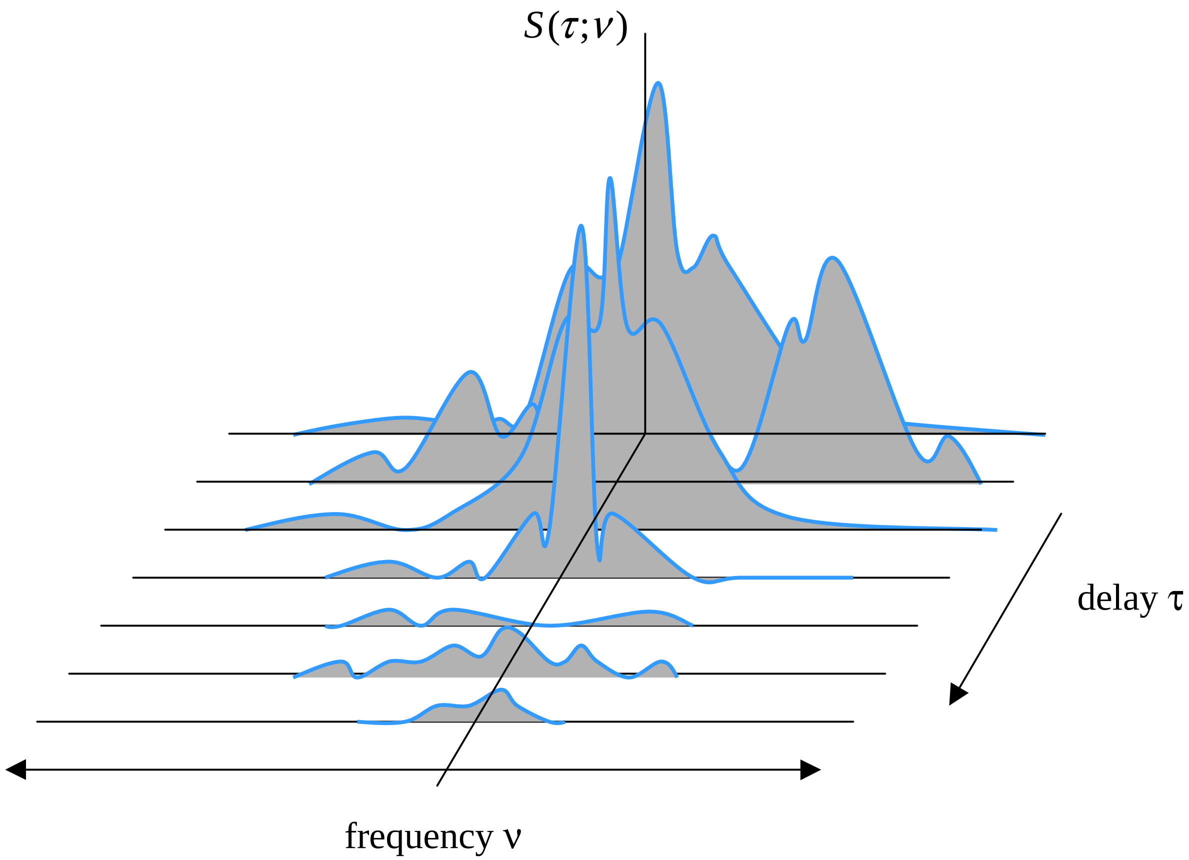
$$S(\nu) = \begin{cases} \dfrac{\sigma_a^2}{v/\lambda} & -v/\lambda < \nu < v/\lambda \\ 0 & \text{otherwise} \end{cases}$$



Putting it all together…



Scattering Function



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