Signal Processing Techniques for Spectrum Sensing and Communications in Cognitive Radios

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Organization (1/2)

- **Introduction**
  - Cognitive radio
  - Primary and Secondary Users (PUs and SUs)
  - Spectrum sensing and sharing
  - Spectrum leakage
- **FFT-based OFDM**
  - Subcarrier leakage
    - Solutions and limitations
- **Spectrum analysis methods**
  - Periodogram Spectral Estimator (PSE)
  - Blackman-Tukey Spectral Estimator (BTSE)
  - Minimum Variance Spectral Estimator (MVSE)
  - Maltitaper Method (MTM)
  - Filter Bank Spectral Estimator (FBSE)

Organization (2/2)

- **Filterbank as multicarrier communication tools**
  - Filtered multitone (FMT)
  - Offset QAM/Staggered modulated multitone (SMT)
  - Cosine modulated multitone (CMT)
- **Implementation of Filterbank Multicarrier Systems**
  - Polyphase structures
- **Conclusions**

Introduction: What is Cognitive Radio?

Cognitive Radios:

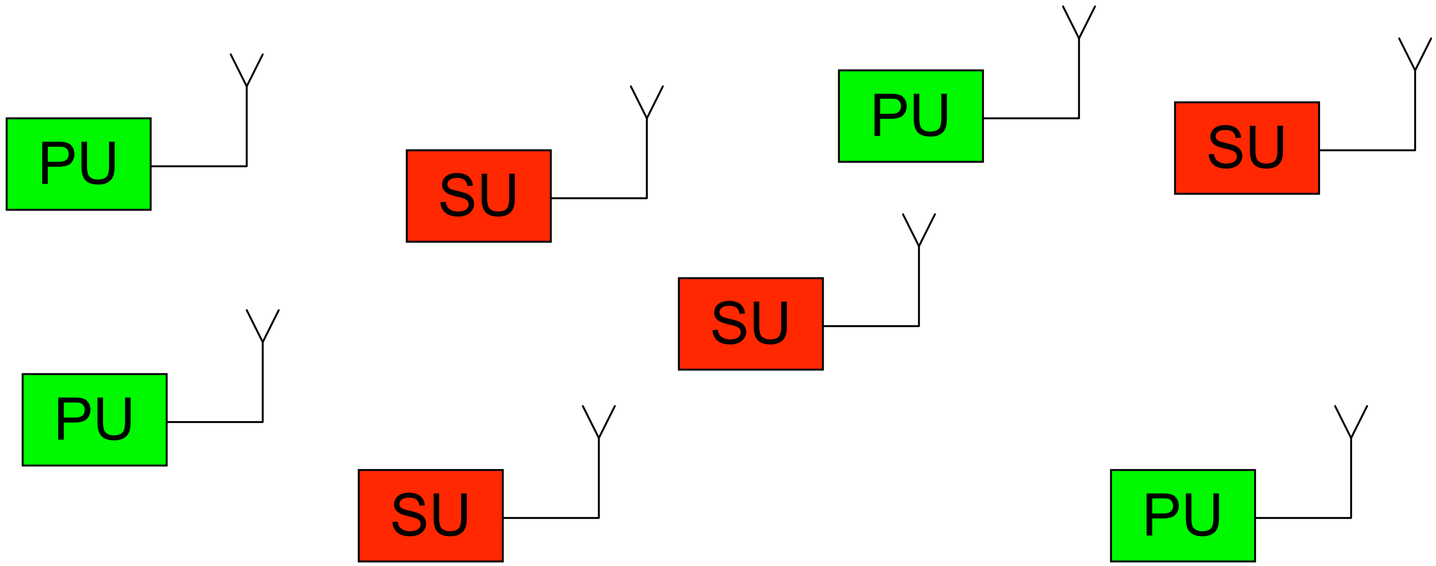
- Have the capability to be aware of their surrounding environment
- Can change PHY depending on environment
- Can change PHY depending on traffic needs
- Can alter higher layer behavior as needed
- Learn from past experiences

⇨ Capable of complex adaptation on lower layers

[Mit99] J. Mitola, III and G.Q. Maguire, Jr., **"Cognitive Radio: Making Software Radios More Personal,"** *IEEE Personal Communications, vol. 6, no. 4, 1999.*

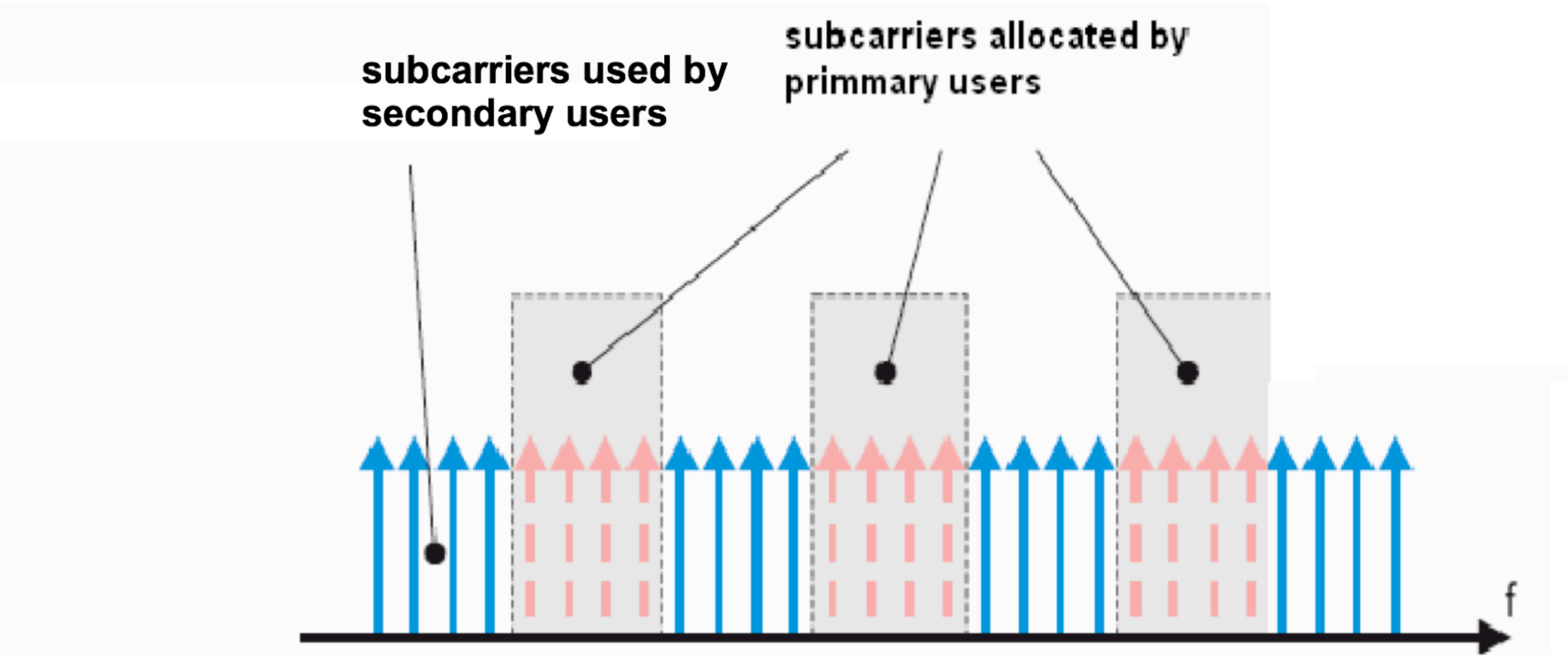
Introduction: Primary and secondary users

- Primary (licensed) and secondary (unlicensed) users coexist and share the same spectrum
- PUs have priority and thus SUs must back-off as soon as PUs begin a communication
  - This requires channel sensing



Introduction

Multicarrier has been proposed for channel sensing and co-existence of PUs and SUs



[WeiJon04] T.A. Weiss and F.K. Jondral, **"Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency,"** *IEEE Commun. Magazine, Vol. 42, No. 3, March 2004, pp. S8 - S14.*

Introduction

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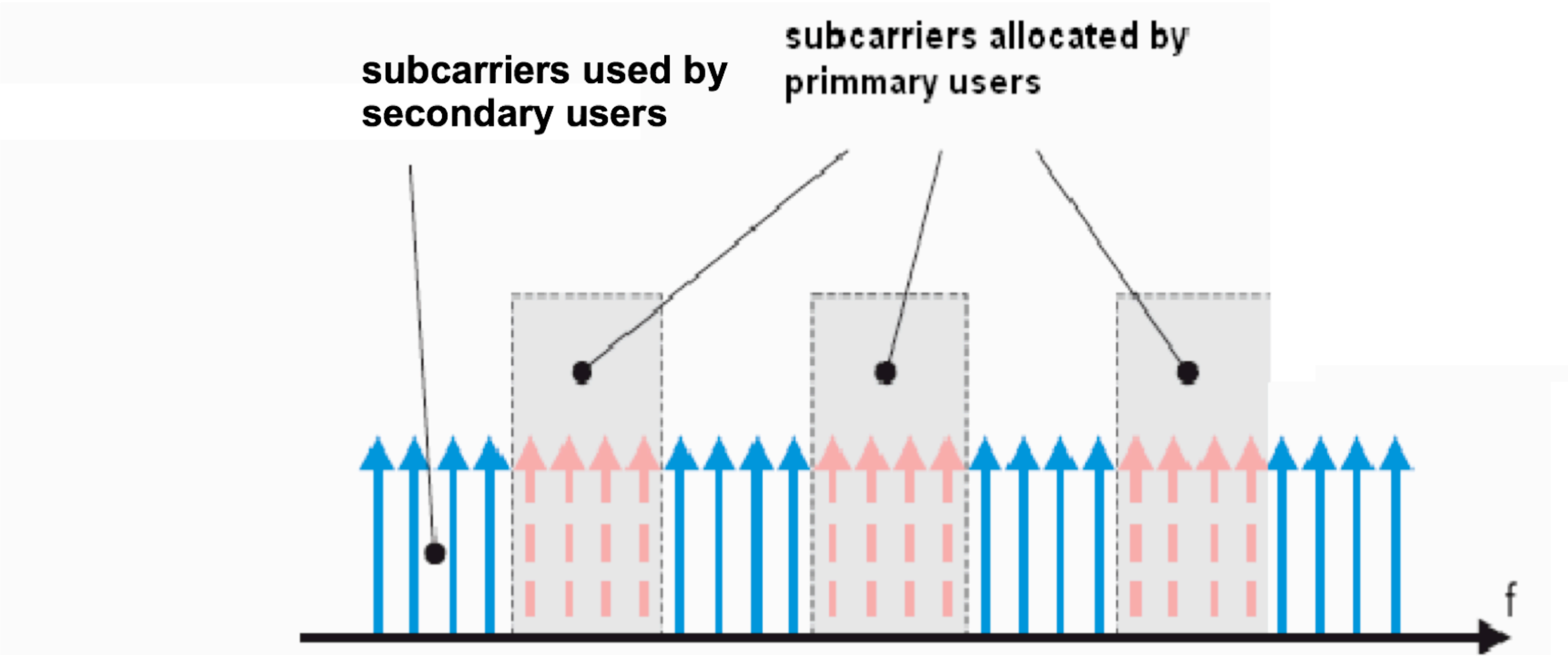


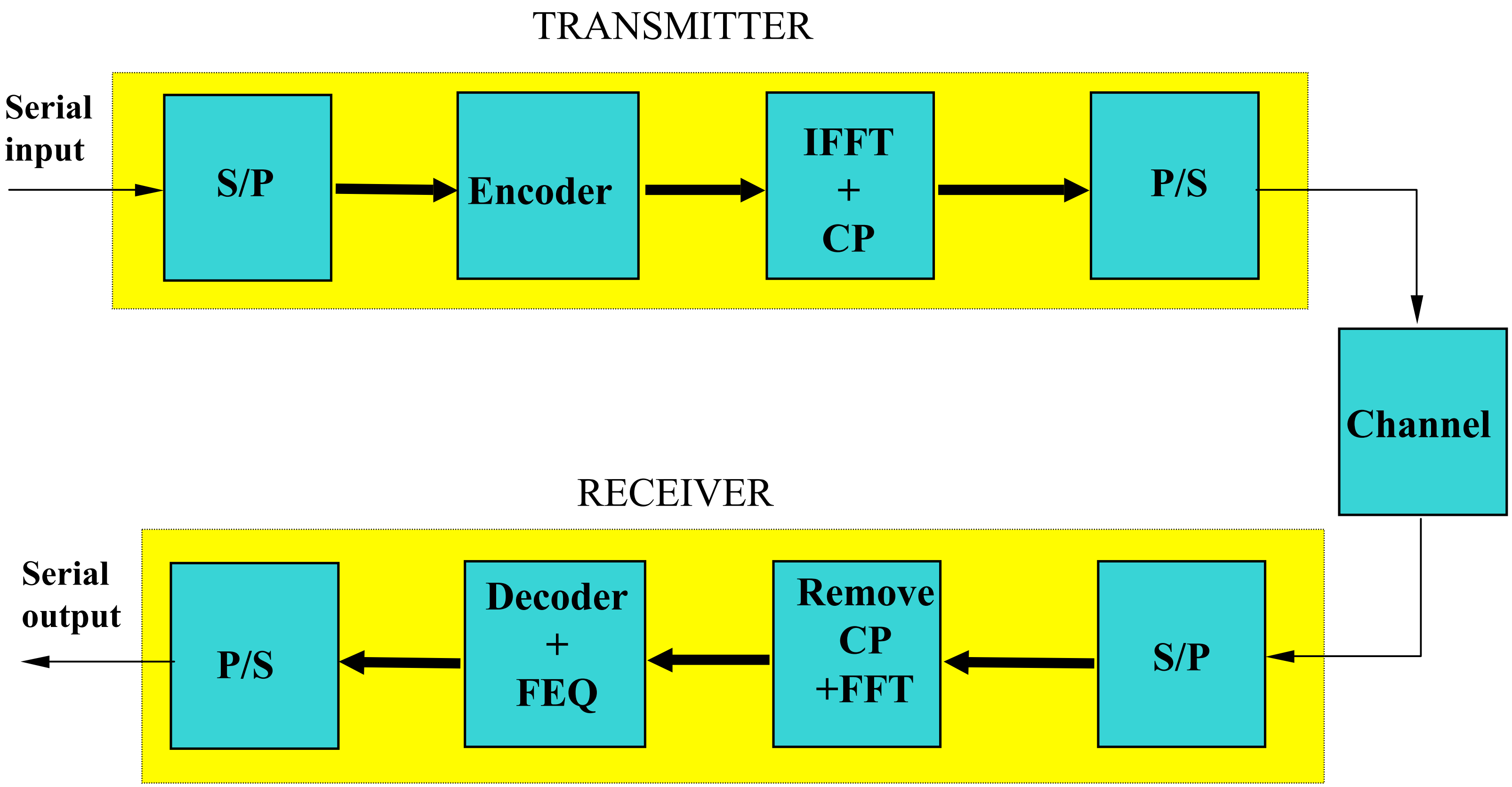
To avoid interference among primary and secondary users good separation/filtering of different subcarriers is necessary.

[WeiJon04] T.A. Weiss and F.K. Jondral, **"Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency,"** *IEEE Commun. Magazine, Vol. 42, No. 3, March 2004, pp. S8 - S14.*

Multicarrier Methods:

Conventional OFDM

FFT-based OFDM: Transceiver Structure



S/P: Serial-to-Parallel
P/S: Parallel-to-Serial

CP: Cyclic Prefix
TEQ: Time Domain Equalizer
FEQ: Frequency Domain Equalizer

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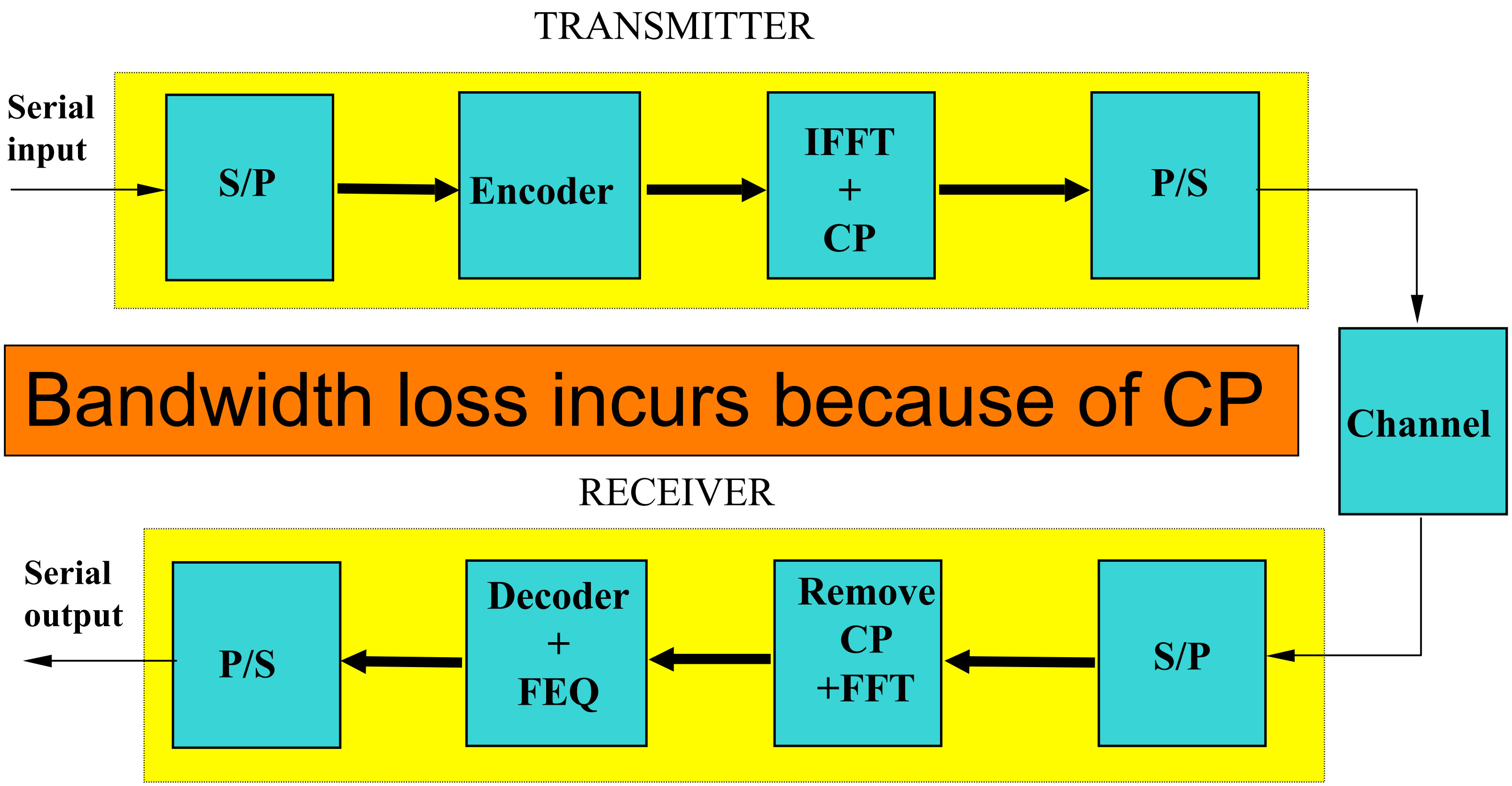
FFT-based OFDM: Transceiver Structure



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FFT-based OFDM: Parameters definition

N: The maximum number of subcarriers / FFT length

C: The number of cyclic prefix samples

T_s: The sample interval (in sec.)

$T = NT_s$: The duration of each FFT block

$T_G = CT_s$: The duration of each cyclic prefix / guard interval

$T_S = T + T_G$: The duration of each OFDM symbol

f_i: The center frequency of the *i*th subcarrier

X_i: The data symbol at the *i*th subcarrier (i.e., in freq. domain)

g(n): The symbol shaping window (n = 0, 1, …, N+C-1)

$x_i(n) = X_i g(n) e^{j2\pi(n-C-1)i/N}/N$: The ith subcarrier signal samples in time domain (before modulation to RF band); n is time index

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FFT-based OFDM: Transmit signal and its spectrum

Power spectrum of the ith subcarrier:

$$\Phi_{x_i x_i}(f) = K|G(f - f_i)|^2$$

Adding up power spectra of all active subcarriers, the power spectrum of transmit signal *x(t)* is obtained as:

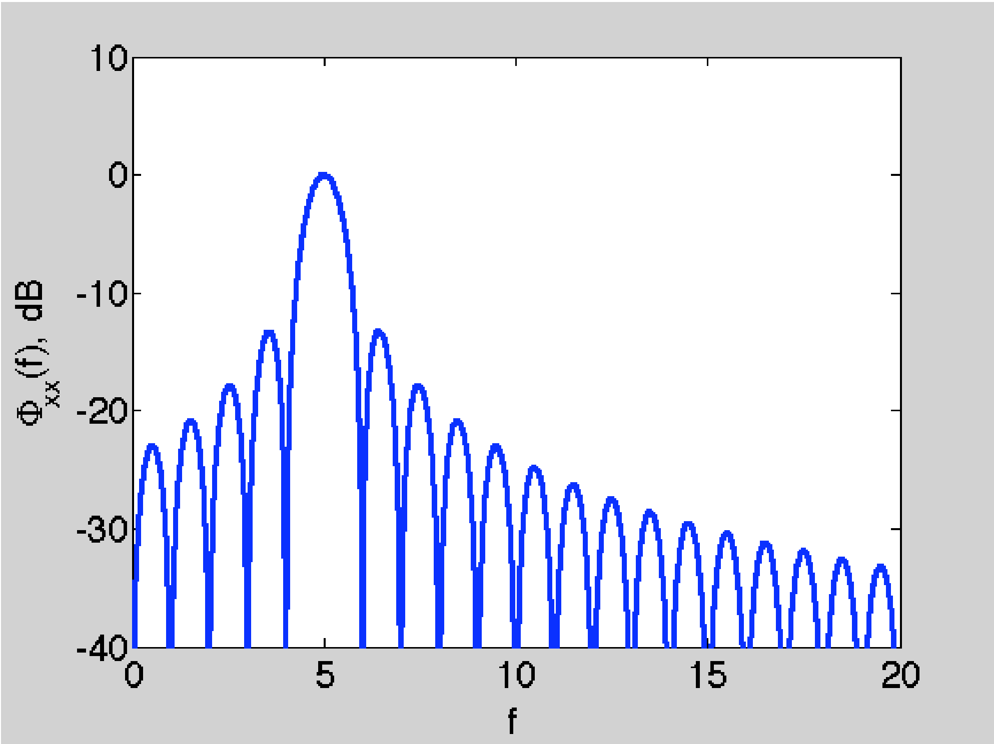
$$\Phi_{xx}(f) = \sum_i \Phi_{x_i x_i}(f)$$

[FarKem08] B. Farhang-Boroujeny and R. Kempter, "**Multicarrier communication techniques for spectrum sensing and communication in cognitive radios,**" IEEE Commun. Magazine, April 2008.

FFT-based OFDM: Transmit signal and its spectrum

In conventional OFDM, where g(n) = 1, for n = 0, 1, …, N+C-1,

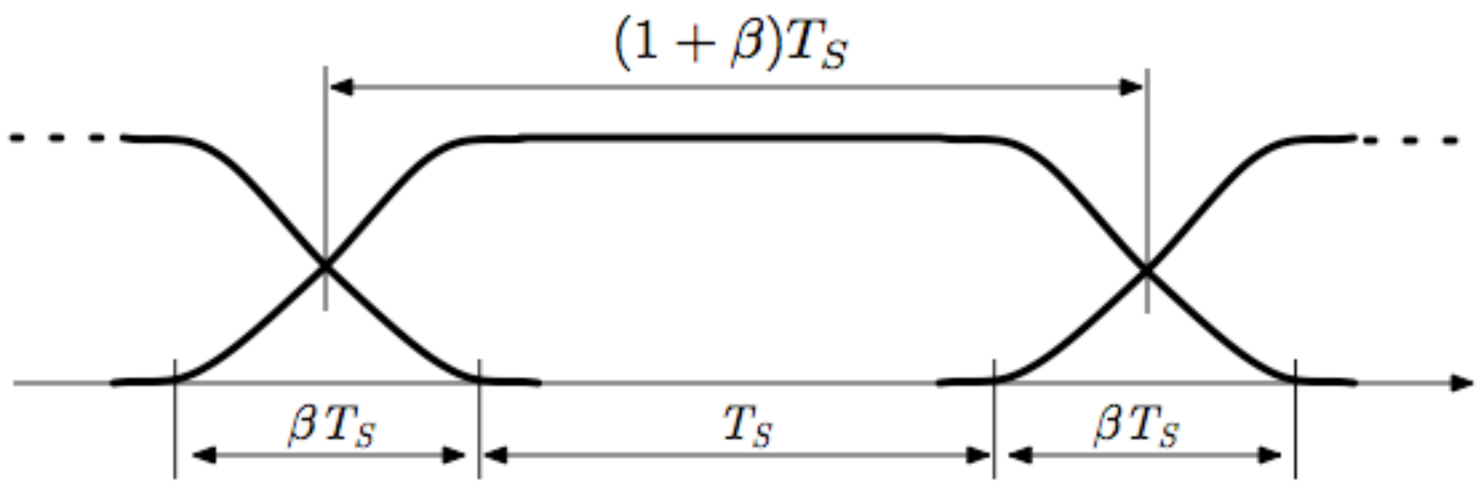
$$\Phi_{x_i x_i}(f) = K\left|\mathrm{sinc}((f - f_i)T_S)\right|^2, \text{ where } \mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$



- Poor side lobes
  - ➢ interference with PUs
- First side lobe is at -13 dB!

FFT-based OFDM: Improving the spectrum of transmit signal

- The large side lobes in the PSD of each subcarrier is a direct consequence of using a rectangular window
- The side lobes can be suppressed significantly by using a window that roles-off gently
- This increases the duration each OFDM symbol from T_S to $(1+\beta)T_S$
  - Hence, further loss in bandwidth efficiency



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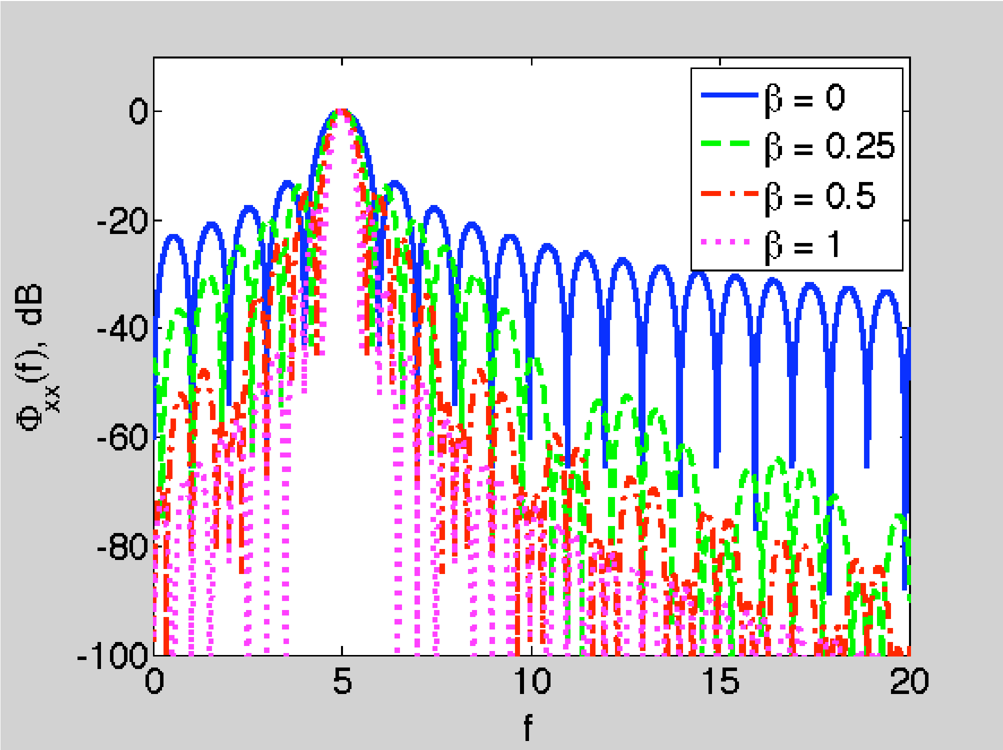
FFT-based OFDM: Improving the spectrum of transmit signal

- Side lobes are suppressed by increasing β
- To suppress side lobes sufficiently, β values of 0.5 or greater may be required (Wiess et al. (2004))
  - ➢ Hence, significant loss in bandwidth efficiency



Notes:

- The side lobes adjacent to the main lobe remain significant, even for $\beta = 1$
- To solve this problem one may introduced guard bands between PU and SU bands
  - ➢ Hence, further loss in bandwidth efficiency

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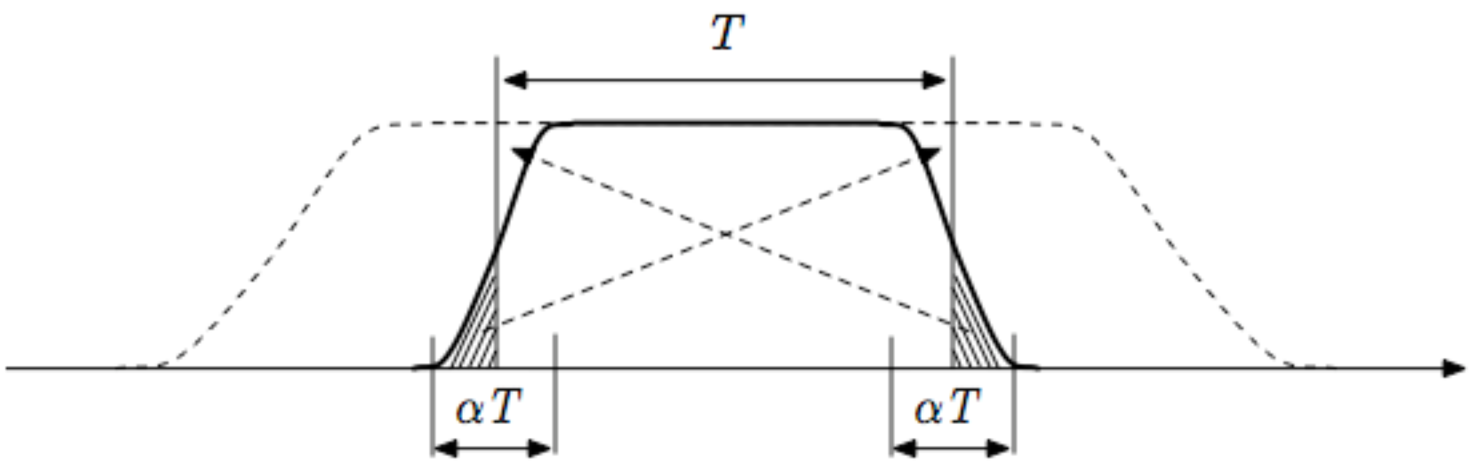
FFT-based OFDM: Interference introduced by PUs and other SUs on a SU

- Traditional OFDM applies FFT to a length N, CP-removed (rectangular) window, of the received signal.
  - This is equivalent to applying a bank of bandpass filters with modulated sinc frequency responses.
  - The large side-lobes of the sinc responses result in significant energy pick up from the bands that are unsynchronized with the intended bands.
    - Hence, significant interference will be picked up.
- **Solution:** apply a window function with gentle transition to zero, before applying FFT.
    - Further reduction in bandwidth efficiency

FFT-based OFDM: Reducing interference from PUs and other SUs on a SU



- Filtering is performed on $(1+\alpha)$N samples and the result is decimated to N samples.
  - This is achieved by performing aliasing in time domain (as shown above) and then applying an N-point FFT.

[SiIsNil99] F. Sjoberg, M. Isaksson, R. Nilsson, P. Odling, S.K. Wilson, and P.O. Borjesson, "**Zipper: A duplex method for VDSL based on DMT**," *IEEE Transactions on Communications, vol. 47, No. 8, pp. 1245-1252, Aug. 1999.*

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FFT-based OFDM: SUMMARY

Advantages

- OFDM is a well-studied method.
- OFDM chip-sets are already developed/available.
- Perfect cancellation of ISI and ICI is achieved, thanks to CP.

Disadvantages

- Hard to synchronize when subcarriers are shared among different transmitters.
- Small asynchronicity between different transmitters results in significant intercarrier interference.
- In cognitive radio, significant overhead should be added to avoid interference between primary and secondary users.

Spectral Estimation

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Spectral Estimation Methods: Parametric spectral estimation

The signal $x(n)$ whose spectrum is desired is treated as a random process and modeled as in the following figure.

- The input $\nu(n)$ is a white random process with variance of unity.
- The parameters a_k and b_k are optimized such that $x(n)$ and $\hat{x}(n)$ have the closet autocorrelation coefficients.

$\nu(n)$

$$H(z) = \frac{\sum_{k=0}^{N-1} b_k z^{-k}}{\sum_{k=0}^{M-1} a_k z^{-k}}$$

$\hat{x}(n)$

$$\Phi_{xx}(f) = \Phi_{\nu\nu}(f) \left| H\left(e^{j2\pi f}\right) \right|^2$$

$$= \left| H\left(e^{j2\pi f}\right) \right|^2$$

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Spectral Estimation Methods: Non-parametric spectral estimation

Different types of non-parametric spectral estimators:

- Periodogram Spectral Estimator (PSE)
- Blackman-Tukey Spectral Estimator (BTSE)
- Minimum Variance Spectral Estimator (MVSE)
- Maltitaper Method (MTM)
- Filter Bank Spectral Estimator (FBSE)

[LimOpp88] J.S. Lim and A.V. Oppenheim (1988), **Advanced Topics in Signal Processing**. *Prentice Hall, Englewood Cliffs, New Jersey, 1988.*

[Thom82] D.J. Thomson (1982), **"Spectrum estimation and harmonic analysis,"** *Proceedings of the IEEE, vol. 70, no. 9, pp. 1055-1096, Sept. 1982.*

[Hay05] S. Haykin (2005), **"Cognitive radio: brain-empowered wireless communications,"** *IEEE Journal Selected Areas in Communications, vol. 23, no. 3, pp. 201-220, Feb. 2005.*

Spectral Estimation Methods: Non-parametric spectral estimation

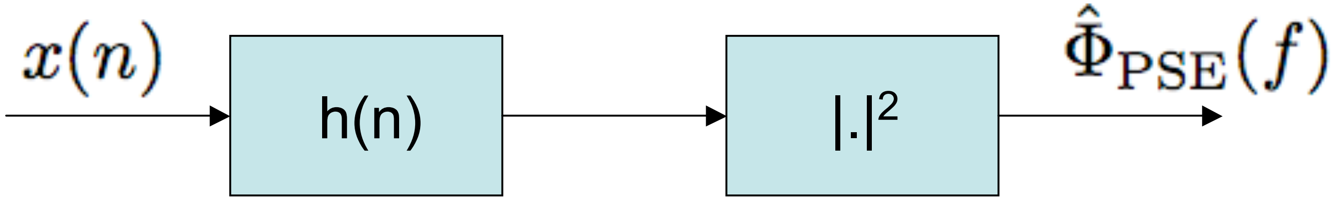
Periodogram Spectral Estimator (PSE): obtains the spectrum of a random process x(n), based on the observed samples {x(n), x(n-1), x(n-2), …, x(n-N+1)}, by evaluating the amplitude of the DFT of the observed vector

$$\mathbf{x}(n) = \begin{bmatrix} x(n) \\ x(n-1) \\ x(n-2) \\ \vdots \\ x(n-N+1) \end{bmatrix}$$

PSE as a filterbank: The process of applying DFT to the observed vector **x**(n), may be also formulated as

$$\hat{\Phi}_{\mathrm{PSE}}(f) = \left| \sum_{k=0}^{N-1} h(k)x(n-k) \right|^2$$

where $$h(k) = \frac{1}{\sqrt{N}} e^{-j2\pi f k}$$



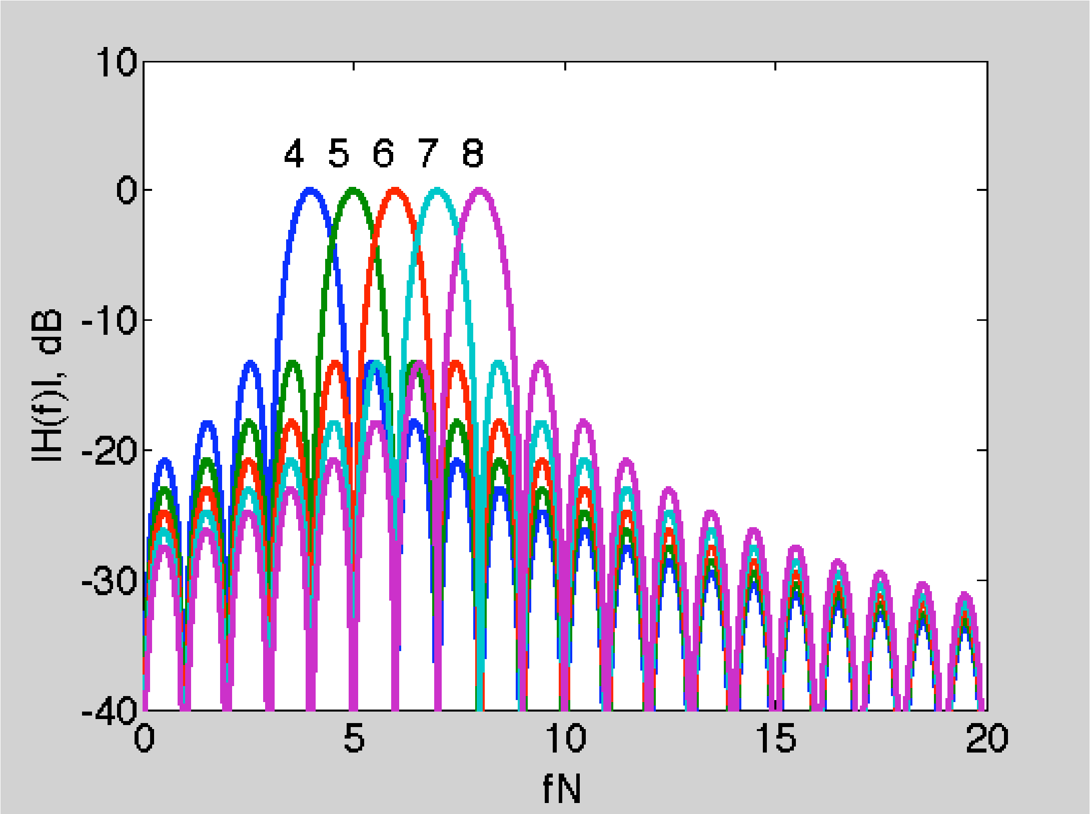
For DFT we have a bank of filters centered at

$$f = 0, \frac{1}{N}, \frac{2}{N}, \cdots, \frac{N-1}{N}$$

Spectral Estimation Methods: Non-parametric spectral estimation

DFT filters:

$$|H(f)| = \frac{1}{\sqrt{N}} \left| \frac{\sin(N\pi(f - f_i))}{\sin(\pi(f - f_i))} \right| \approx \sqrt{N}|\text{sinc}(N(f - f_i))|$$

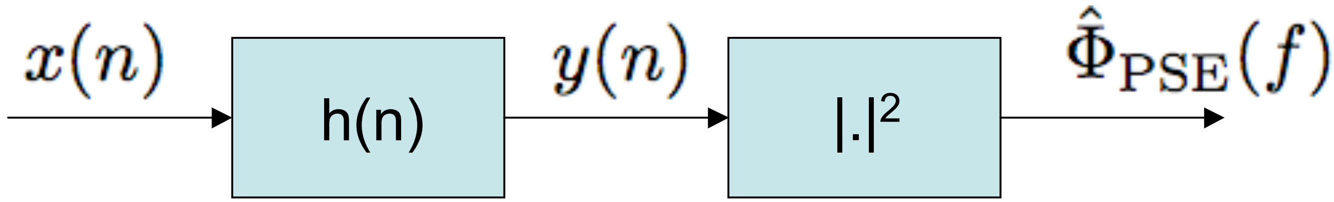


Relatively large side-lobes result in significant leakage of spectral power among different bands.

- Hence, reduces the spectral dynamic range.

Spectral Estimation Methods: Non-parametric spectral estimation

Resolution of the estimates of PSD in PSE:



Since x(n) is a random process, the filter output y(n) is a random variable.

Moreover, y(n), in general, can be approximated by a Gaussian, since it is constructed by linearly combining (a large) set of samples of x(n). Accordingly, $|y(n)|^2$ has a chi-square distribution with 2 degree of freedom, viz.,

$$\hat{\Phi}_{\mathrm{PSE}}(f) \sim \chi_2^2$$

Thus,

$$\mathrm{VAR}[\hat{\Phi}_{\mathrm{PSE}}(f)] = 2E[\hat{\Phi}_{\mathrm{PSE}}(f)]$$

Because of their large variance, the PSD estimates are **not reliable**.

Prototype filter of the DFT filterbank:

- The protoype filter in DFT has the coefficient vector

$$h_0(n) = \frac{1}{\sqrt{N}}, \quad \text{for } n = 0, 1, \cdots, N-1$$

  - The ith-band filter in DFT has the coefficient vector

$$h_i(n) = h_0(n)e^{-j2\pi in/N} = \frac{1}{\sqrt{N}}e^{-j2\pi in/N}, \quad \text{for } n = 0, 1, \cdots, N-1$$

  - The prototype filter is a lowpass filter and the ith-band filter is obtained by modulating it.
  - The prototype filter is also the 0th-band filter in the filterbank.

Spectral Estimation Methods: Non-parametric spectral estimation

Matrix formulation the DFT filterbank:

$$\begin{bmatrix} X_n(0) \\ X_n(1) \\ X_n(2) \\ \vdots \\ X_n(N-1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & e^{-i2\pi/N} & e^{-i4\pi/N} & \cdots & e^{-i2\pi(N-1)/N} \\ 1 & e^{-i4\pi/N} & e^{-i8\pi/N} & \cdots & e^{-i4\pi(N-1)/N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-i2\pi(N-1)/N} & e^{-i4\pi(N-1)/N} & \cdots & e^{-i\pi(N-1)^2/N} \end{bmatrix}}_{\text{DFT matrix}}$$

$$\times \left(\underbrace{\begin{bmatrix} \frac{1}{\sqrt{N}} \\ \frac{1}{\sqrt{N}} \\ \frac{1}{\sqrt{N}} \\ \vdots \\ \frac{1}{\sqrt{N}} \end{bmatrix}} \cdot \begin{bmatrix} x(n) \\ x(n-1) \\ x(n-2) \\ \vdots \\ x(n-N+1) \end{bmatrix} \right)$$

DFT matrix

May be viewed as a rectangular window/prototype filter coefficients that are applied to input samples before taking the Fourier transform.

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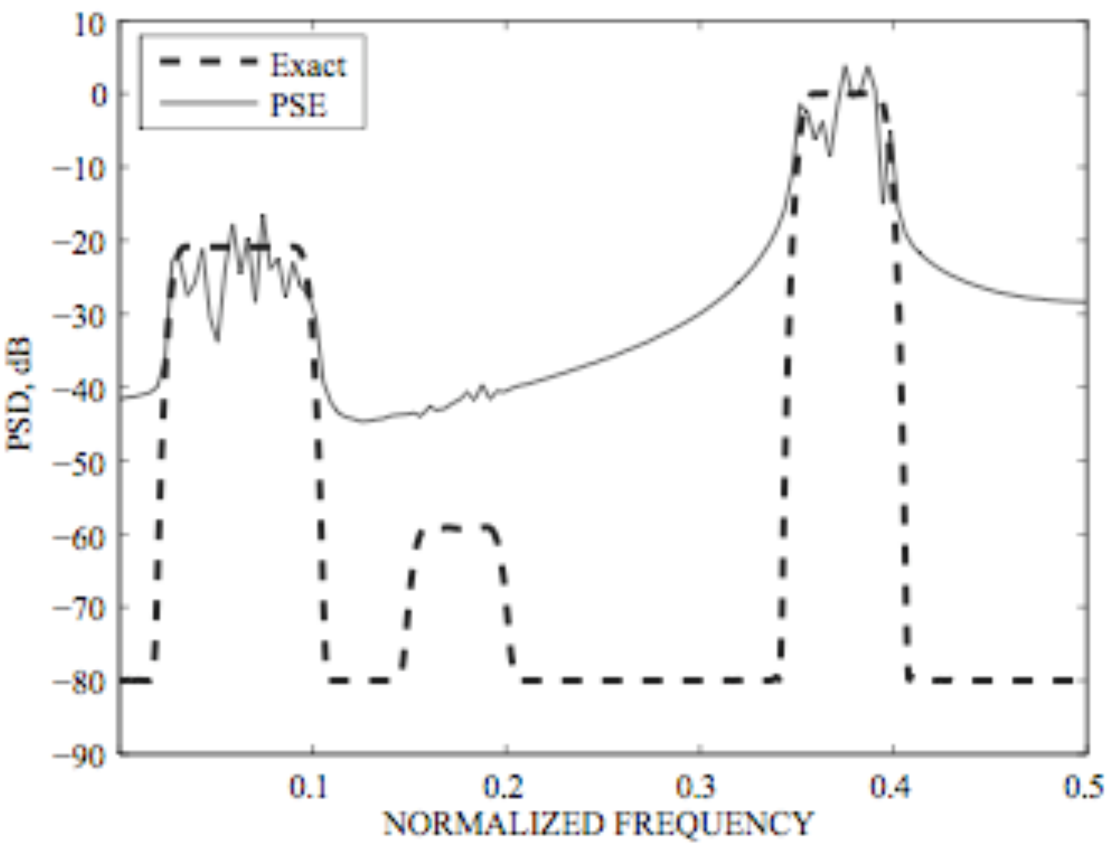
Spectral Estimation Methods: Non-parametric spectral estimation

A snapshot of PSE:



Notes:

- DFT size N = 256.
- The expected large variance of the estimates is seen.
- Small spectral dynamic range of PSE is a direct consequence of large side lobes in DFT filters.

Blackman-Tukey Spectral Estimator (BTSE):

- The large side lobes in PSE arises because of the use of a poor prototype filter.
  - The problem can be resolved by improving the prototype filter, equivalent to applying a window function before filtering.
  - Window function can be directly applied to the signals samples or to the auto-correlation coefficients of the signal samples.
  - When window function is applied to the auto-correlation coefficients of the signal samples, the resulting method called BTSE.

Blackman-Tukey Spectral Estimator (BTSE): Examples of common windows (length input signal sample = M+1, N = 2M+1)

- Rectangular:

$$w(k) = \begin{cases} \frac{1}{\sqrt{2M+1}}, & |k| \le M \\ 0, & |k| > M \end{cases} \qquad W(\omega) = W_{\mathrm{rect}}(\omega) = \frac{\sin\frac{\omega}{2}(2M+1)}{\sqrt{2M+1}\sin\frac{\omega}{2}}$$

- Hanning:

$$w(k) = \begin{cases} \frac{1}{2\sqrt{2M+1}}(1+\cos\frac{\pi k}{M}), & |k| \le M \\ 0, & |k| > M \end{cases}$$

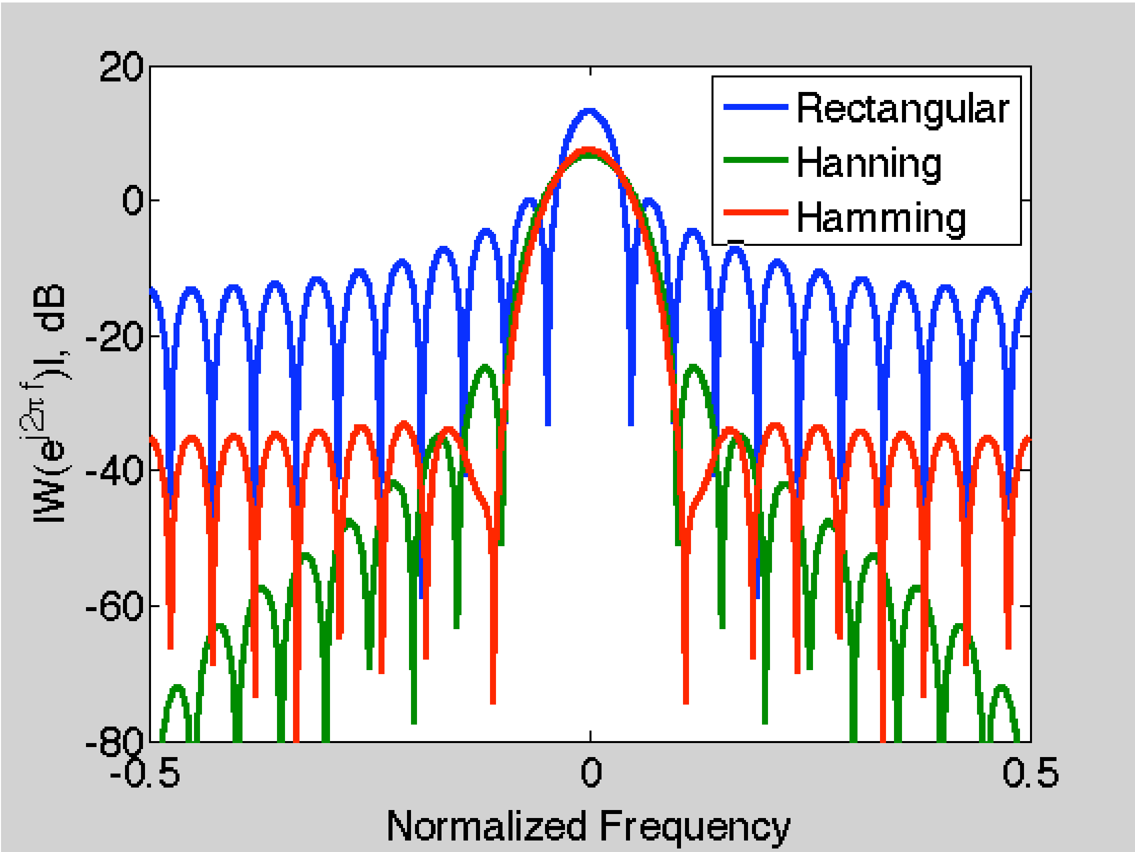
$$W(\omega) = \frac{1}{4}W_{\mathrm{rect}}(\omega - \frac{\pi}{M}) + \frac{1}{2}W_{\mathrm{rect}}(\omega) + \frac{1}{4}W_{\mathrm{rect}}(\omega + \frac{\pi}{M})$$

- Hamming:

$$w(k) = \begin{cases} \frac{1}{\sqrt{2M+1}}(0.54+0.46\cos\frac{\pi k}{M}), & |k| \le M \\ 0, & |k| > M \end{cases}$$

$$W(\omega) = 0.23W_{\mathrm{rect}}(\omega - \frac{\pi}{M}) + 0.54W_{\mathrm{rect}}(\omega) + 0.23W_{\mathrm{rect}}(\omega + \frac{\pi}{M})$$

Blackman-Tukey Spectral Estimator (BTSE): Examples of common windows



Note: side lobes have decreased at the cost of wider main lobe.

- Reduction of Leakage among different bands, i.e., increased spectral dynamic range, is traded at the cost of a lower resolution in frequency.

Minimum Variance Spectral Estimator (MVSE):

- Each point of PSD is estimated using a different filter.
- These filters are adopted to the spectrum whose estimate is desired.
- Each filter is selected to have a gain of unity at the center of the passband, while the side lobes are optimized for minimum leakage of energy from other bands.
- **We do not explore this method as a good candidate for spectrum sensing in CR, mostly because of its computational complexity.**

Multitaper Method (MTM):

- This method replaces the single (prototype) filter in the previous methods by a few filters for measurement of each point of PSD.
- All filters have the same passband. However, by design they are orthogonal, hence, their outputs are a set of uncorrelated random variables.
- The output power of the filters are averaged to reduce the variance of the estimates.
- A set of prototype filters are used for all the bands and polyphase architecture is used for efficient implementation.
- The prototype filters are a set of **prolate filters** that satisfy some desirable properties/optimality conditions, as discussed in the next slide.

[Thom82] D.J. Thomson (1982), **"Spectrum estimation and harmonic analysis,"** *Proceedings of the IEEE, vol. 70, no. 9, pp. 1055-1096, Sept. 1982.*

Origin of the Prolate Filters: *Slepian Sequences*

- The Slepian sequences $\mathbf{s}_k=[s_k(1), s_k(2), \ldots, s_k(N)]^T$, k=1, 2, …, K constitute a set of K unit length orthogonal bases vectors which are used to obtain an optimal expansion of the time sequence

$$\mathbf{x}(n)=[x(n-N+1), x(n-N+2), \ldots, x(n-1), x(n)]^T$$

over the frequency band $[f_i-\Delta f/2, f_i+\Delta f/2]$.

- The expansion of x(n) has the form of

$$\mathbf{x}(n) = \kappa_1\mathbf{s}_1+ \kappa_2\mathbf{s}_2+\ldots+ \kappa_K\mathbf{s}_K, \qquad \kappa_k = \mathbf{s}_k^H\mathbf{x}(n)$$

- To maximize the accuracy of the estimate, the Slepian sequences are chosen such that their spectrum is maximally concentrated over the desired band $[f_i-\Delta f/2, f_i+\Delta f/2]$. This can be related to the ***minimax theorem.***

Minimax Theorem:

The distinct eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_N$ of the correlation matrix $\mathbf{R}$ of an observation vector x(n), and their corresponding eigenvectors, $\mathbf{q}_1$, $\mathbf{q}_2$, …, $\mathbf{q}_N$, may be obtained through the following optimization procedure:

$$\lambda_1 = \max E[|\mathbf{q}_1^H \mathbf{x}(n)|^2], \text{ subject to } \mathbf{q}_1^H \mathbf{q}_1 = 1$$

and for k = 2,3, …, N

$$\lambda_k = \max E[|\mathbf{q}_k^H \mathbf{x}(n)|^2], \text{ subject to } \mathbf{q}_k^H \mathbf{q}_k = 1 \text{ and } \mathbf{q}_k^H \mathbf{q}_i = 1, \text{ for } i=1,2,\ldots,k-1$$

Alternatively, the following procedure may also be used:

$$\lambda_N = \min E[|\mathbf{q}_N^H \mathbf{x}(n)|^2], \text{ subject to } \mathbf{q}_N^H \mathbf{q}_N = 1$$

and for k = N-1,…, 2,1

$$\lambda_k = \min E[|\mathbf{q}_k^H \mathbf{x}(n)|^2], \text{ subject to } \mathbf{q}_k^H \mathbf{q}_k = 1 \text{ and } \mathbf{q}_k^H \mathbf{q}_i = 1, \text{ for } i=N,N-1,\ldots,k+1$$

[Far98] B. Farhang-Boroujeny, **Adaptive Filters: Theory and Applications.** John Wiley & Sons, 1998.

Prolate Filters Design:

1. Construct the correlation matrix **R** of a random process with the power spectral density

$$\Phi_{xx}(f) = \begin{cases} 1, & |f| < \frac{\Delta f}{2} \\ 0, & \text{otherwise} \end{cases}$$

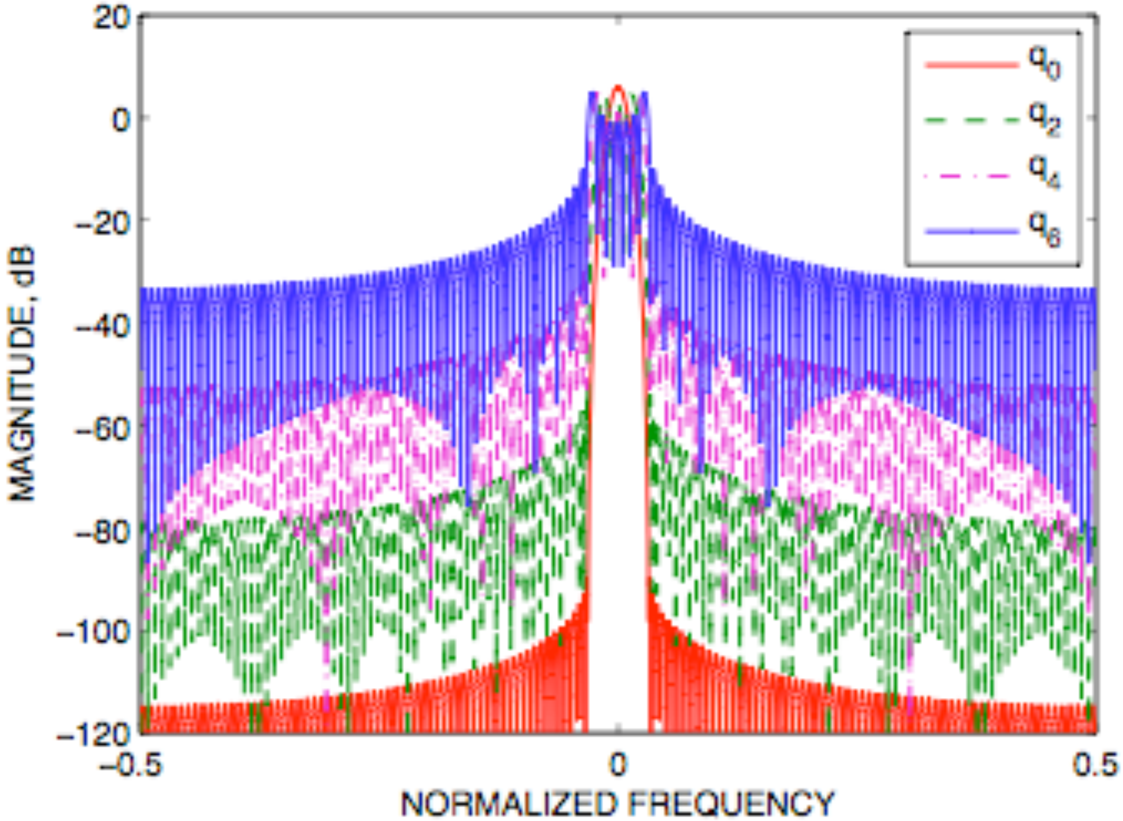
R is a Toeplitz matrix with the first row of

$$[\phi(0)\ \phi(1)\ \cdots,\ \phi(N-1)],\ \text{with}\ \phi(n) = \Delta f \text{sinc}(\Delta f n)$$

2. The first K eigenfilters corresponding to the largest eigenvalues of **R** are the coefficient vectors of the prolate filters.

Spectral Estimation Methods: Non-parametric spectral estimation

An Example of Prolate Filters:



Filter parameters:

- Number of subbands: N=16
- Filter length: L=8N=128

Observations:

- Only the first few filters have good stopband attenuation.
  - ➢Hence, to have a good spectral dynamic range, only the outputs of the first few prolate filters can be used.

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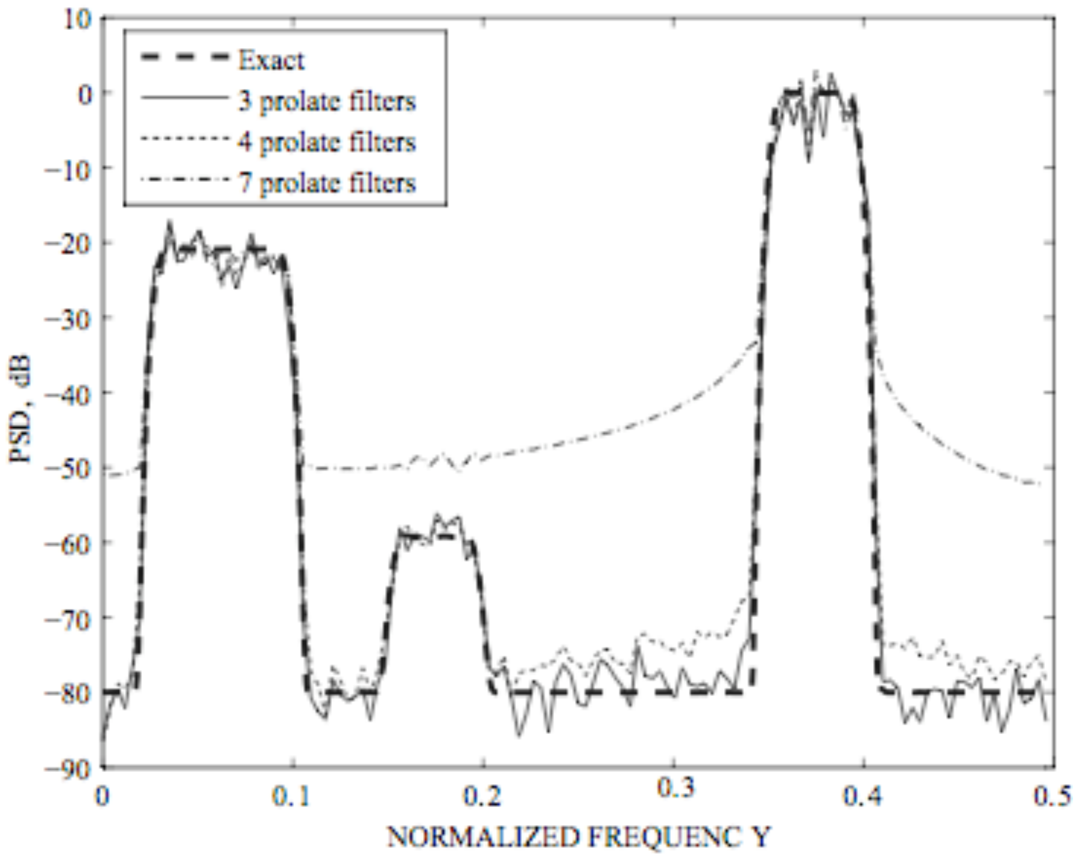
An Example of Spectrum Sensing Using Prolate Filters:



Filter parameters:

- Number of subbands: N=128
- Filter length: L=8N=1024

Observation:

- As predicted, to have a good spectral dynamic range (say, 60 dB or better), not more than three prolate filters should be used.

Adaptive MTM: Thompson proposed the following formulae in order to reduce the impact of poor side lobes of higher numbered prolate filters:

$$\hat{S}(f) = \frac{\sum_{k=0}^{K-1} |d_k(f)|^2 \hat{S}_k(f)}{\sum_{k=0}^{K-1} |d_k(f)|^2} \quad \text{and} \quad d_k(f) = \frac{\sqrt{\lambda_k} S(f)}{\lambda_k S(f) + B_k(f)}$$

Leakage power from other bands

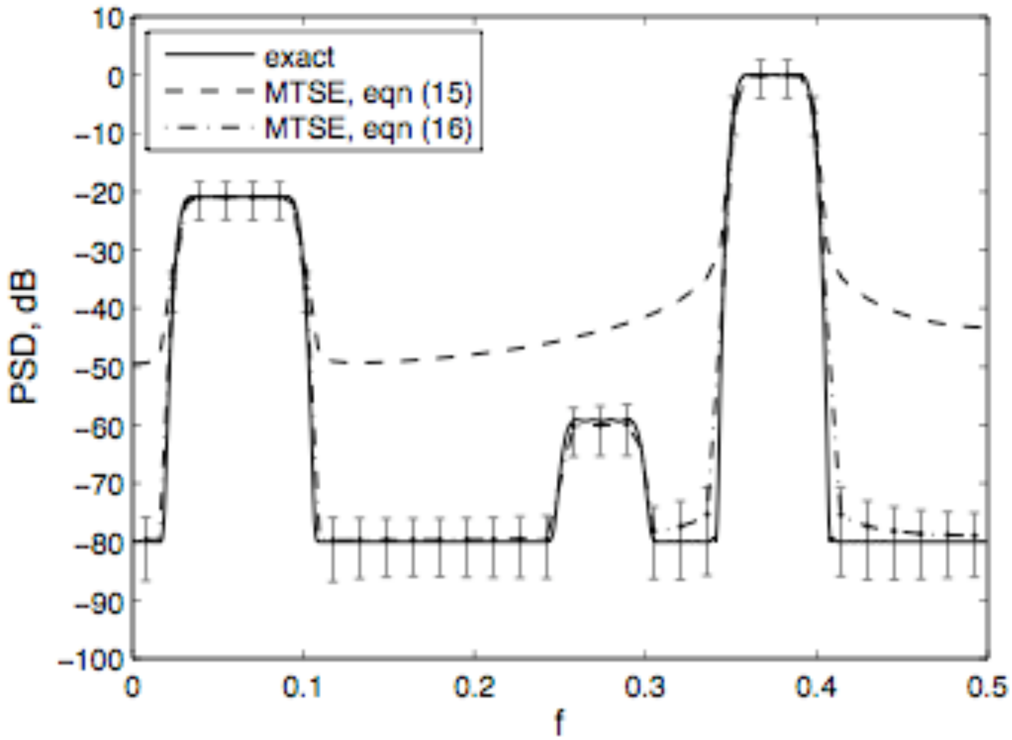
Starting with coarse estimate of S(f), $d_k(f)$ is estimated and used to improve the estimate of S(f). Iterations continue until S(f) converges.

A very complex procedure!

[Thom82] D.J. Thomson (1982), **"Spectrum estimation and harmonic analysis,"** *Proceedings of the IEEE, vol. 70, no. 9, pp. 1055-1096, Sept. 1982.*

Spectral Estimation Methods: Non-parametric spectral estimation

An Sample result of adaptive MTM:



- Results are average of 10,000 snapshots.
- Vertical lines indicate 95% confidence intervals.
- Variances are larger at lower levels of PSD.

An Sample result of adaptive MTM (continued):

Effective Degree of Freedon (EDF):

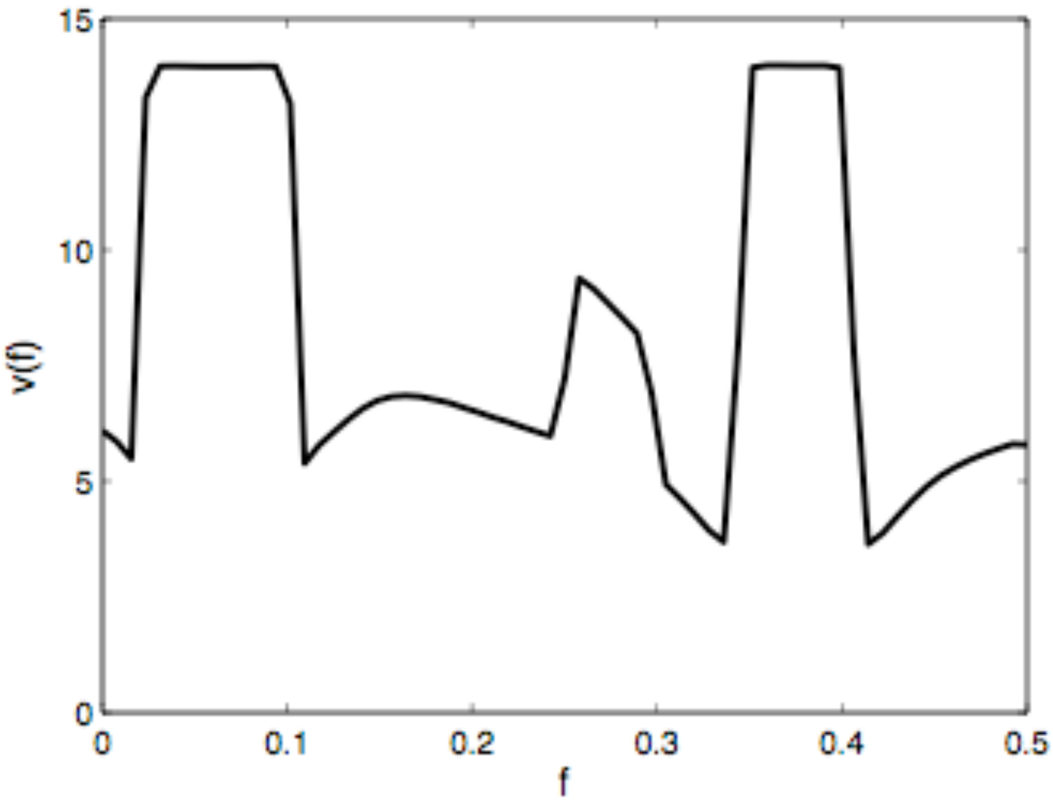
$$v(f) = 2\sum_{k=0}^{K-1} |d_k(f)|^2$$

When all the averaging factors $|d_k(f)|^2$ are equal, there are K complex Gaussian random variables ($S_k(f)$'s) whose magnitudes are averaged. This results in a chi-square random variable with 2K degrees of freedom: $v(f) = 2K$.

When a few of the factors $|d_k(f)|^2$ are zero, the degree of the freedom decreases accodingly, and thus the variance of the estimated spectrum, S(f), increases.

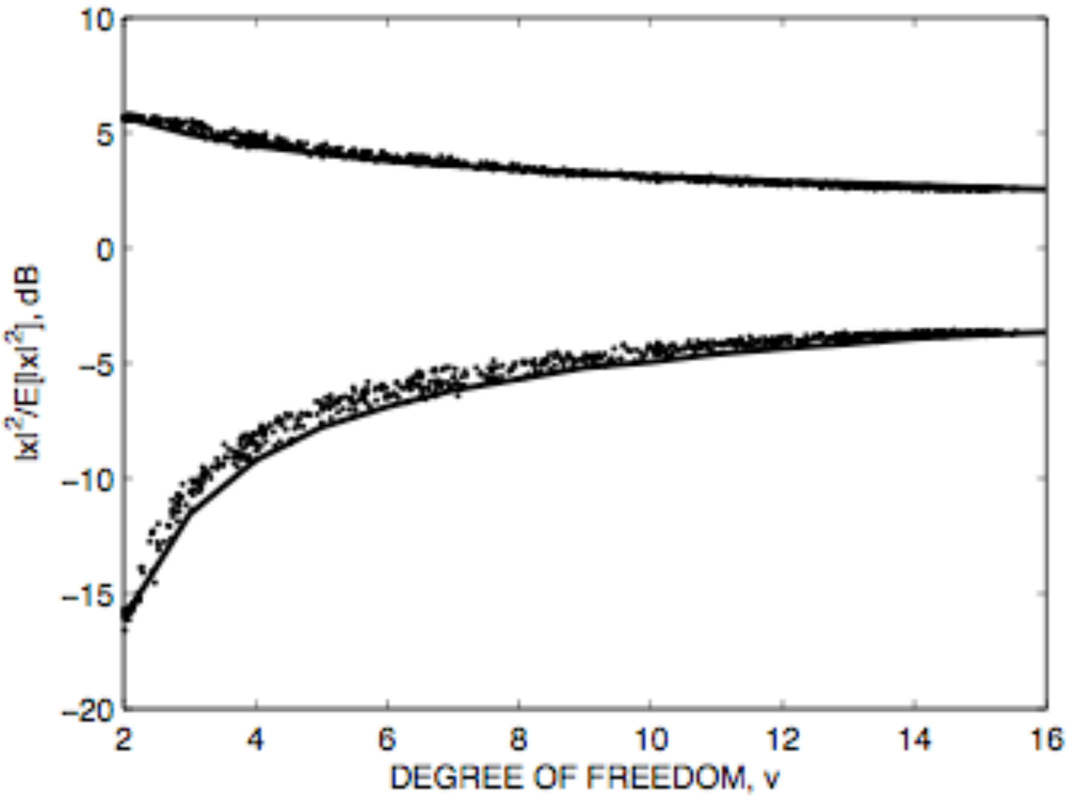
An Sample result of adaptive MTM (continued):

Effective Degree of Freedon (EDF) for the PSD results presented earlier.



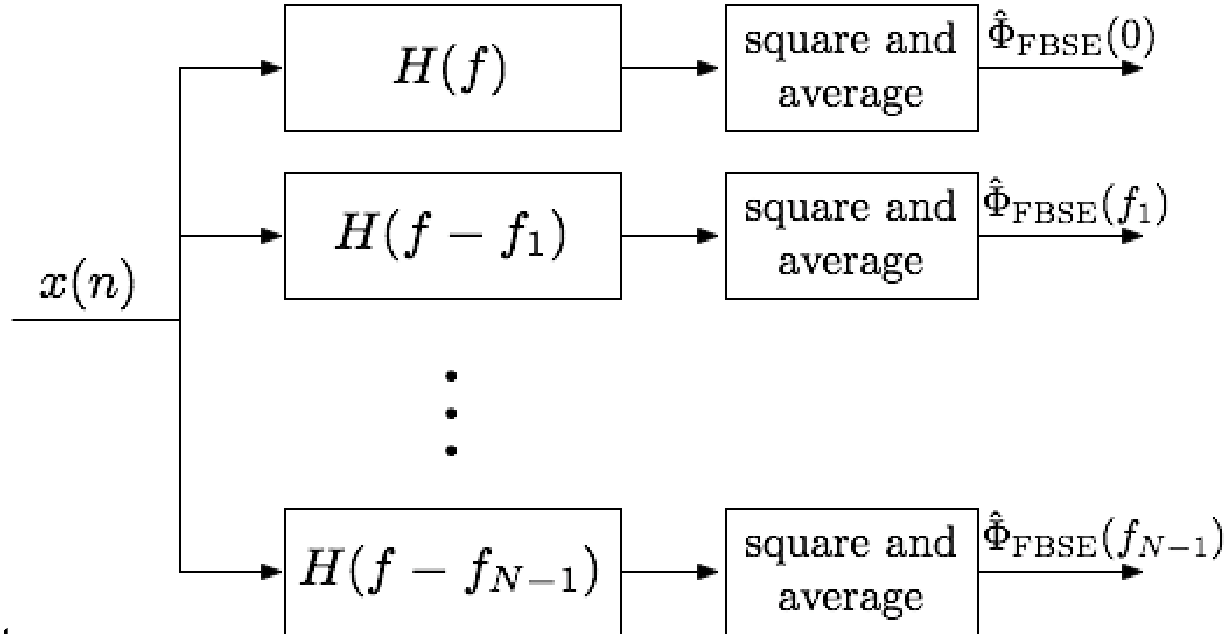
An Sample result of adaptive MTM (continued):

The 95% boundary limits of a chi-square distribution with various degrees of freedom.



Spectral Estimation Methods: Non-parametric spectral estimation

Filter Bank Spectral Estimator (FBSE):



Notes:

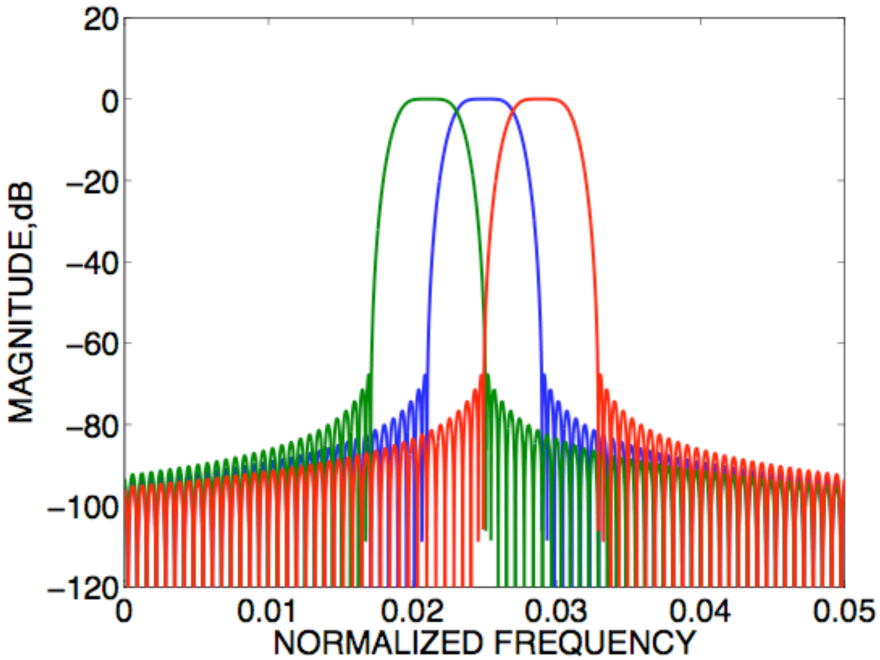
- H(f), known as prototype filter, is centered around f = 0.
- The rest of the filters are frequency-shifted/modulated copies of H(f).

[Far08]	B. Farhang-Boroujeny, **"Filter bank spectrum sensing for cognitive radios,"** IEEE Trans. On Signal Processing, May 2008.

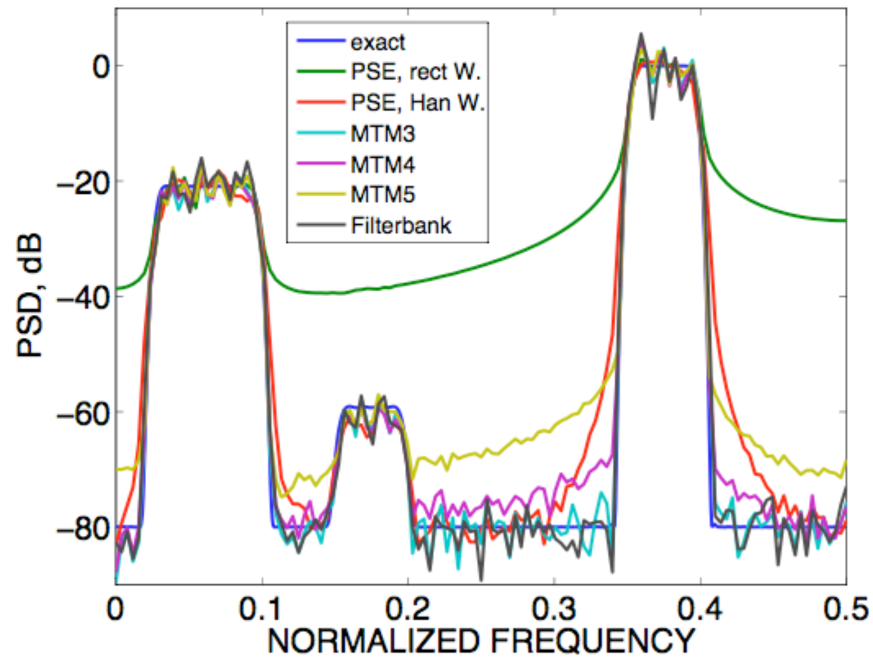
Spectral Estimation Methods: Non-parametric spectral estimation

An Example of Spectrum Sensing Using FBSE:

Magnitude responses of three bandpass filters in a filter bank



Snapshots of FBSE and other techniques



Filter parameters:

- Number of subbands: N=256; Filter length: L=6N=1536;
- The data length for FBSE is 8N = 2048; Output power of each subband is measured by averaging over three samples at spacing 256.
- The data lengths for PSE and MTM are 256 and 1024 samples, respectively.

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CONCLUSIONS:

- Periodogram spectral estimation may be insufficient to achieve the required spectral dynamic range.
- The multitaper method gives excellent results, with limited length of data. However, it is too complex to implement.
- Filter bank spectral estimation gives excellent result. Yet, it may cost very little if filter bank multicarrier is adopted for signal modulation.

Filter Bank Multicarrier (FBMC) Methods

Filter Bank Multicarrier (FBMC) Methods

- **Filtered Multitone (FMT):** Uses subcarrier bands with no overlap. Data symbols are quadrature amplitude modulated (QAM).
  - Guard bands are used to separate subcarrier bands. This results in some loss in bandwidth efficiency

- **Multicarrier with Offset QAM/Staggered Modulated Multitone (SMT):** Subcarrier bands are maximally overlapped/minimally spaced.
  - Carrier spacing = symbol rate

- **Cosine Modulated Multitone (CMT):** Uses pulse amplitude modulated (PAM) symbols with vestigial sideband modulation. Subcarrier bands are maximally overlapped / minimally spaced.
  - Carrier spacing = one half of symbol rate

Both SMT and CMT achieve **maximum bandwidth efficiency**

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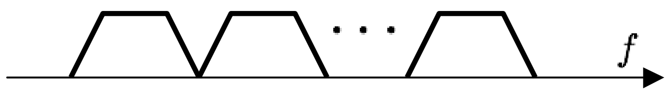
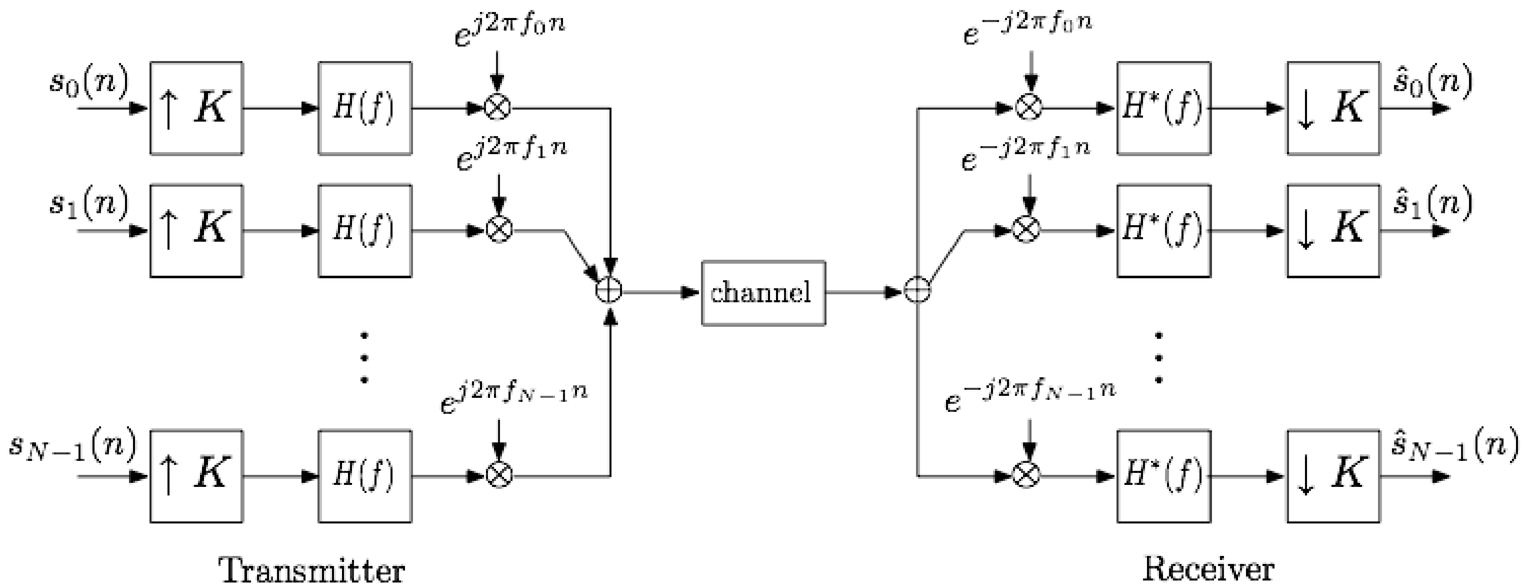
Filter Bank Multicarrier (FBMC) Methods: FMT (Summary)

- FMT follows the simple principles of the conventional frequency division multiplexing (FDM).
  - ➢ Subcarrier bands have no overlap.



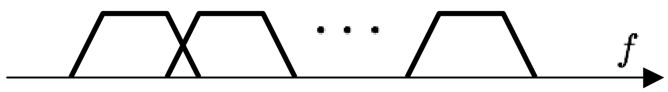
- To allow an efficient implementation based on polyphase structures, (i) some restrictions on the position of subcarrier bands are imposed; (ii) a prototype filter is used for all subcarrier bands.
- Transmit symbols, in general, are QAM and H(f) and H*(f) are a pair of root-Nyquist filters.
- Equalizers are needed after decimators at the receiver.
- A choice of K > N allows addition of guard bands between subcarrier bands.



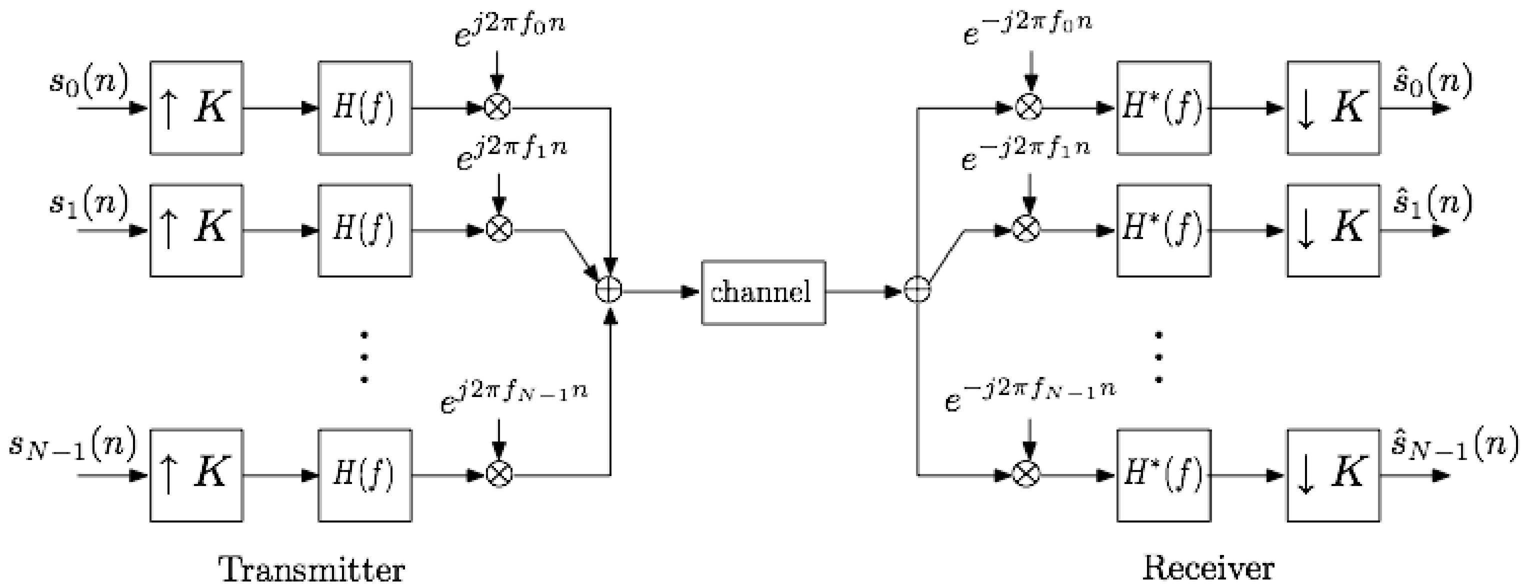
[CherEle99] G. Cherubini, E. Eleftheriou, S. Olcer (1999), **"Filtered multitone modulation for VDSL,"** *in Proc. IEEE Globecom 99, vol. 2, pp. 1139-1144, 1999.*

Filter Bank Multicarrier (FBMC) Methods: SMT (Summary)

- SMT allows overlap of adjacent bands.
  - Maximizes bandwidth efficiency (K = N)



- Transmit symbols are offset QAM: in-phase and quadrature components have a time offset of half symbol interval (not shown below), i.e., time staggered.
- If the overlaps are limited to adjacent bands and H(f) and H*(f) are a pair of root-Nyquist filters, separation of data symbols at the receiver output is guaranteed.
- Equalizers are needed after decimators at the receiver.



[Sal67] B.R. Saltzberg (1967), **"Performance of an efficient parallel data transmission system,"** *IEEE Trans. Comm. Tech.,vol. 15, no. 6, pp. 805-811, Dec. 1967.*

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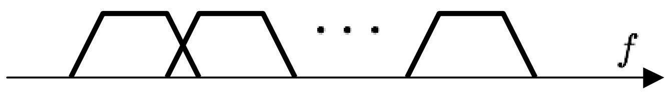
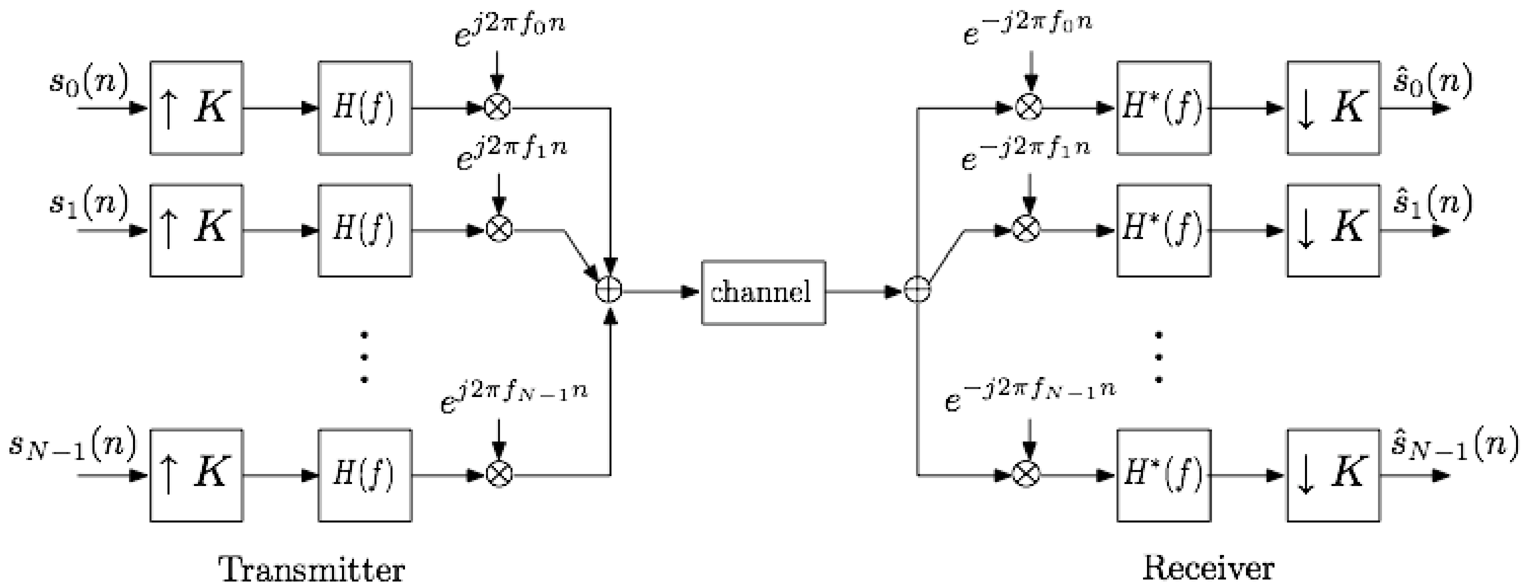
Filter Bank Multicarrier (FBMC) Methods: CMT (Summary)

- CMT allows overlap of adjacent bands.
  - ➢ Maximizes bandwidth efficiency (K = N)



- Transmit symbols are PAM (pulse amplitude modulated). To allow maximum bandwidth efficiency, vestigial sideband modulation is adopted.
- The overlaps are limited to adjacent bands to simplify filter designs. Here, also, selection of root-Nyquist filters for H(f) and H*(f) guarantees separation of data symbols at the receiver.
- Equalizers are needed after decimators at the receiver.



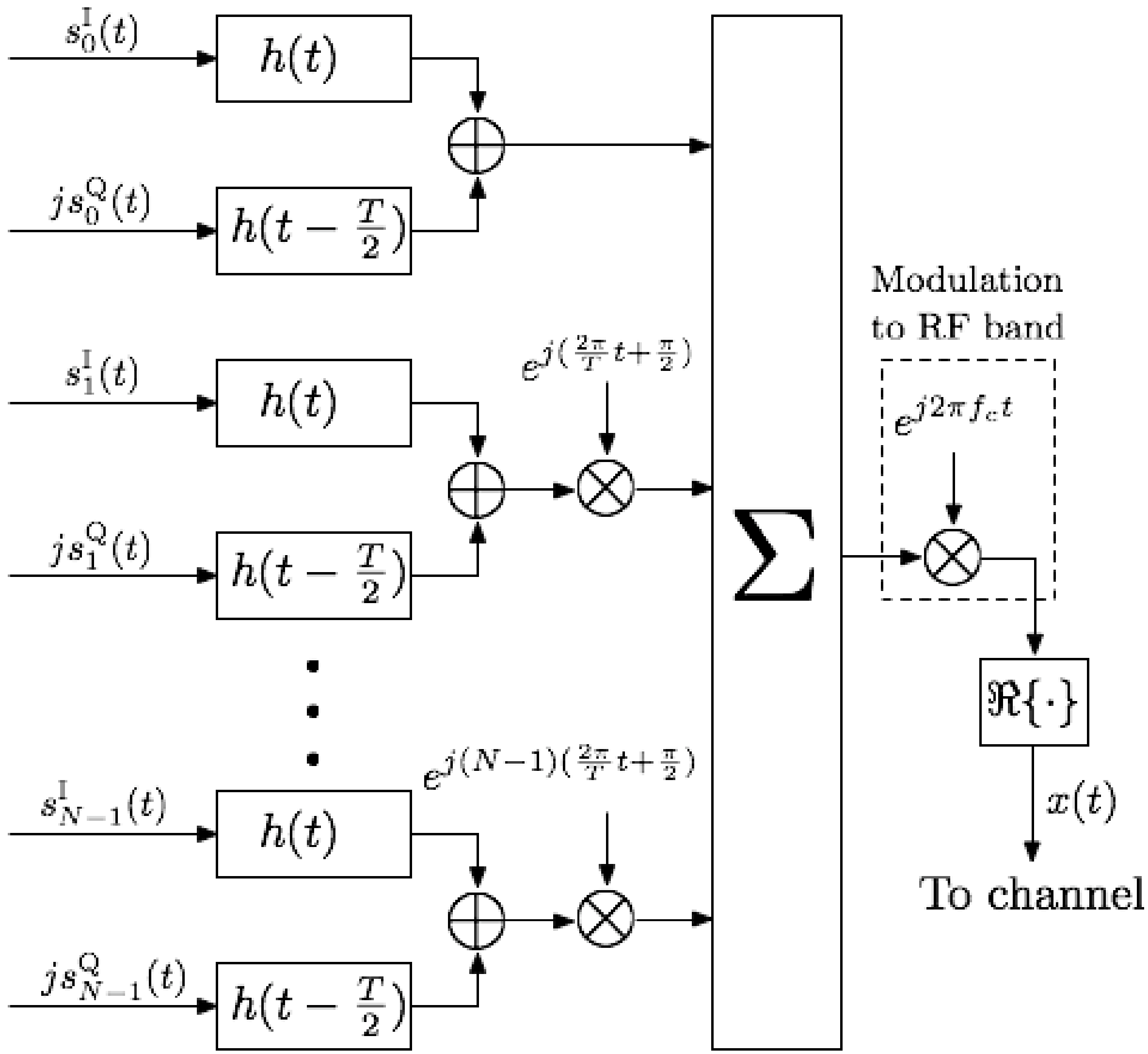
[Far03] B. Farhang-Boroujeny (2003), **"Multicarrier modulation with blind detection capability using cosine modulated filter banks,"** *IEEE Trans. Commun., vol. 51, pp. 2057-2070, Dec. 2003.*

Staggered Modulated Multitone (SMT) Details

Filter Bank Multicarrier (FBMC) Methods: SMT (Synthesis/TX)

$$s_k(t) = \sum_{n=-\infty}^{\infty} s_k[n]\delta(t - nT)$$

$$s_k[n] = s_k^{\mathrm{I}}[n] + js_k^{\mathrm{Q}}[n]$$



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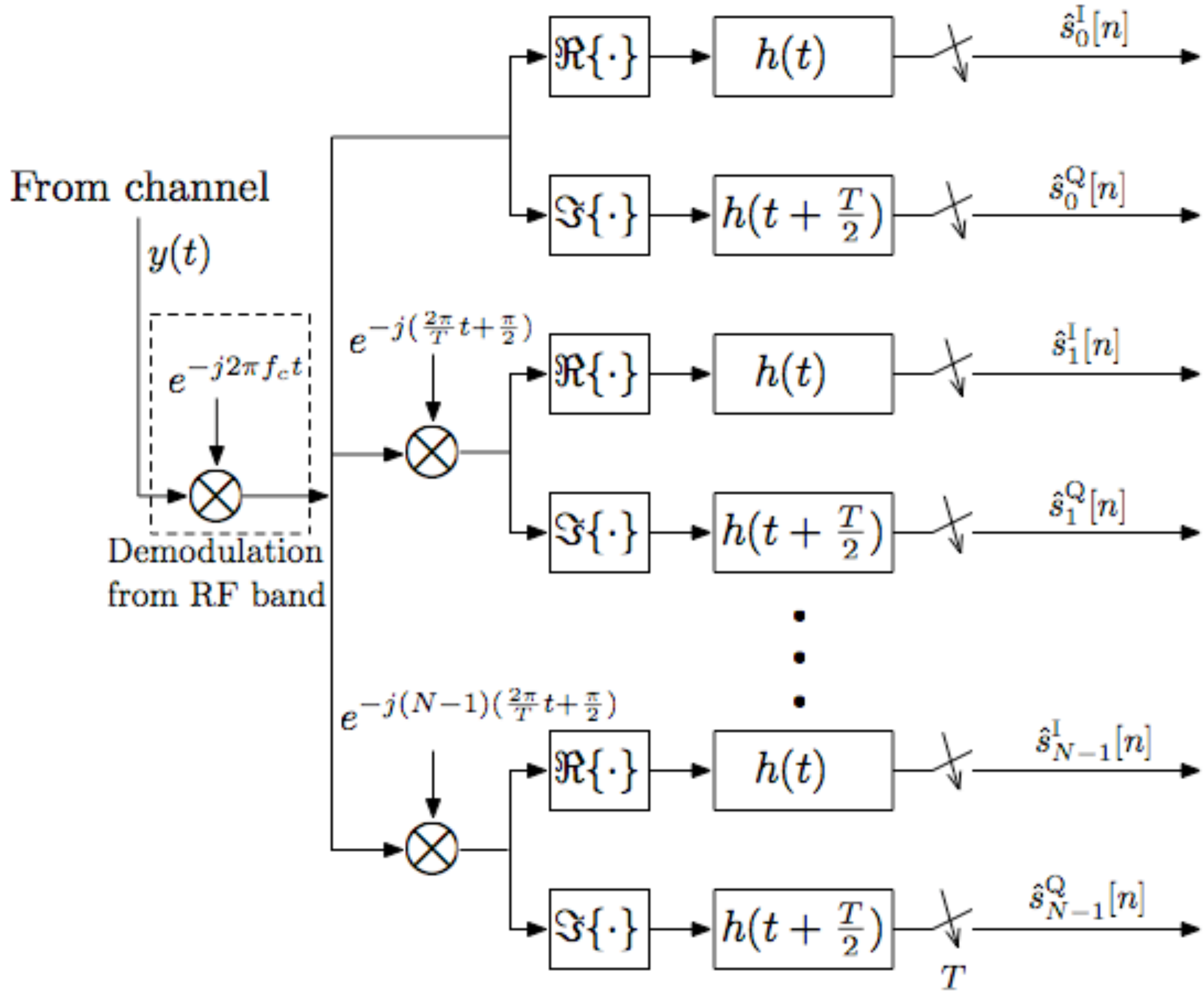
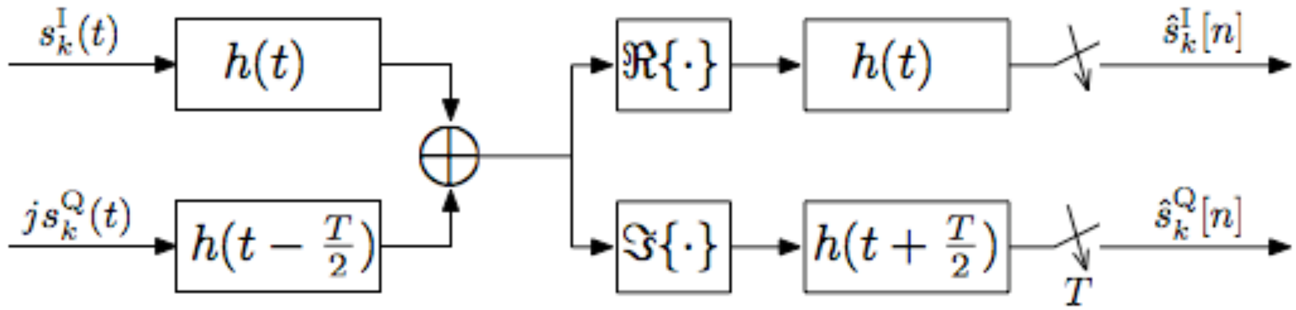
Filter Bank Multicarrier (FBMC) Methods: SMT (Analysis/RX)

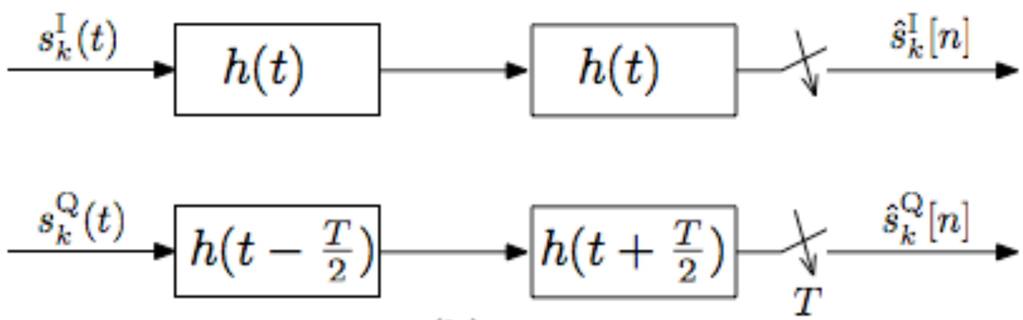


Filter Bank Multicarrier (FBMC) Methods: SMT (Details)

Intersymbol Interference (ISI):



(a)



(b)

- (b) follows from (a) because h(t) is a real-valued function.
- To achieve ISI free transmission h(t) must be a square-root Nyquist and symmetric pulse shape.

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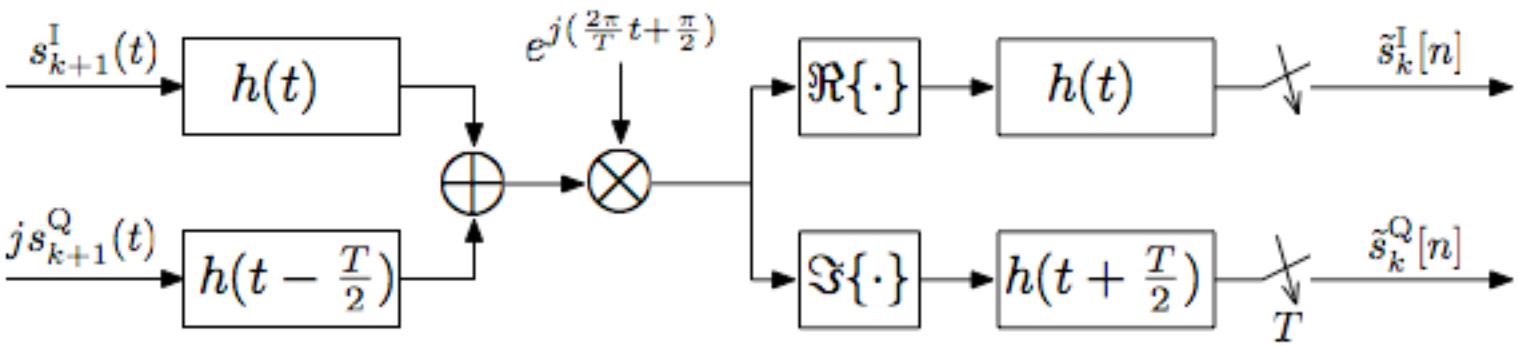
Filter Bank Multicarrier (FBMC) Methods: SMT (Details)

Intercarrier Interference (ICI):

The path between the k+1th and kth subcarrier:



Here, the outputs are ICI terms from k+1th to the kth subcarrier. Thus, for ICI free transmission the output samples must be zero.

Equations that show this are presented in the next slide.

One of the four different cases is presented. The rest are similar.

Filter Bank Multicarrier (FBMC) Methods: SMT (Details)

The impulse response between the input $s^{I}_{k+1}(t)$ and the output before the sampler in the upper-right branch of the figure in the previous slide is given by

$$\begin{aligned} g_1(t) &= \Re\left\{h(t)e^{j\left(\frac{2\pi}{T}t+\frac{\pi}{2}\right)}\right\} \star h(t) = \left(h(t)\cos\left(\frac{2\pi}{T}t+\frac{\pi}{2}\right)\right) \star h(t) \\ &= -\left(h(t)\sin\left(\frac{2\pi}{T}t\right)\right) \star h(t) = -\int_{-\infty}^{\infty} h(\tau)\sin\left(\frac{2\pi}{T}\tau\right)h(t-\tau)d\tau. \end{aligned}$$

Substituting t=nT, we get

$$g_1(nT) = -\int_{-\infty}^{\infty} h(\tau)\sin\left(\frac{2\pi}{T}\tau\right)h(nT-\tau)d\tau.$$

Applying the change of variable τ to $\frac{nT}{2}+\tau$

$$\begin{aligned} g_1(nT) &= -\int_{-\infty}^{\infty} h\left(\frac{nT}{2}+\tau\right)\sin\left(\frac{2\pi}{T}\tau+n\pi\right)h\left(\frac{nT}{2}-\tau\right)d\tau \\ &= (-1)^{n+1}\int_{-\infty}^{\infty} \underbrace{h\left(\frac{nT}{2}+\tau\right)h\left(\frac{nT}{2}-\tau\right)}_{\text{even}}\underbrace{\sin\left(\frac{2\pi}{T}\tau\right)}_{\text{odd}}d\tau = 0 \end{aligned}$$

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Filter Bank Multicarrier (FBMC) Methods: SMT (Details)

Channel Impact:

- In general, channel introduces a gain that varies across the channel.
- However, if the number of subcarriers is large, the gain over each subcarrier band may be approximated by a complex-valued constant, say h_k.
- When this approximation holds, the channel effect can be compensated for each subcarrier by using a single tap equalizer whose gain is set equal to $1/h_k$.
- Hirosaki (1981), has explored the problem for the more general case where channel gain varies across each subcarrier band and suggested a fractionally-spaced transversal equalizer for each subcarrier.

[Hir81] B. Hirosaki, **"An orthogonally multiplexed QAM system using the discrete Fourier transform",** *IEEE Trans. Commun., Volume 29, Issue 7, Jul 1981, pp. 982 – 989.*

Cosine Modulated Multitone (CMT) Details

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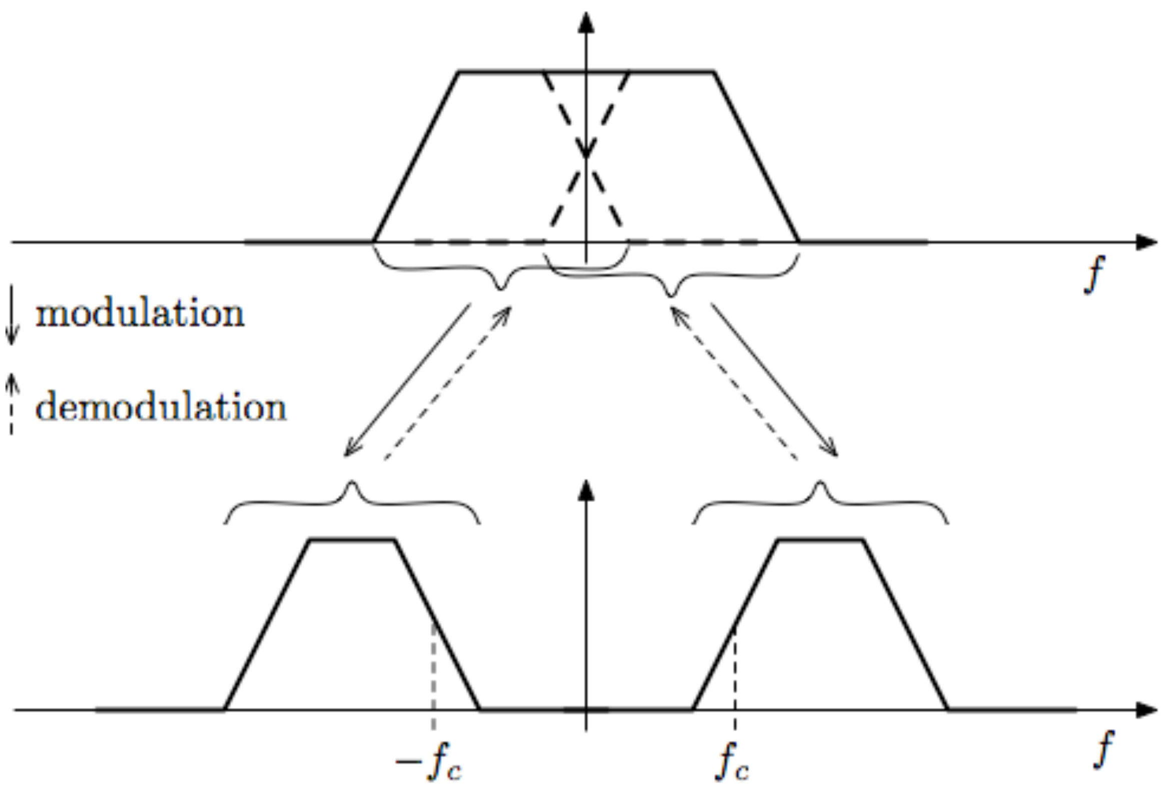
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Modulation type: Vestigial Side-Band (VSB)

Data symbols: PAM (Pulse Amplitude Modulation)

VSB



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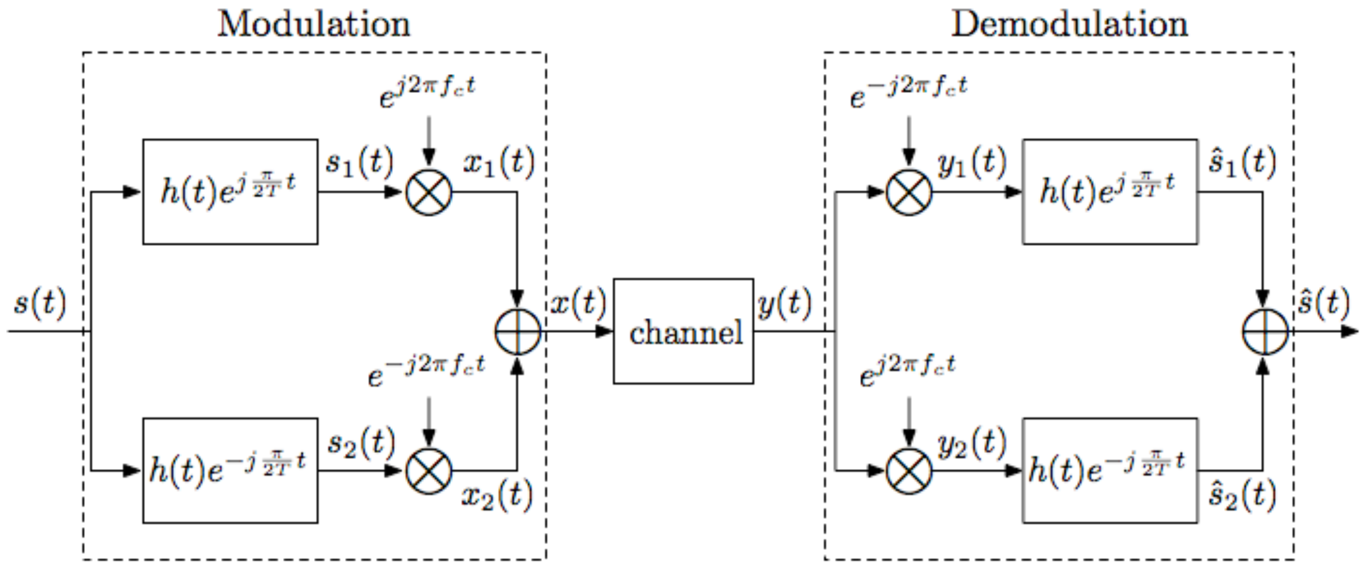
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

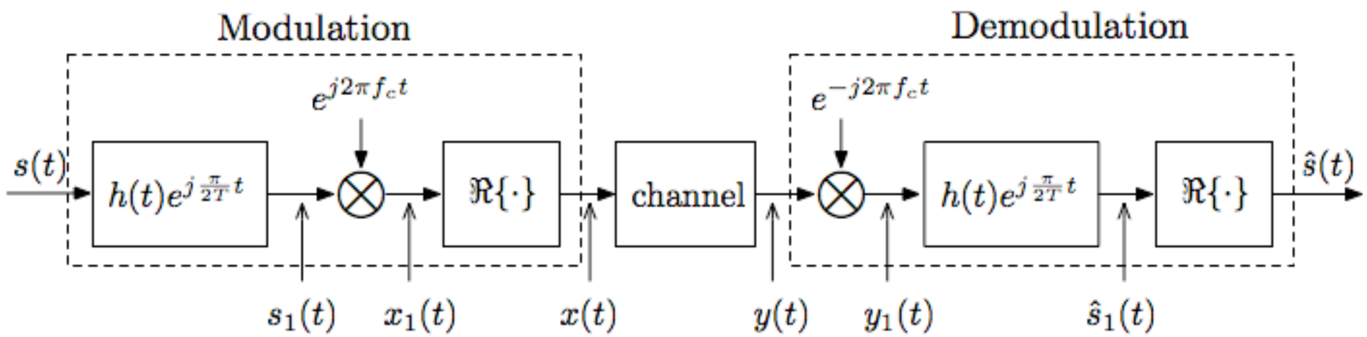
Block diagram of a VSB transceiver.

Detailed:



(a)

Simplified:

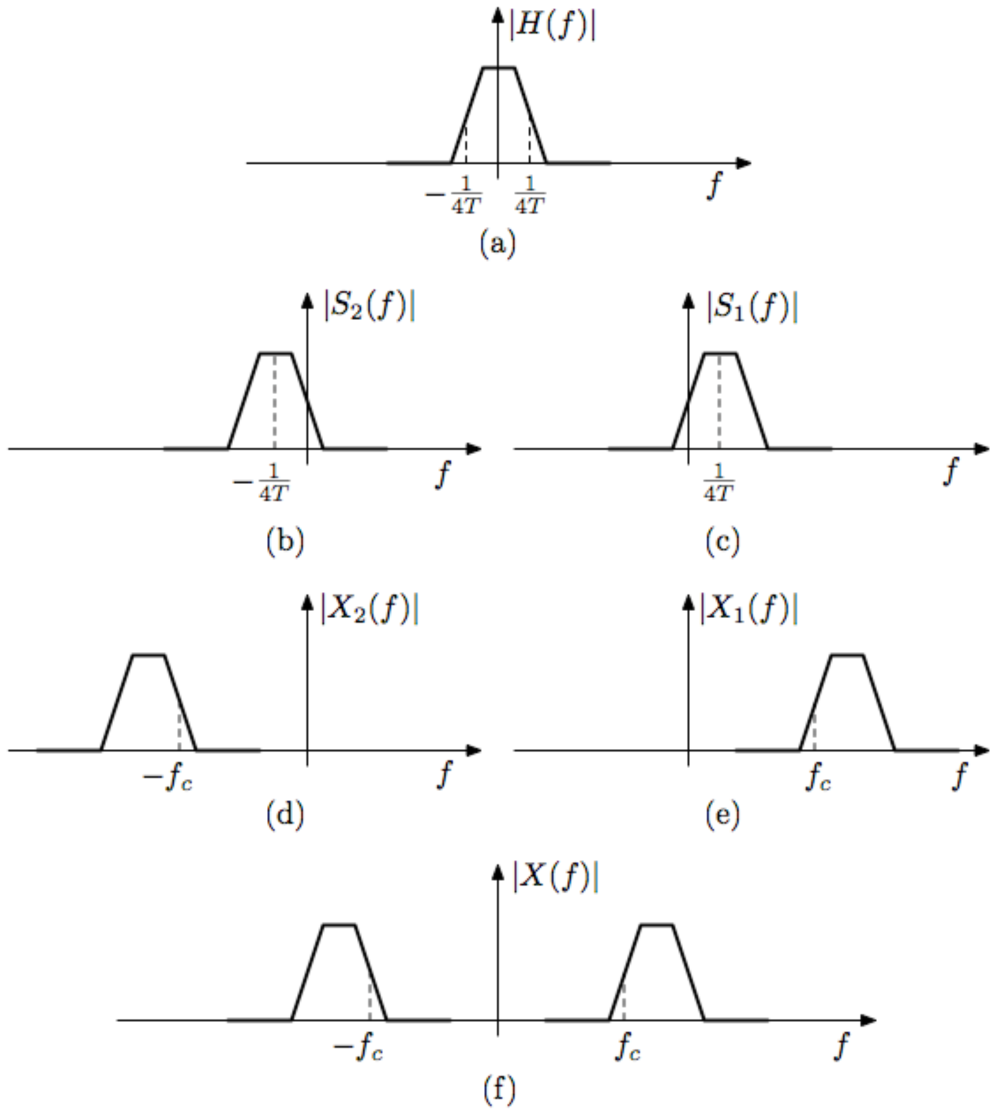


(b)

Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Spectra of various signals in Figure (a) of the previous slide.

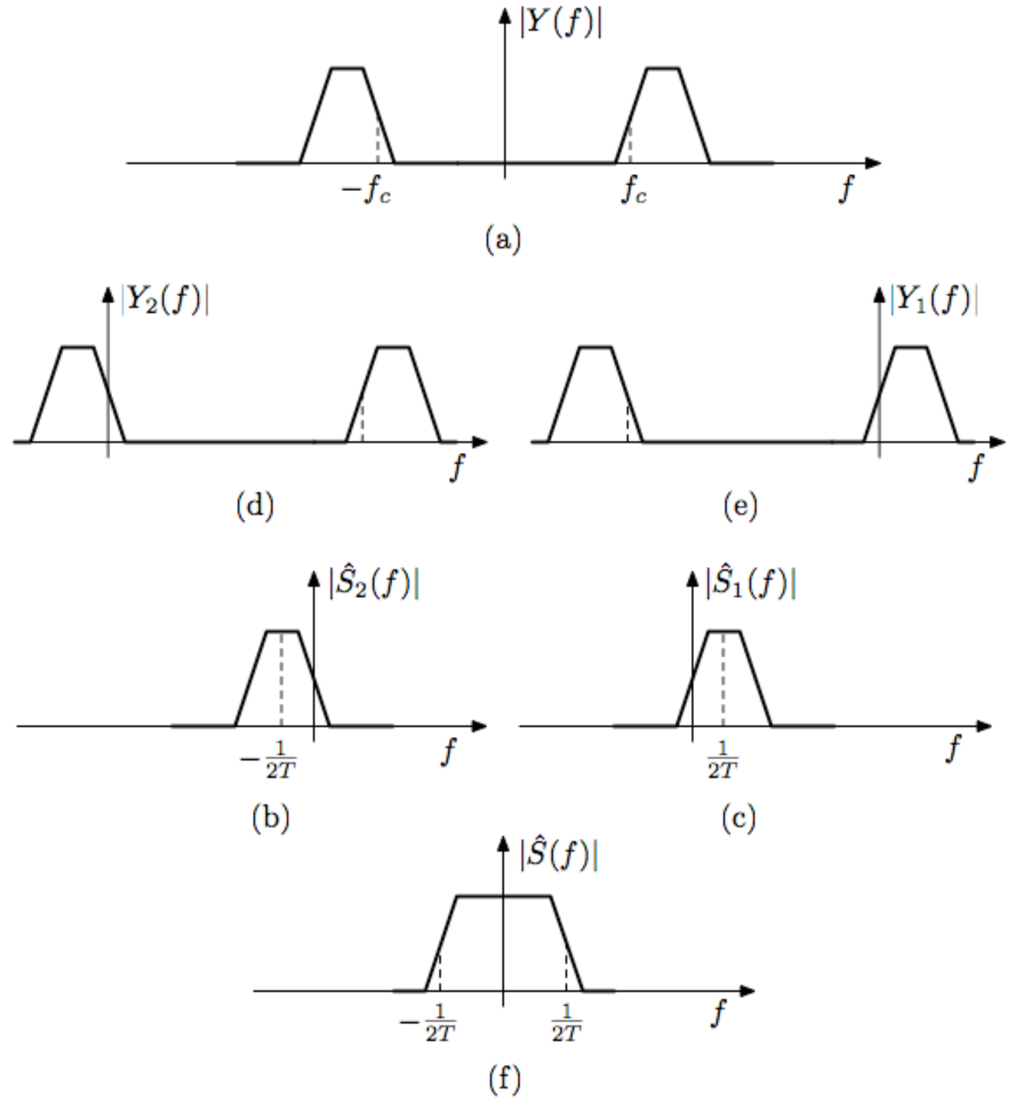
Set 1:



Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Spectra of various signals in Figure (a) of the previous slide.

Set 2:



Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Intersymbol interference (ISI): Ignoring the channel distortion, the impulse response across a VSB channel is obtained as

$$\begin{aligned} g(t) &= \Re\{h(t)e^{j\frac{\pi}{2T}t} \star h(t)e^{j\frac{\pi}{2T}t}\} \\ &= \Re\left\{\int_{-\infty}^{\infty} h(\tau)e^{j\frac{\pi}{2T}\tau} h(t-\tau)e^{j\frac{\pi}{2T}(t-\tau)}d\tau\right\} \\ &= \Re\left\{e^{j\frac{\pi}{2T}t}\int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau\right\}. \end{aligned}$$

Letting $\int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau = p(t)$, this simplifies to

$$g(t) = p(t)\cos\left(\frac{\pi}{2T}t\right).$$

Thus,

$$g(nT) = p(nT)\cos\left(\frac{\pi}{2T}nT\right) = p(nT)\cos\left(\frac{n\pi}{2}\right).$$

Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Intersymbol interference (ISI): Ignoring the channel distortion, the impulse response across a VSB channel is obtained as

$$\begin{aligned} g(t) &= \Re\{h(t)e^{j\frac{\pi}{2T}t} \star h(t)e^{j\frac{\pi}{2T}t}\} \\ &= \Re\left\{\int_{-\infty}^{\infty} h(\tau)e^{j\frac{\pi}{2T}\tau}h(t-\tau)e^{j\frac{\pi}{2T}(t-\tau)}d\tau\right\} \\ &= \Re\left\{e^{j\frac{\pi}{2T}t}\int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau\right\}. \end{aligned}$$

Letting $\int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau = p(t)$, this simplifies to

$$g(t) = p(t)\cos\left(\frac{\pi}{2T}t\right).$$

Thus,

$$g(nT) = p(nT)\cos\left(\frac{\pi}{2T}nT\right) = p(nT)\cos\left(\frac{n\pi}{2}\right).$$

Equal to zero for *n* non-zero even integer   equal to zero for *n* odd

Intersymbol interference (ISI): Ignoring the channel distortion, the impulse response across a VSB channel is obtained as

$$\begin{aligned} g(t) &= \Re\{h(t)e^{j\frac{\pi}{2T}t} \star h(t)e^{j\frac{\pi}{2T}t}\} \\ &= \Re\left\{\int_{-\infty}^{\infty} h(\tau)e^{j\frac{\pi}{2T}\tau}h(t-\tau)e^{j\frac{\pi}{2T}(t-\tau)}d\tau\right\} \\ &= \Re\left\{e^{j\frac{\pi}{2T}t}\int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau\right\}. \end{aligned}$$

Letting $\int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau = p(t)$, this simplifies to

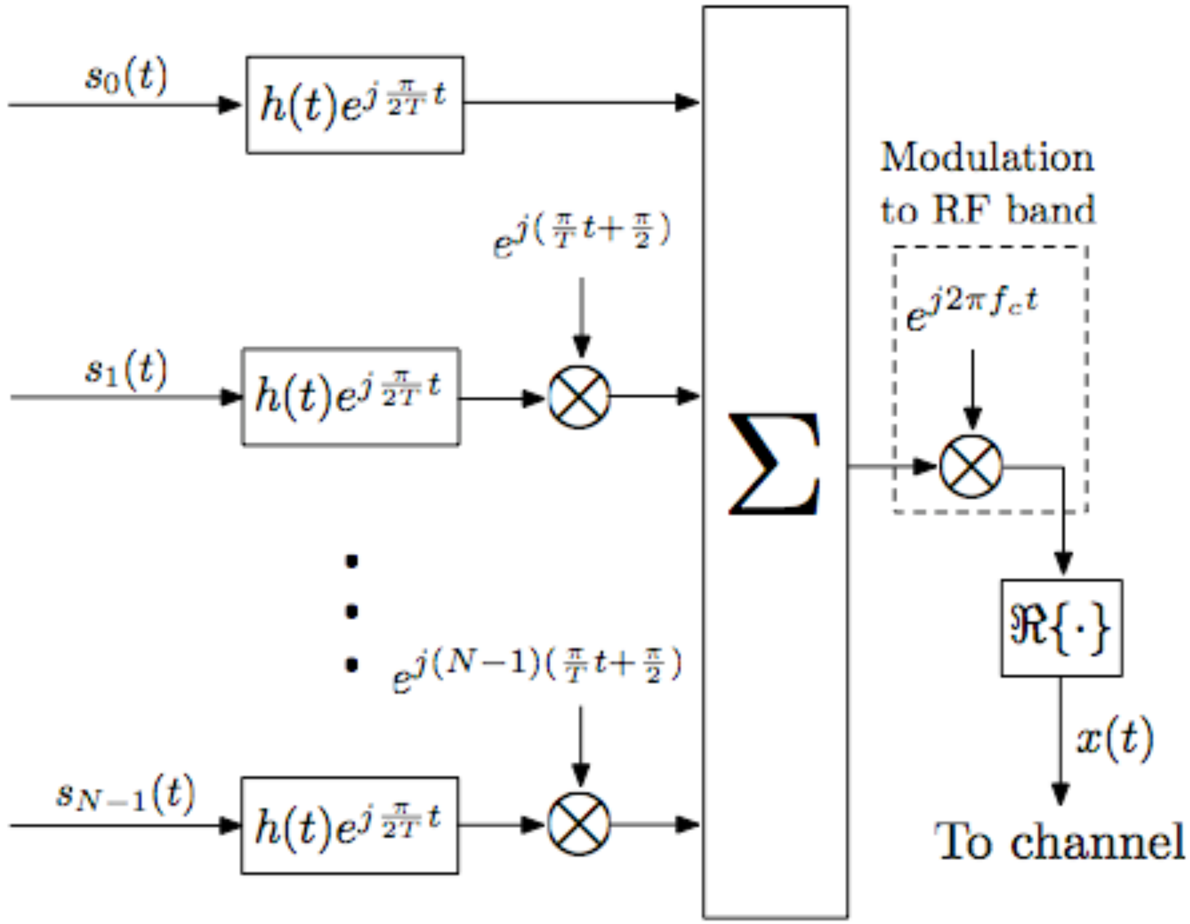
$$g(t) = p(t)\cos\left(\frac{\pi}{2T}t\right).$$

Thus,

$$g(nT) = p(nT)\cos\left(\frac{\pi}{2T}nT\right) = p(nT)\cos\left(\frac{n\pi}{2}\right).$$

Equal to zero for *n* non-zero even integer equal to zero for *n* odd
(i.e., p(t) is Nyquist pulse with zero crossing at the interval 2T)

Filter Bank Multicarrier (FBMC) Methods: CMT (Synthesis/TX)



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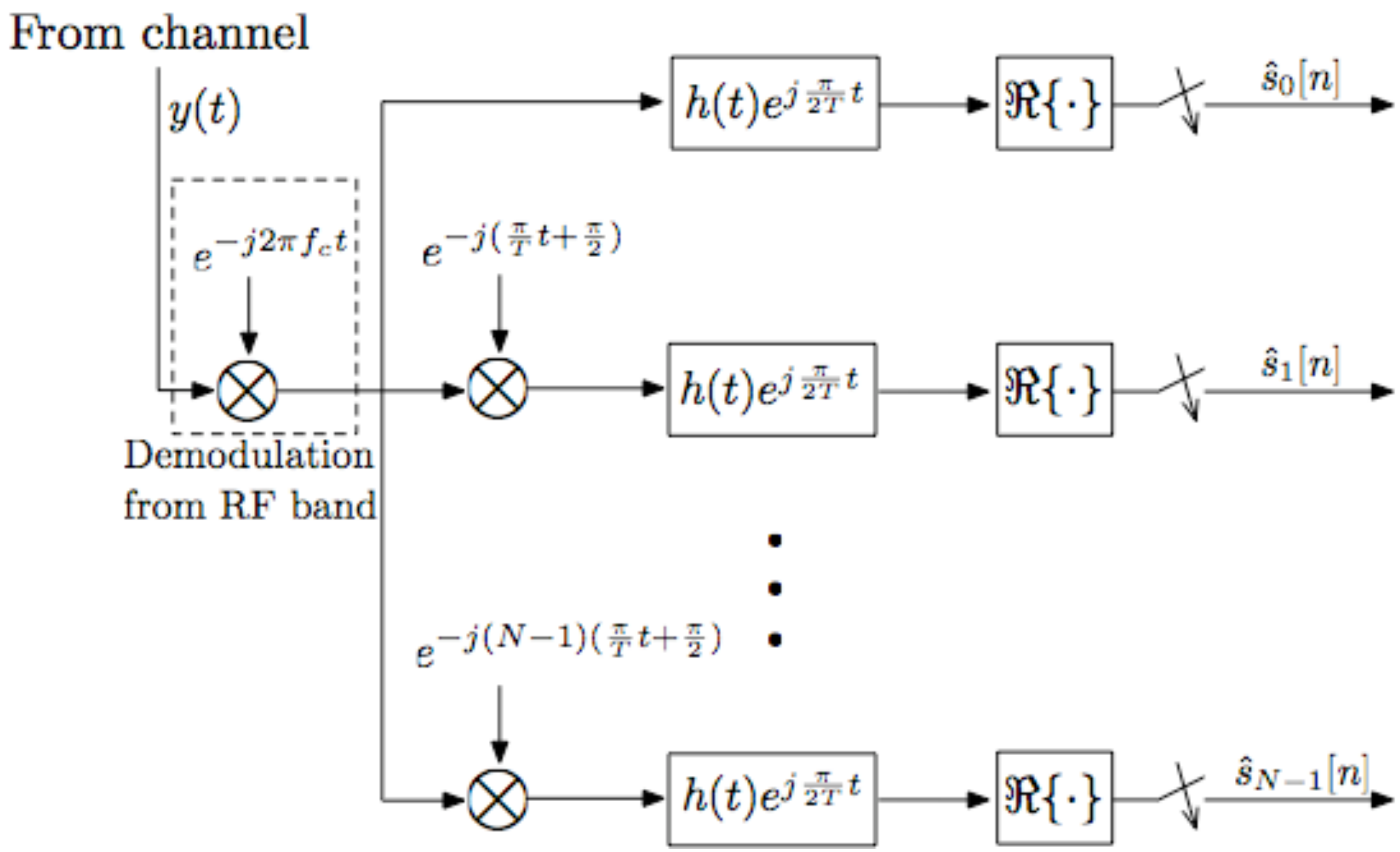
Filter Bank Multicarrier (FBMC) Methods: CMT (Receiver/RX)



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Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

The impulse response between the input $s^{I}_{k+1}(t)$ at the transmitter and the output before the sampler at the kth subcarrier of the receiver is

$$\begin{aligned} g_1(t) &= \Re\{h(t)e^{j(\frac{3\pi}{2T}t+\frac{\pi}{2})} \star h(t)e^{j\frac{\pi}{2T}t}\} = \Re\left\{\int_{-\infty}^{\infty} h(\tau)e^{j(\frac{3\pi}{2T}\tau+\frac{\pi}{2})}h(t-\tau)e^{j\frac{\pi}{2T}(t-\tau)}d\tau\right\} \\ &= \Re\left\{e^{j(\frac{\pi}{2T}t+\frac{\pi}{2})}\int_{-\infty}^{\infty} h(\tau)e^{j\frac{\pi}{T}\tau}h(t-\tau)d\tau\right\}. \end{aligned}$$

Noting that

$$e^{j(\frac{\pi}{2T}t+\frac{\pi}{2})} = -\sin\left(\frac{\pi}{2T}t\right) + j\cos\left(\frac{\pi}{2T}t\right)$$

and, since $h(t)$ is a real function,

$$\int_{-\infty}^{\infty} h(\tau)e^{j\frac{\pi}{T}\tau}h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)h(t-\tau)\cos\left(\frac{\pi}{T}\tau\right)d\tau + j\int_{-\infty}^{\infty} h(\tau)h(t-\tau)\sin\left(\frac{\pi}{T}\tau\right)d\tau.$$

Hence,

$$g_1(t) = -\sin\left(\frac{\pi}{2T}t\right)\int_{-\infty}^{\infty} h(\tau)h(t-\tau)\cos\left(\frac{\pi}{T}\tau\right)d\tau - \cos\left(\frac{\pi}{2T}t\right)\int_{-\infty}^{\infty} h(\tau)h(t-\tau)\sin\left(\frac{\pi}{T}\tau\right)d\tau.$$

We are interested in the sample values of $g_1(t)$ at the time instants nT, for integer values of n.

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Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

For an even value of $n = 2k$, one finds that $\sin\left(\frac{\pi}{2T}2kT\right) = \sin(k\pi) = 0$ and, thus

$$g_1(2kT) = (-1)^{k+1} \int_{-\infty}^{\infty} h(\tau)h(2kT - \tau) \sin\left(\frac{\pi}{T}\tau\right) d\tau$$

where we have noted that $\cos(k\pi) = (-1)^k$. Applying a change of variable τ to $kT + \tau$, we get

$$g_1(2kT) \quad = \quad -\int_{-\infty}^{\infty} h(kT + \tau)h(kT - \tau) \sin\left(\frac{\pi}{T}\tau\right) d\tau = 0.$$

where the second identity follows since the expression under the integral is an odd function of τ.

Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

For an odd values of $n = 2k+1$, one finds that $\cos\left(\frac{\pi}{2T}(2k+1)T\right) = \cos(k\pi + \pi/2) = 0$ and, thus,

$$g_1((2k+1)T) = (-1)^{k+1} \int_{-\infty}^{\infty} h(\tau)h((2k+1)T - \tau) \cos\left(\frac{\pi}{T}\tau\right) d\tau.$$

where we have noted that $\sin(k\pi + \pi/2) = (-1)^k$. Applying a change of variable τ to $\frac{(2k+1)T}{2} + \tau$, we get

$$g_1(\tfrac{(2k+1)T}{2}) = \int_{-\infty}^{\infty} h(\tfrac{(2k+1)T}{2} + \tau)h(\tfrac{(2k+1)T}{2} - \tau) \sin\left(\frac{\pi}{T}\tau\right) d\tau = 0.$$

where the second identity follows since the expression under the integral is an odd function of τ.

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Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

For an odd values of $n = 2k+1$, one finds that $\cos\left(\frac{\pi}{2T}(2k+1)T\right) = \cos(k\pi + \pi/2) = 0$ and, thus,

$$g_1((2k+1)T) = (-1)^{k+1}\int_{-\infty}^{\infty} h(\tau)h((2k+1)T-\tau)\cos\left(\frac{\pi}{T}\tau\right)d\tau.$$

where we have noted that $\sin(k\pi+\pi/2) = (-1)^k$. Applying a change of variable τ to $\frac{(2k+1)T}{2}+\tau$, we get

$$g_1(\tfrac{(2k+1)T}{2}) = \int_{-\infty}^{\infty} h(\tfrac{(2k+1)T}{2}+\tau)h(\tfrac{(2k+1)T}{2}-\tau)\sin\left(\frac{\pi}{T}\tau\right)d\tau = 0.$$

where the second identity follows since the expression under the integral is an odd function of τ.

Combining the above results, one finds that

$$g_1(nT) = 0$$

for all integer values of n, which is the condition we required for ICI free transmission.

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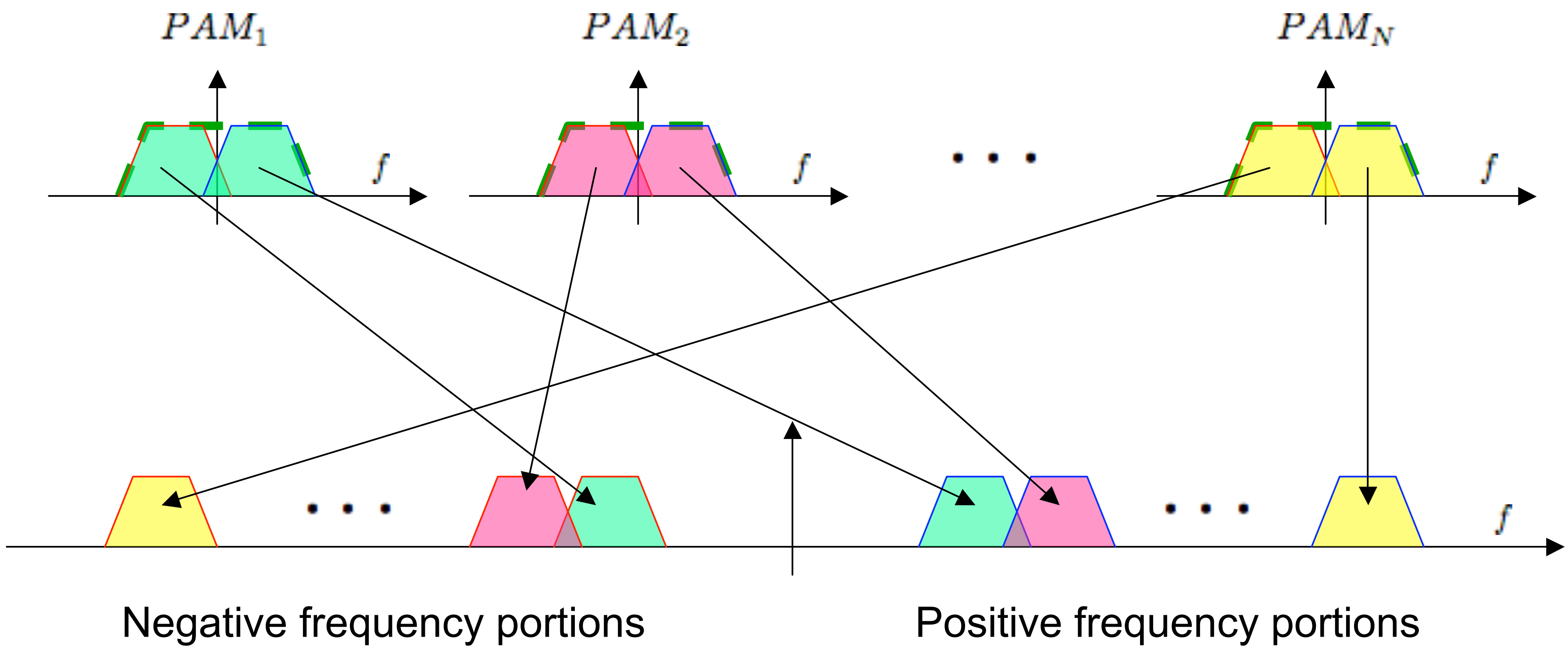
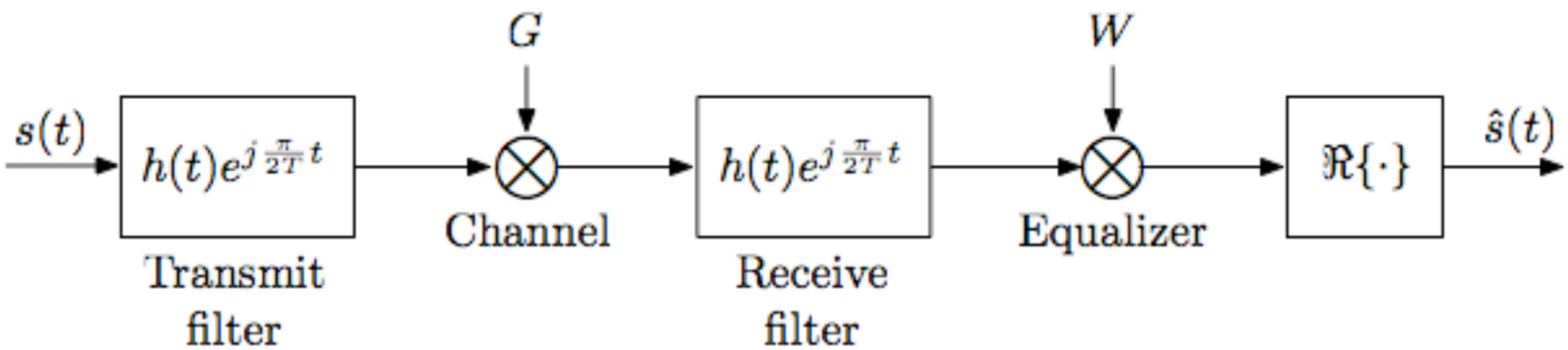
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

In CMT data symbols are PAM and modulated into a set of vestigial sideband subcarrier channels.

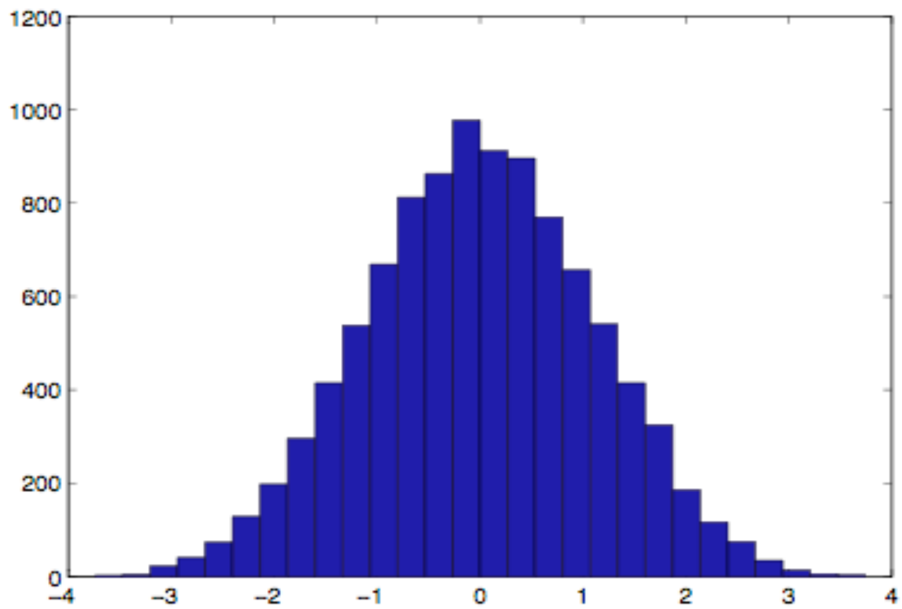


Equalization:

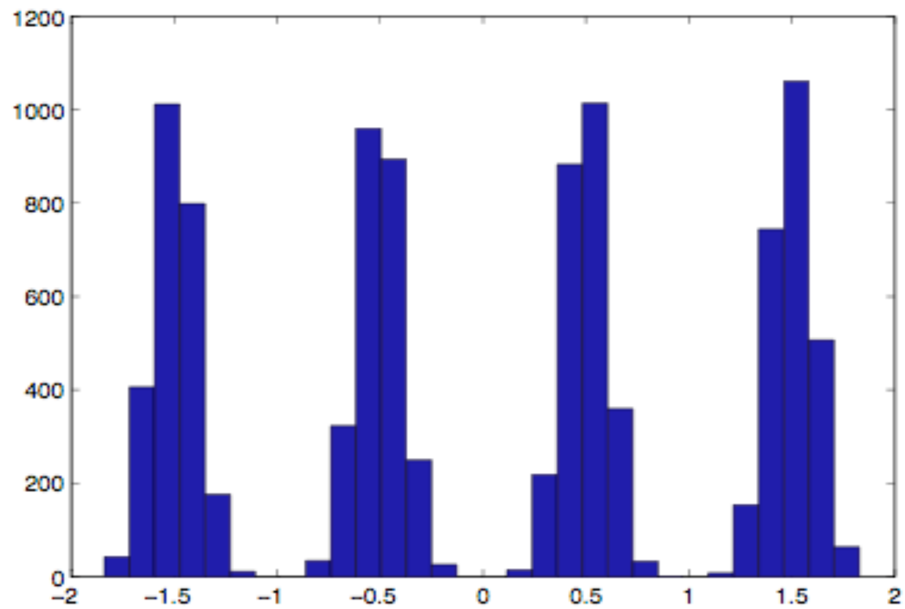


Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Blind equalization



Before equalization



After equalization

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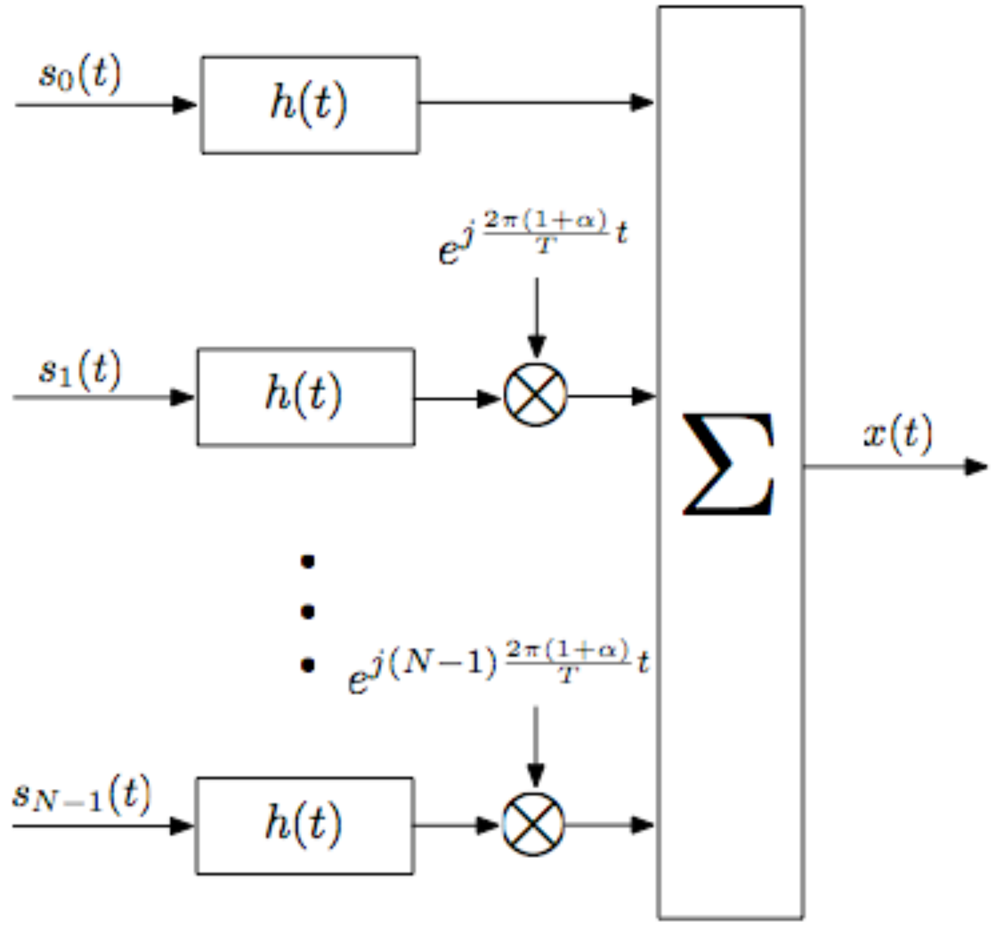
Filtered Multitone (Details)

Filter Bank Multicarrier (FBMC) Methods: FMT (Synthesis/TX)

Transmitter:



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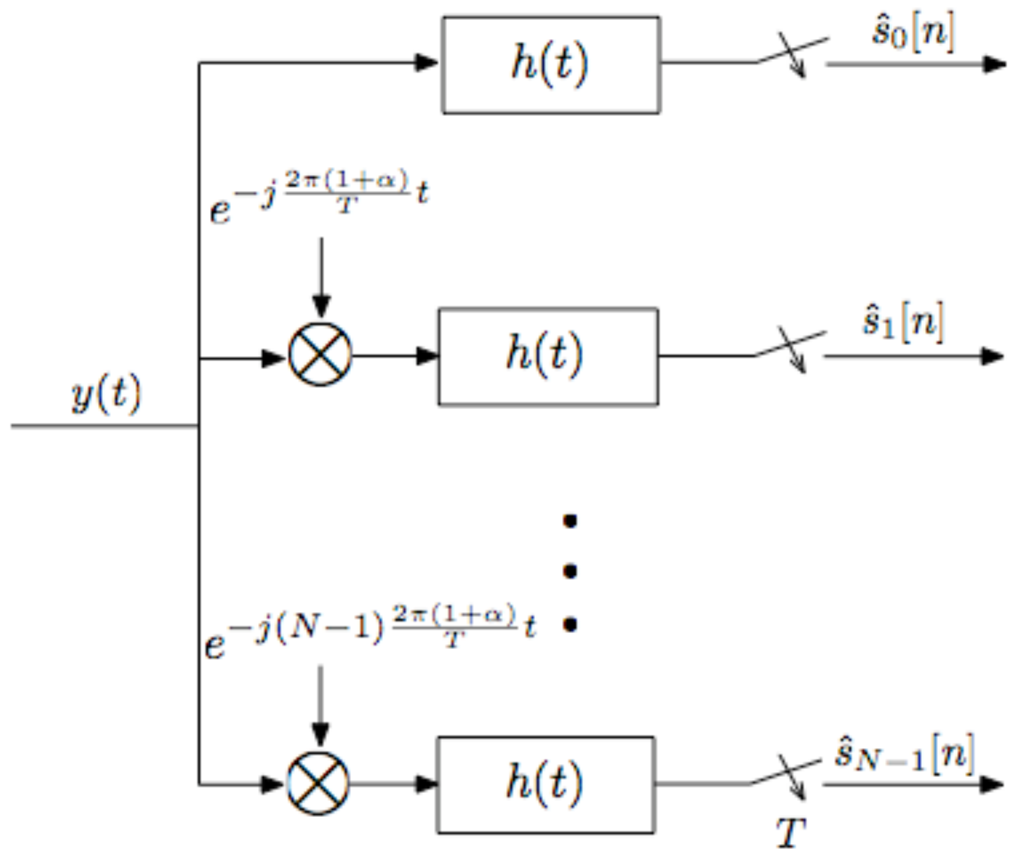
Filter Bank Multicarrier (FBMC) Methods: FMT (Analysis/RX)

Receiver:



Implementation of Multicarrier Filter Banks

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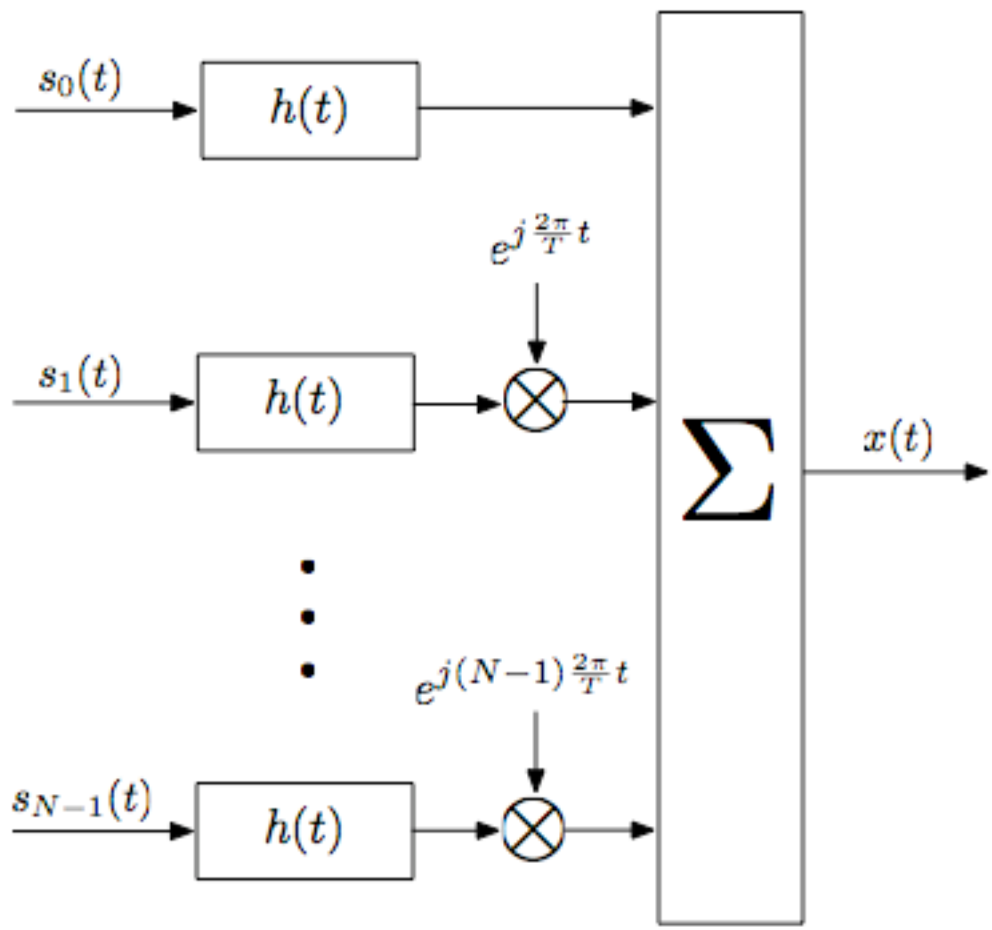
Implementation of MCFB: Basic structure (synthesis/TX)

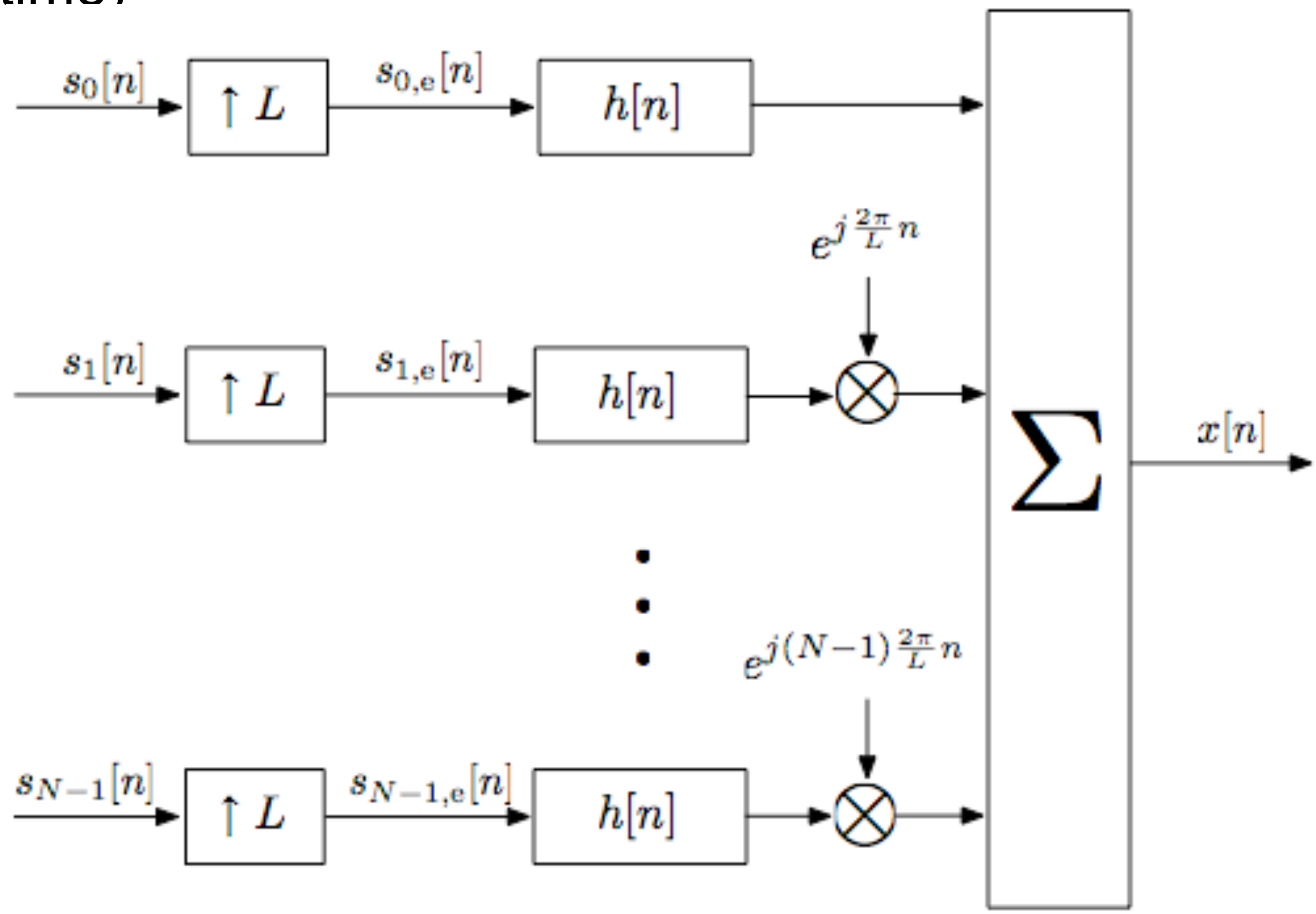
(continuous-time)



$$s_k(t) = \sum_{n=-\infty}^{\infty} s_k[n]\delta(t-nT), \quad \text{for } k = 0, 1, \cdots, N-1$$

Implementation of MCFB: Basic structure (synthesis/TX)

(discrete-time)



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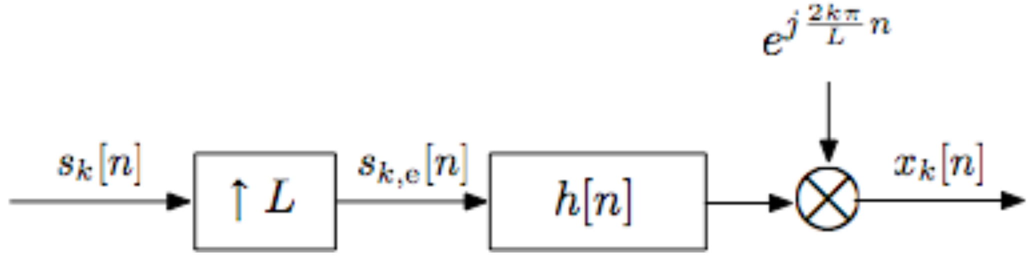
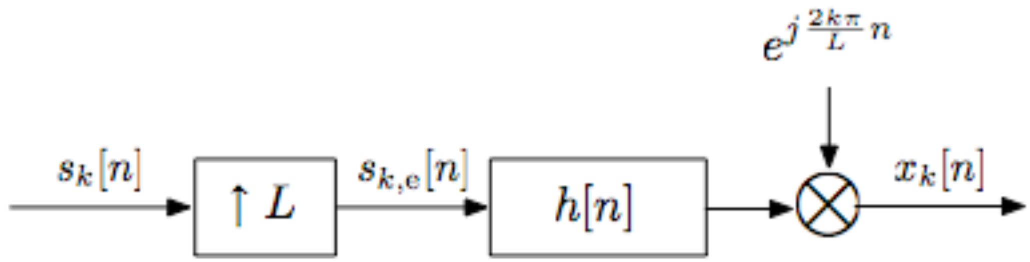
Implementation of MCFB: Basic structure (synthesis/TX)

Development of the polyphase structure: consider the kth branch of the synthesis filter bank of the previous slide



Implementation of MCFB: Basic structure (synthesis/TX)

Development of the polyphase structure: consider the kth branch of the synthesis filter bank of the previous slide

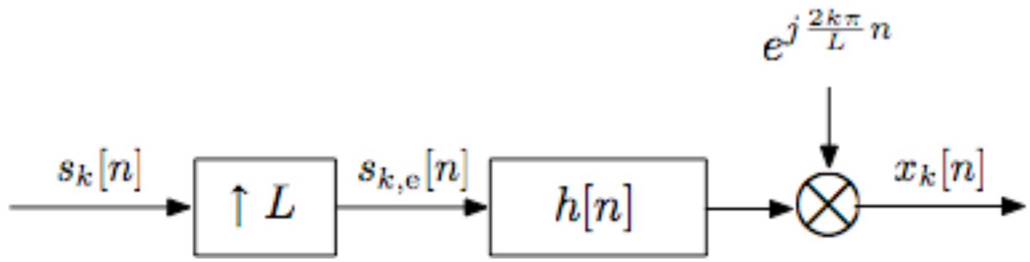


$$x_k[n] = \left(\sum_{m=-\infty}^{\infty} s_k[m]h[n-mL] \right) e^{j\frac{2k\pi}{L}n}.$$

Noting that $e^{j\frac{2k\pi}{L}n} = e^{j\frac{2k\pi}{L}(n-mL)}$, we obtain $x_k[n] = \sum_{m=-\infty}^{\infty} s_k[m]h_k[n-mL]$, where $h_k[n] = h[n]e^{j\frac{2k\pi}{L}n}$. We note that $h_k[n]$ is a modulated version of $h[n]$.

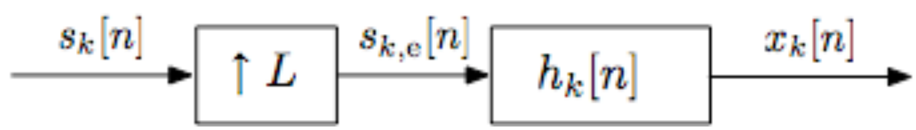
Implementation of MCFB: Basic structure (synthesis/TX)

Development of the polyphase structure: consider the kth branch of the synthesis filter bank of the previous slide



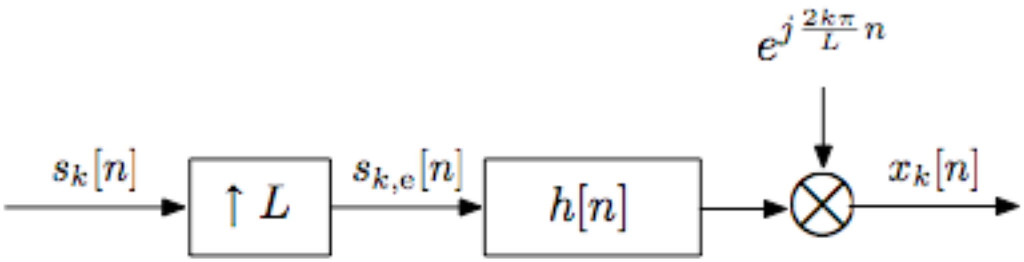
$$x_k[n] = \left(\sum_{m=-\infty}^{\infty} s_k[m]h[n-mL] \right) e^{j\frac{2k\pi}{L}n}.$$

Noting that $e^{j\frac{2k\pi}{L}n} = e^{j\frac{2k\pi}{L}(n-mL)}$, we obtain $x_k[n] = \sum_{m=-\infty}^{\infty} s_k[m]h_k[n-mL]$, where $h_k[n] = h[n]e^{j\frac{2k\pi}{L}n}$. We note that $h_k[n]$ is a modulated version of $h[n]$.



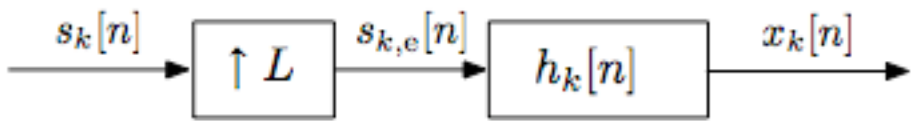
Implementation of MCFB: Basic structure (synthesis/TX)

Development of the polyphase structure: consider the kth branch of the synthesis filter bank of the previous slide



$$x_k[n] = \left(\sum_{m=-\infty}^{\infty} s_k[m] h[n - mL] \right) e^{j\frac{2k\pi}{L}n}.$$

Noting that $e^{j\frac{2k\pi}{L}n} = e^{j\frac{2k\pi}{L}(n-mL)}$, we obtain $x_k[n] = \sum_{m=-\infty}^{\infty} s_k[m] h_k[n-mL]$, where $h_k[n] = h[n]e^{j\frac{2k\pi}{L}n}$. We note that $h_k[n]$ is a modulated version of $h[n]$.



$$X_k(z) = S_k\left(z^L\right) H_k(z)$$

Implementation of MCFB: Basic structure (synthesis/TX)

Derivation of the synthesis polyphase filter bank

We note that $X(z) = \sum_{k=0}^{N-1} X_k(z)$. Also, $H_k(z) = \sum_n h[n]z^{-n}W_L^{kn}$, where $W_L = e^{-j2\pi/L}$.

From these, we obtain

$$X(z) = \sum_{k=0}^{N-1} \sum_n S_k\left(z^L\right) h[n]z^{-n}W_L^{kn}.$$

Next, we introduce the change of variable $n = mL + l$, to obtain

$$X(z) = \sum_{k=0}^{N-1} \sum_{l=0}^{L-1} S_k\left(z^L\right) W_L^{kl} E_l(z^L)z^{-l}$$

where $E_l(z)$ is the lth polyphase component of $H(z)$, defined as

$$E_l(z) = \cdots + h[l-L]z + h[l] + h[l+L]z^{-1} + \cdots$$

From these, we obtain the results presented on the next slide.

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Implementation of MCFB: Basic structure (synthesis/TX)

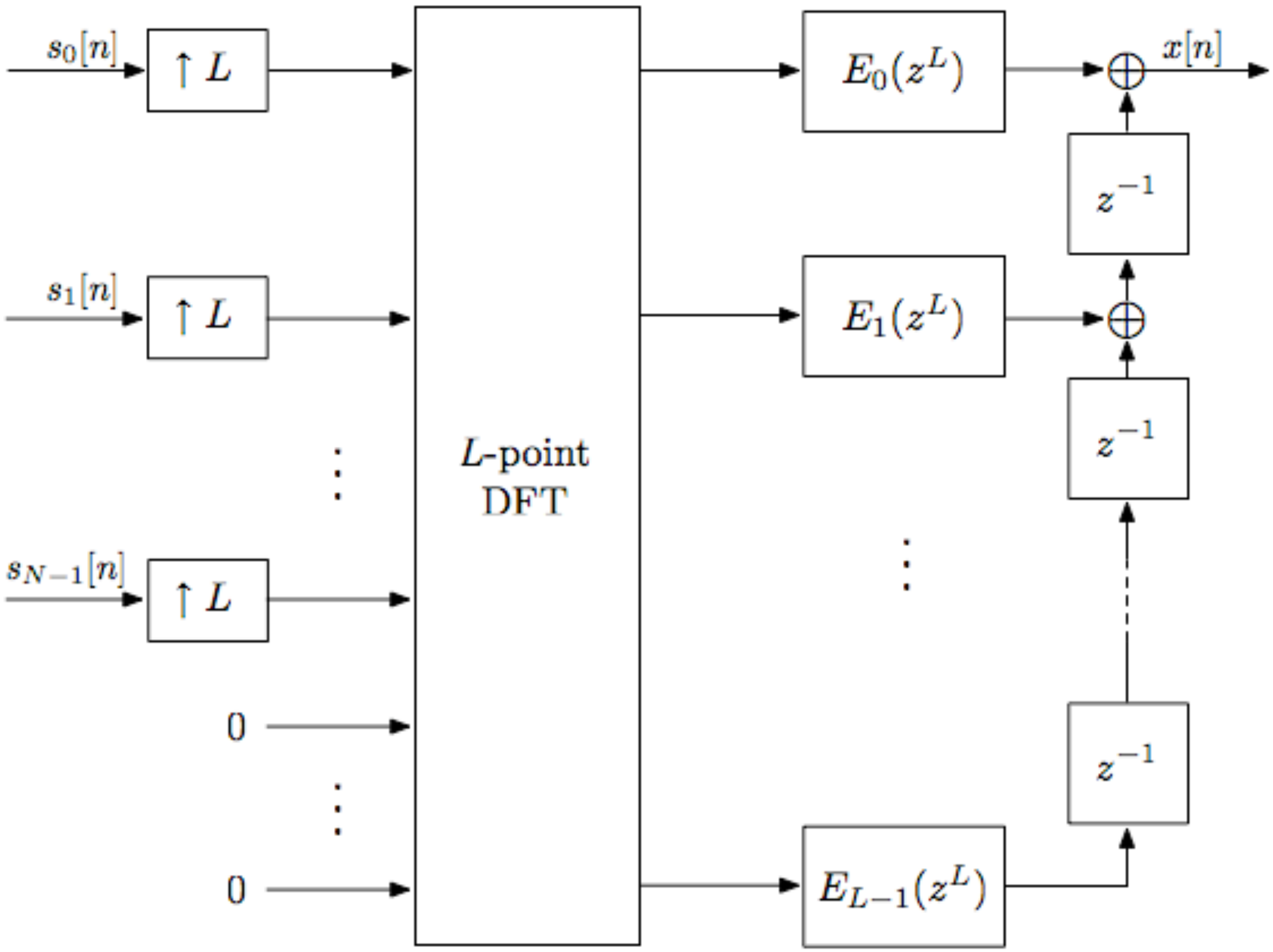
$$X(z) = \overbrace{[S_0(z^L)\ S_1(z^L)\ \cdots\ S_{N-1}(z^L)\ 0\ \cdots\ 0]}^{1\times L}\mathcal{F} \times \begin{bmatrix} E_0(z^L) & 0 & \cdots & 0 \\ 0 & E_1(z^L) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{L-1}(z^L) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(L-1)} \end{bmatrix}.$$

where $\mathcal{F}$ is the $L \times L$ Fourier transform matrix defined as

$$\mathcal{F} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & W_L & \cdots & W_L^{L-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & W_L^{L-1} & \cdots & W_L^{(L-1)(L-1)} \end{bmatrix}.$$

Implementation of MCFB: Basic structure (synthesis/TX)

Using the equations on the previous slide, we obtain the following structure



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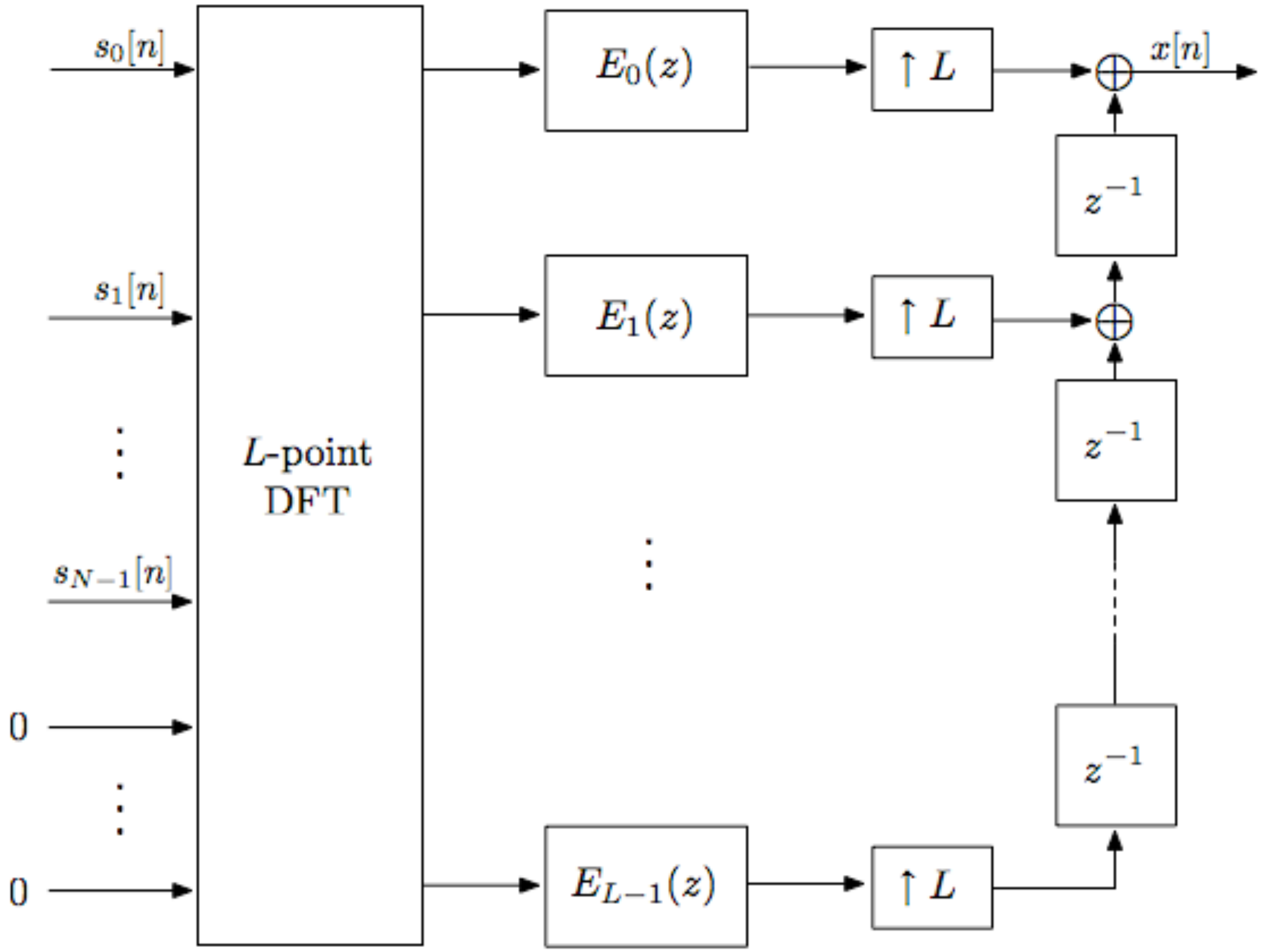
Implementation of MCFB: Basic structure (synthesis/TX)

Moving the expander to the output side:



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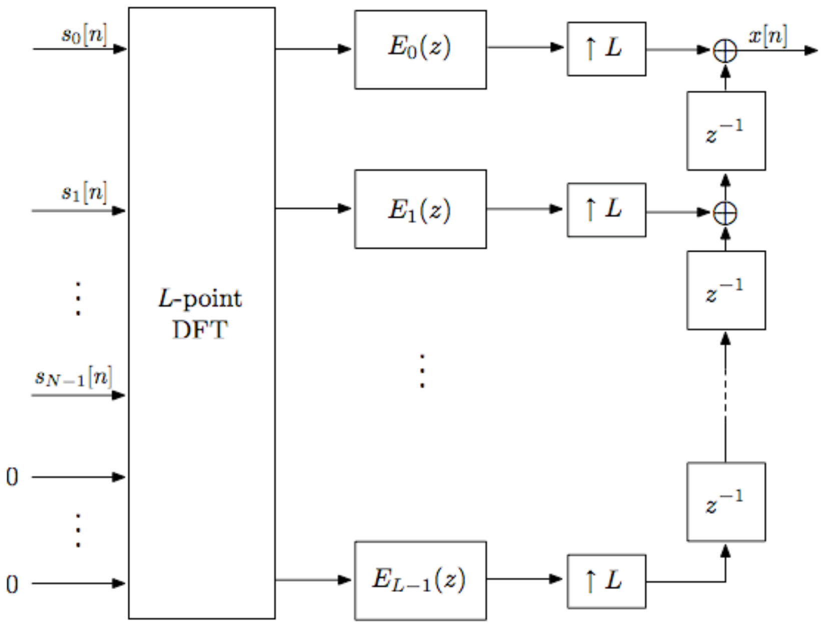
Implementation of MCFB: Basic structure (synthesis/TX)

Moving the expander to the output side:



$$X(z) = \overbrace{[S_0(z^L)\ S_1(z^L)\ \cdots\ S_{N-1}(z^L)\ 0\ \cdots\ 0]}^{1\times L}\mathcal{F} \times \begin{bmatrix} E_0(z^L) & 0 & \cdots & 0 \\ 0 & E_1(z^L) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{L-1}(z^L) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(L-1)} \end{bmatrix}.$$

where $\mathcal{F}$ is the $L \times L$ Fourier transform matrix defined as

$$\mathcal{F} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & W_L & \cdots & W_L^{L-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & W_L^{L-1} & \cdots & W_L^{(L-1)(L-1)} \end{bmatrix}.$$

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Implementation of MCFB: Basic structure (analysis/RX)

Note that $X_k(z) = S_k(z^L)H_k(z)$ and

$$
\begin{aligned}
X(z) &= \sum_{k=0}^{N-1} S_k(z^L)H_k(z) \\
&= \overbrace{[S_0(z^L)\ S_1(z^L)\ \cdots\ S_{N-1}(z^L)\ 0\ \cdots\ 0]}^{1\times L} \begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{L-1}(z) \end{bmatrix}.
\end{aligned}
$$

Hence,

$$
\begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{L-1}(z) \end{bmatrix} = \mathcal{F} \times \begin{bmatrix} E_0(z^L) & 0 & \cdots & 0 \\ 0 & E_1(z^L) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{L-1}(z^L) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(L-1)} \end{bmatrix}.
$$

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Implementation of MCFB: Basic structure (analysis/RX)

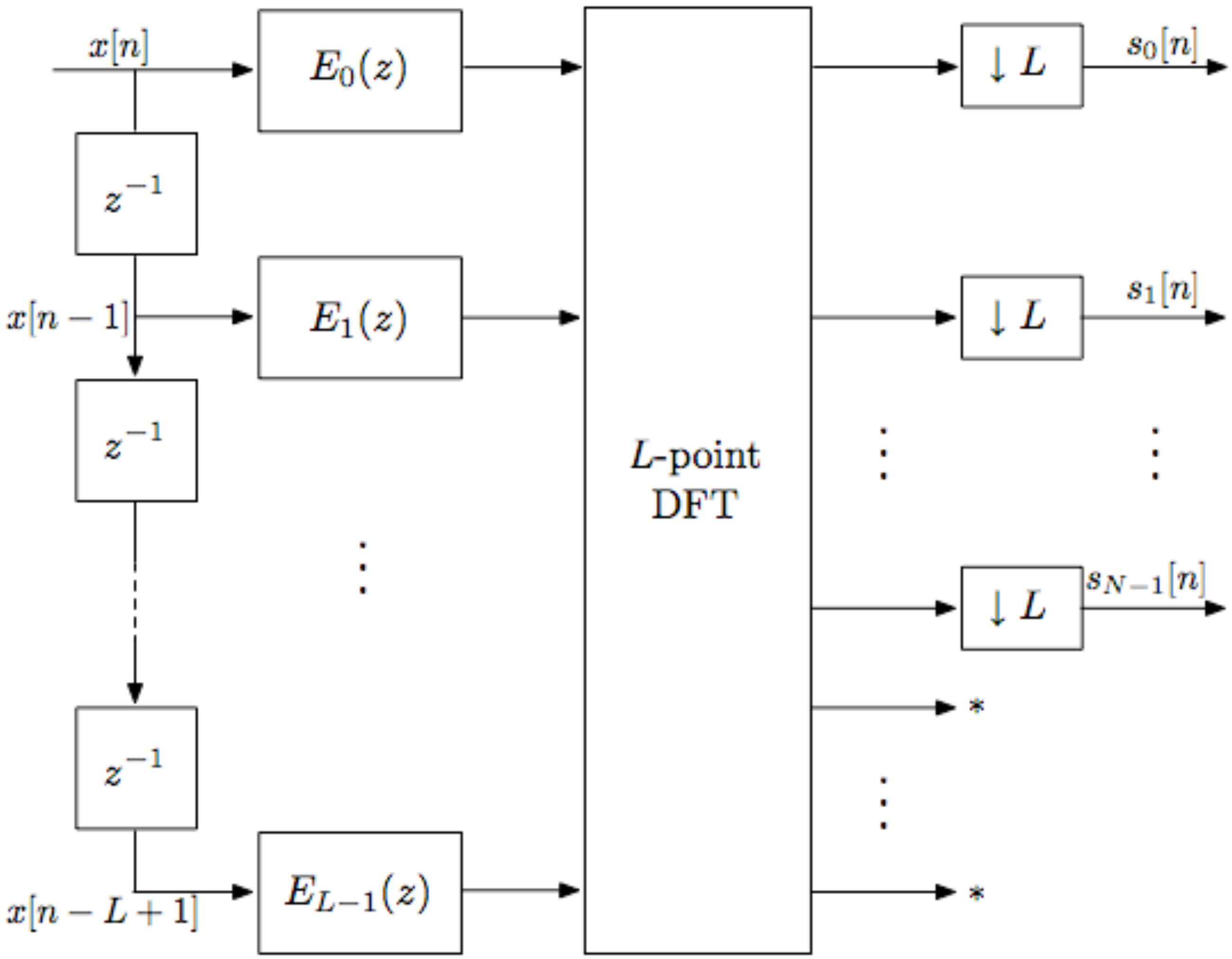
From the results on the previous slide, we get

$$
\begin{bmatrix} S_0(z^L) \\ S_1(z^L) \\ \vdots \\ S_{N-1}(z^L) \\ * \\ \vdots \\ * \end{bmatrix}
=
\begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{L-1}(z) \end{bmatrix} X(z)
$$

$$
= \mathcal{F} \times \begin{bmatrix} E_0(z^L) & 0 & \cdots & 0 \\ 0 & E_1(z^L) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{L-1}(z^L) \end{bmatrix}
\times \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(L-1)} \end{bmatrix} X(z).
$$

Implementation of MCFB: Basic structure (analysis/RX)

Using the equation on the previous slide, the following structure is obtained



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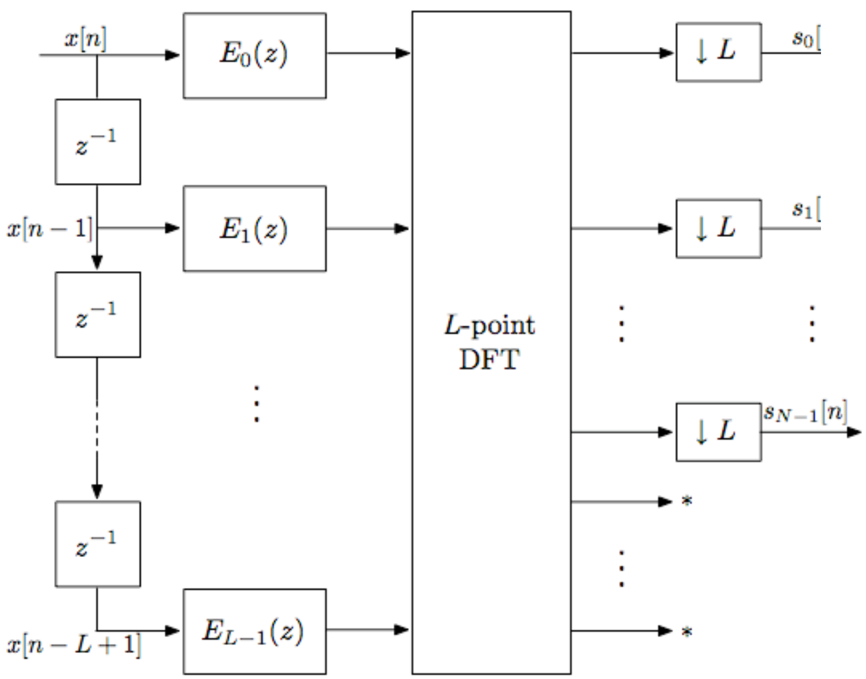
Implementation of MCFB: Basic structure (analysis/RX)

Using the equation on the previous slide, the following structure is obtained



From the results on the previous slide, we get

$$\begin{bmatrix} S_0(z^L) \\ S_1(z^L) \\ \vdots \\ S_{N-1}(z^L) \\ * \\ \vdots \\ * \end{bmatrix} = \begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{L-1}(z) \end{bmatrix} X(z)$$

$$= \mathcal{F} \times \begin{bmatrix} E_0(z^L) & 0 & \cdots & 0 \\ 0 & E_1(z^L) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{L-1}(z^L) \end{bmatrix} \times \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(L-1)} \end{bmatrix} X(z).$$

Staggered Modulated Multitone (SMT)

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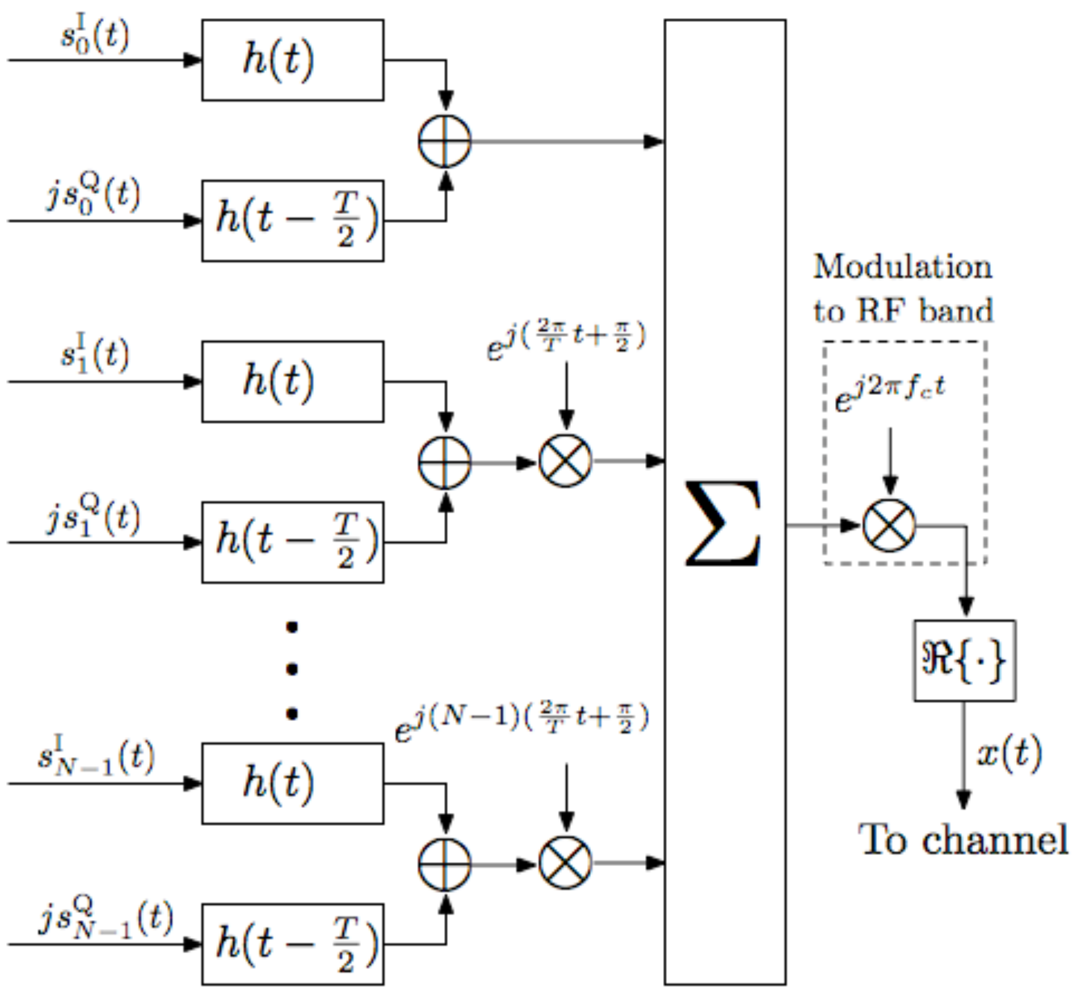
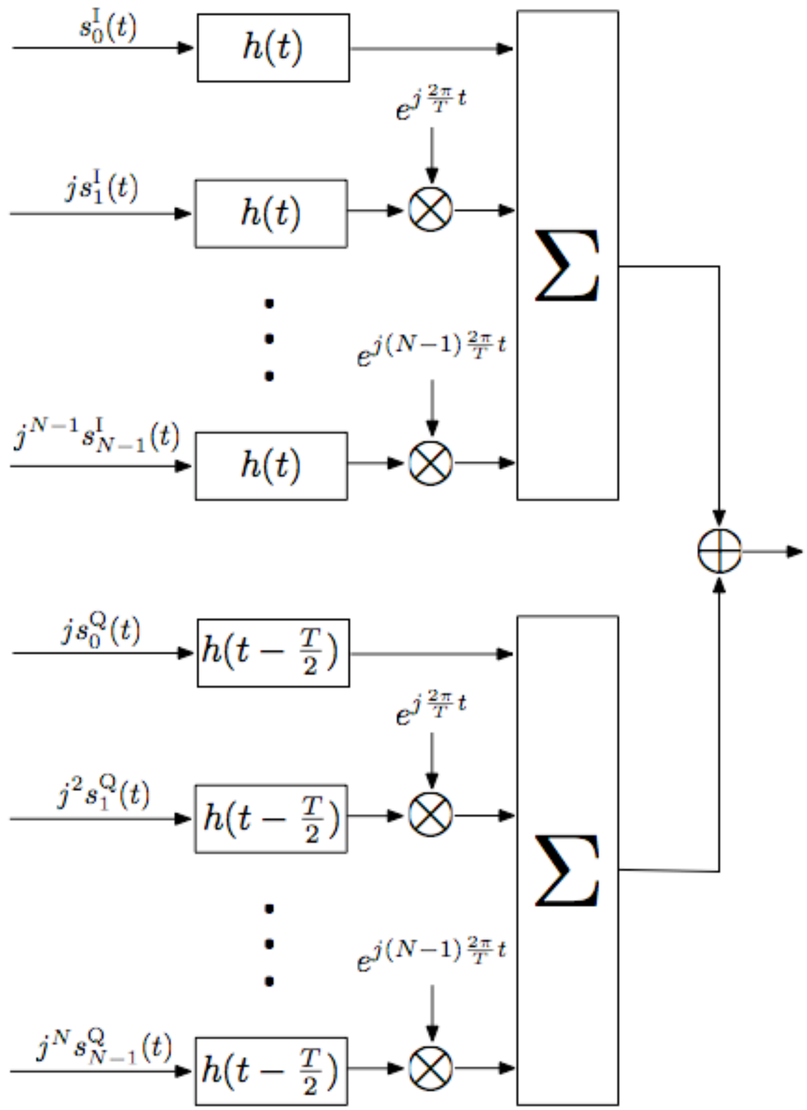
Implementation of MCFB: SMT (synthesis/transmitter)

Recall:



And note that, ignoring modulation to RF band, this can be rearranged as in the next slide.

Implementation of MCFB: SMT (synthesis/transmitter)



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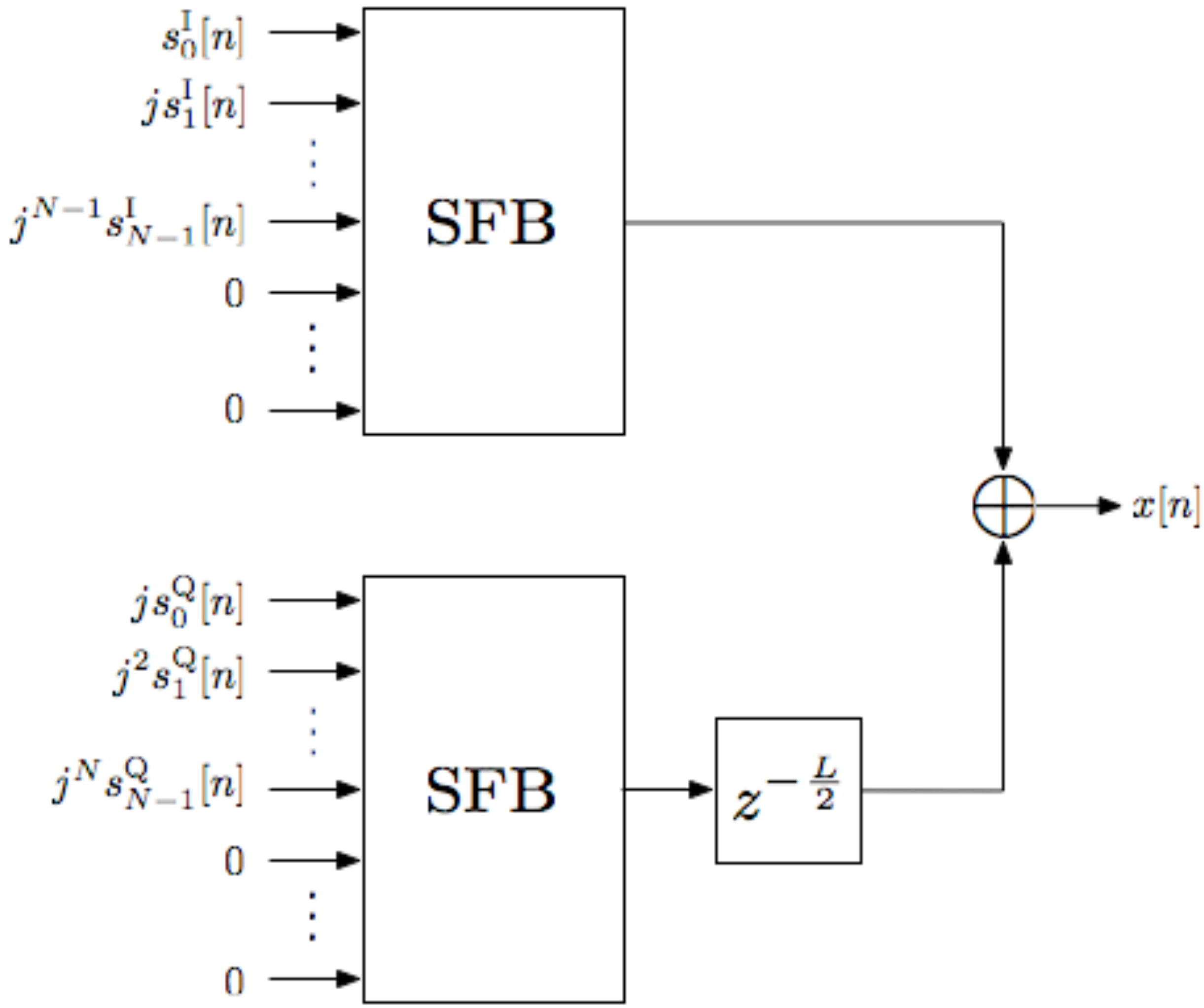
Implementation of MCFB: SMT (synthesis/transmitter)

or



SFB: Symthesis filter bank

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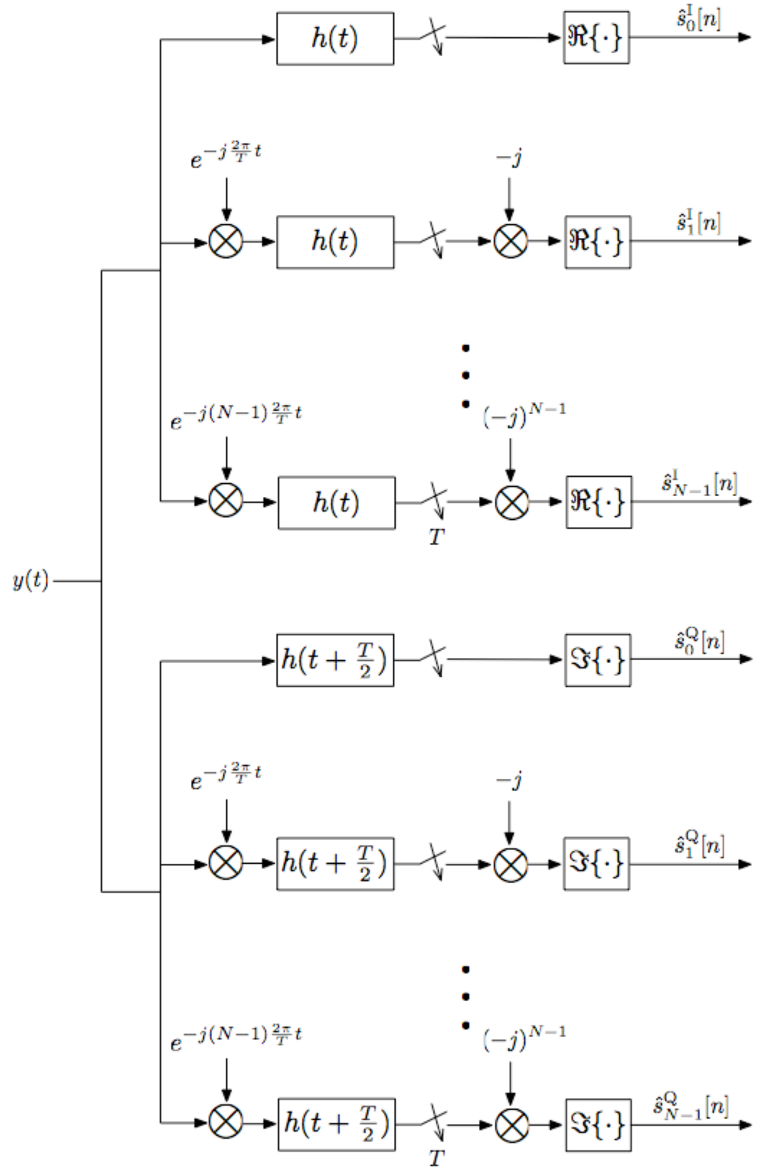
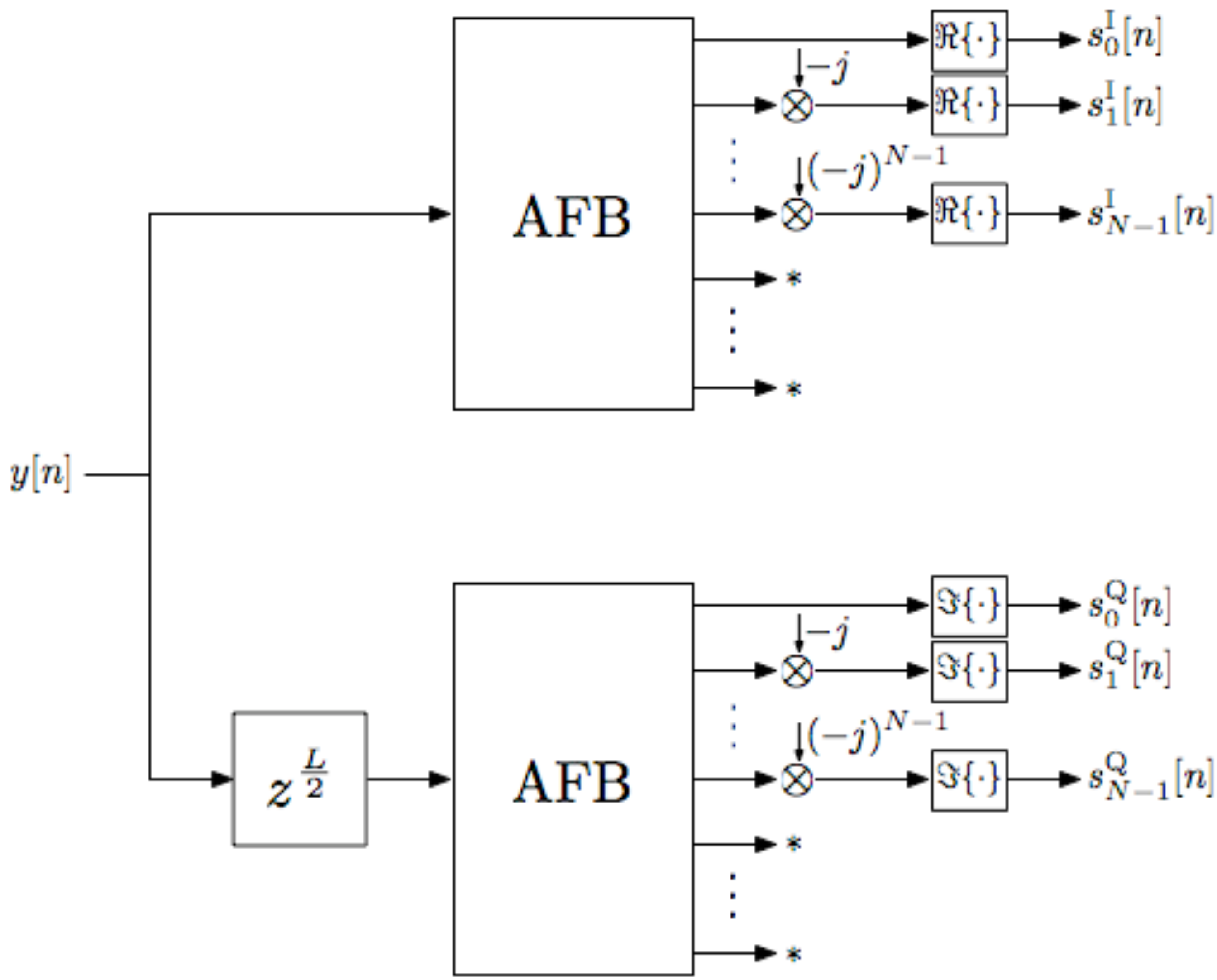
Implementation of MCFB: SMT (analysis/receiver)



Implementation of MCFB: SMT (analysis/receiver)



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Cosine Modulated Multitone (CMT)

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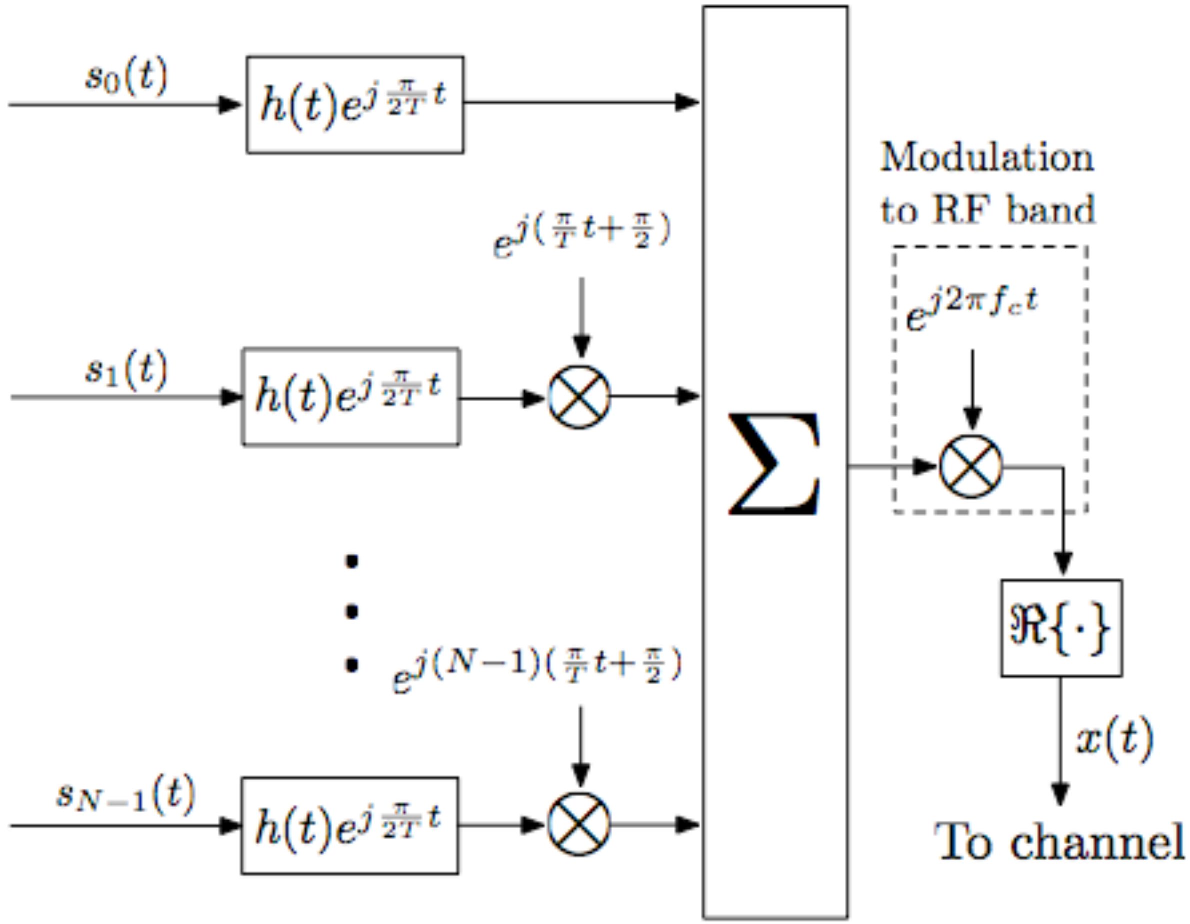
Implementation of MCFB: CMT (synthesis/transmitter)



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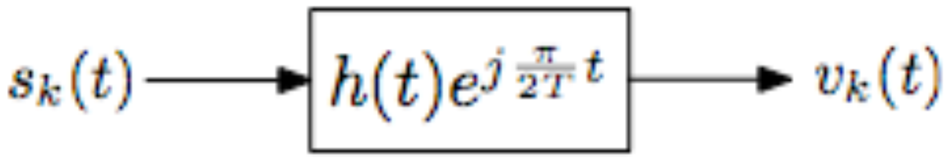
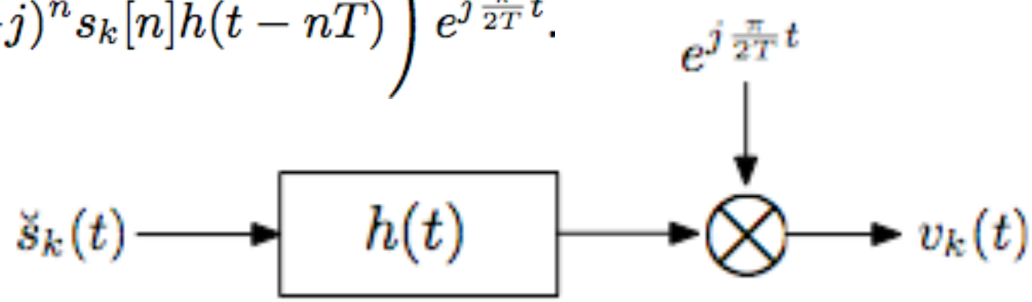
Implementation of MCFB: CMT (synthesis/transmitter)



$$s_k(t) = \sum_{n=-\infty}^{\infty} s_k[n]\delta(t - nT)$$

$$\begin{aligned} v_k(t) &= \sum_{n=-\infty}^{\infty} s_k[n]h(t-nT)e^{j\frac{\pi}{2T}(t-nT)} \\ &= \left(\sum_{n=-\infty}^{\infty} (-j)^n s_k[n]h(t-nT)\right) e^{j\frac{\pi}{2T}t}. \end{aligned}$$



$\breve{s}_k(t) = \sum_{n=-\infty}^{\infty} \breve{s}_k[n]\delta(t-nT)$,
where $\breve{s}_k[n] = (-j)^n s_k[n]$

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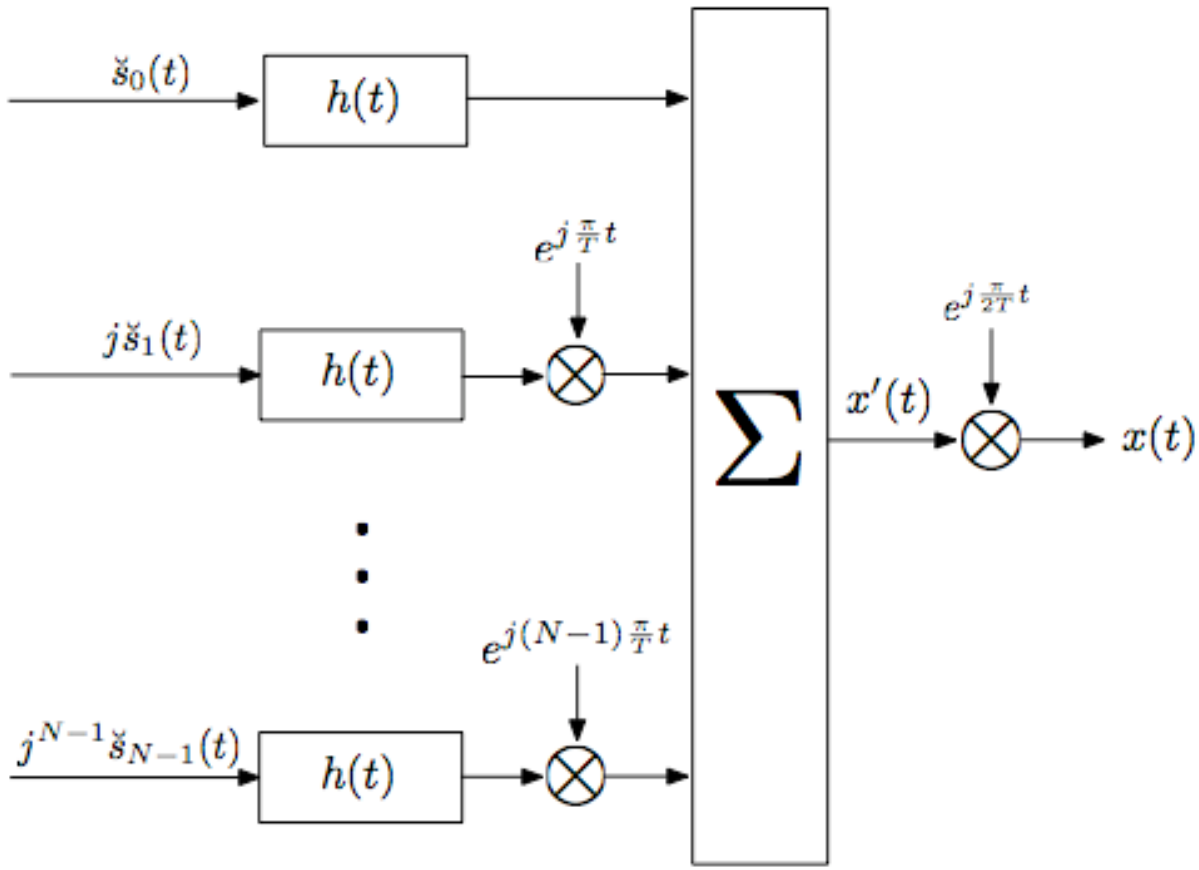
Implementation of MCFB: CMT (synthesis/transmitter)

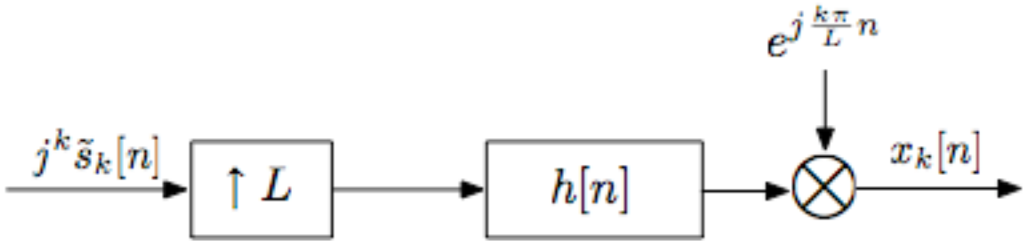
Using the results of the previous slide, the following structure is obtained.



Notes:

- The phase shifts of π/2 in the modulators have been extracted and added to the inputs.
- The subcarrier spacing is π/T (not 2π/T).

Implementation of MCFB: CMT (synthesis/transmitter)



In discrete-time

$$x_k[n] = \left(\sum_{m=-\infty}^{\infty} j^k \breve{s}_k[m] h[n-mL] \right) e^{j\frac{k\pi}{L}n}.$$

Noting that $e^{j\frac{k\pi}{L}n} = (-1)^m e^{j\frac{k\pi}{L}(n-mL)}$, we obtain

$$x_k[n] = \sum_{m=-\infty}^{\infty} j^k \tilde{s}_k[m] h_k[n-mL]$$

where $\tilde{s}_k[n] = (-1)^n \breve{s}_k[n] = j^n s_k[n]$ and $h_k[n] = h[n]e^{j\frac{k\pi}{L}n}$. We note that $h_k[n]$ is a modulated version of $h[n]$. If $h[n]$ is a lowpass filter centered around $f = 0$, $h_k[n]$ will be a bandpass filter centered around $f = \frac{k}{2L}$.

Implementation of MCFB: CMT (synthesis/transmitter)

From the results of the previous slide, we obtain

$$X_k(z) = j^k \tilde{S}_k\left(z^L\right) H_k(z)$$

Implementation of MCFB: CMT (synthesis/transmitter)

In discrete-time, $x'[n]$ and $x[n]$ relate according to the equation $x[n] = x'[n]e^{j\frac{\pi}{2L}n} = x'[n]W_{2L}^{-\frac{1}{2}n}$ which in z-domain implies

$$X(z) = X'(zW_{2L}^{\frac{1}{2}})$$

We also note that

$$X'(z) = \sum_{k=0}^{N-1} X_k(z).$$

Moreover,

$$H_k(z) = \sum_n h[n]z^{-n}W_{2L}^{kn}$$

From these, we obtain

$$X'(z) = \sum_{k=0}^{N-1}\sum_n j^k \tilde{S}_k\left(z^L\right) h[n]z^{-n}W_{2L}^{kn}.$$

Introducing the change of variable $n = 2mL + l$, we get

$$X'(z) = \sum_{k=0}^{N-1}\sum_{l=0}^{2L-1} j^k \tilde{S}_k\left(z^L\right) W_{2L}^{kl} E_l(z^{2L})z^{-l}$$

where $E_l(z)$ is the lth polyphase component of $H(z)$, defined as

$$E_l(z) = \cdots + h[l-2L]z + h[l] + h[l+2L]z^{-1} + \cdots .$$

Note that here the decimation factor is $2L$.

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Implementation of MCFB: CMT (synthesis/transmitter)

Next, we note that

$$X'(z) = \overbrace{[\tilde{S}_0(z^L)\ j\tilde{S}_1(z^L)\ \cdots\ j^{N-1}\tilde{S}_{N-1}(z^L)\ 0\ \cdots\ 0]}^{1\times 2L}\mathcal{F}$$
$$\times \begin{bmatrix} E_0(z^{2L}) & 0 & \cdots & 0 \\ 0 & E_1(z^{2L}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{2L-1}(z^{2L}) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(2L-1)} \end{bmatrix}$$

where $\mathcal{F}$ is the $2L \times 2L$ Fourier transform matrix defined as

$$\mathcal{F} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & W_{2L} & \cdots & W_{2L}^{2L-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & W_{2L}^{2L-1} & \cdots & W_{2L}^{(2L-1)(2L-1)} \end{bmatrix}.$$

Implementation of MCFB: CMT (synthesis/transmitter)

Using the results of the past two slides, we get

$$X(z) = \overbrace{[\tilde{S}_0(jz^L)\ j\tilde{S}_1(jz^L)\ \cdots\ j^{N-1}\tilde{S}_{N-1}(jz^L)\ 0\ \cdots\ 0]}^{1\times 2L}\mathcal{F}$$

$$\times \begin{bmatrix} E_0(-z^{2L}) & 0 & \cdots & 0 \\ 0 & E_1(-z^{2L}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{2L-1}(-z^{2L}) \end{bmatrix} \times \begin{bmatrix} 1 \\ W_{2L}^{-\frac{1}{2}} z^{-1} \\ \vdots \\ W_{2L}^{-\frac{2L-1}{2}} z^{-(2L-1)} \end{bmatrix}.$$

Also, noting that $\tilde{S}(z) = S(z/j)$, we get

$$\tilde{S}(jz) = S(z).$$

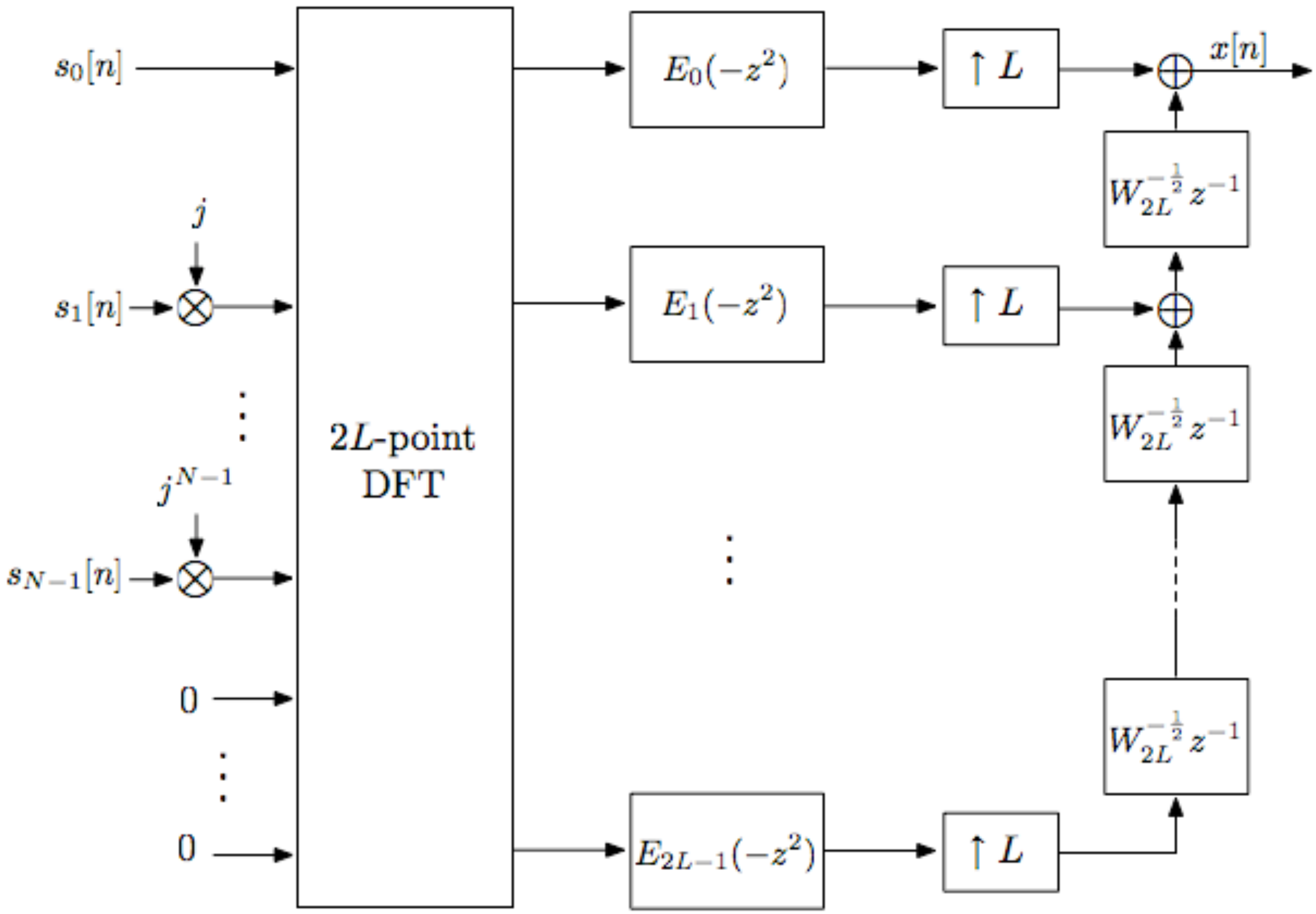
Substituting z by z^L, we obtain

$$X(z) = \overbrace{[S_0(z^L)\ jS_1(z^L)\ \cdots\ j^{N-1}S_{N-1}(z^L)\ 0\ \cdots\ 0]}^{1\times 2L}\mathcal{F}$$

$$\times \begin{bmatrix} E_0(-z^{2L}) & 0 & \cdots & 0 \\ 0 & E_1(-z^{2L}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{2L-1}(-z^{2L}) \end{bmatrix} \times \begin{bmatrix} 1 \\ W_{2L}^{-\frac{1}{2}} z^{-1} \\ \vdots \\ W_{2L}^{-\frac{2L-1}{2}} z^{-(2L-1)} \end{bmatrix}.$$

Implementation of MCFB: CMT (synthesis/transmitter)

The above results leads to the synthesis/transmitter structure:



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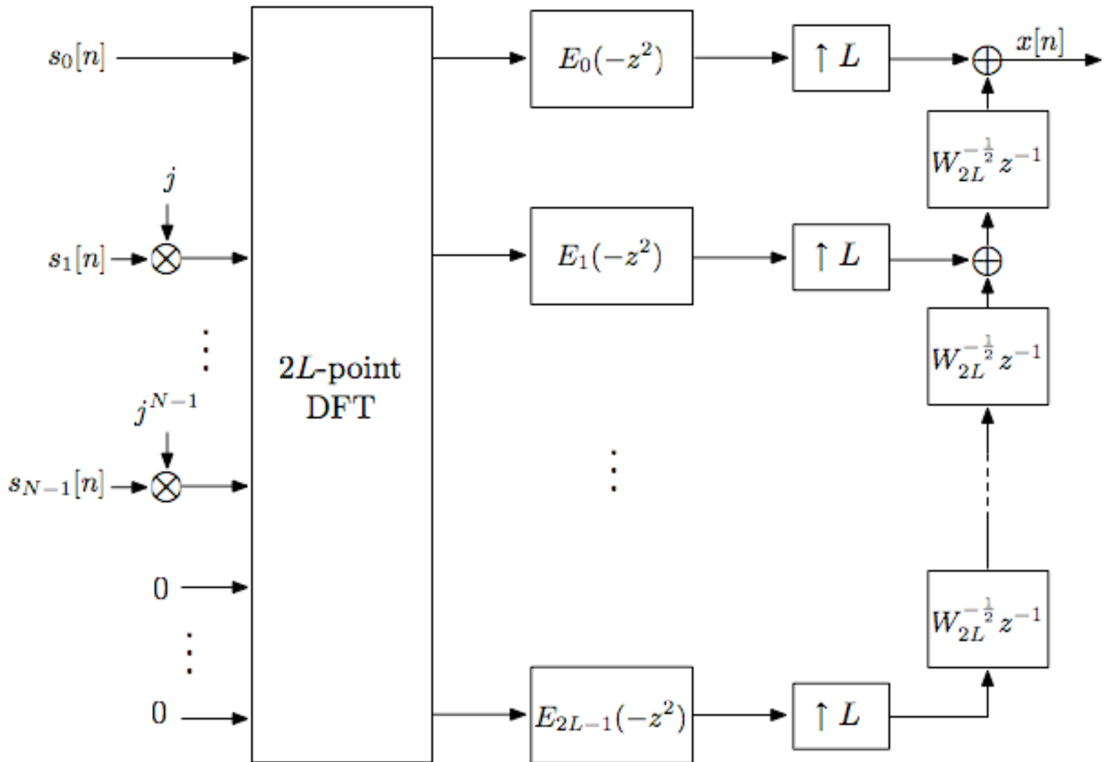
Implementation of MCFB: CMT (synthesis/transmitter)

The above results leads to the synthesis/transmitter structure:



$$X(z) = \overbrace{[S_0(z^L)\ jS_1(z^L)\ \cdots\ j^{N-1}S_{N-1}(z^L)\ 0\ \cdots\ 0]}^{1\times 2L}\mathcal{F} \times \begin{bmatrix} E_0(-z^{2L}) & 0 & \cdots & 0 \\ 0 & E_1(-z^{2L}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{2L-1}(-z^{2L}) \end{bmatrix} \times \begin{bmatrix} 1 \\ W_{2L}^{-\frac{1}{2}} z^{-1} \\ \vdots \\ W_{2L}^{-\frac{2L-1}{2}} z^{-(2L-1)} \end{bmatrix}.$$

Implementation of MCFB: CMT (analysis/receiver)

In CMT, the synthesis is performed through a set of expanders followed by VSB filters:

$$X(z) = [S_0(z^L)\ S_1(z^L)\ \cdots\ S_{N-1}(z^L)\ 0\ \cdots\ 0] \begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{2L-1}(z) \end{bmatrix}.$$

Note that here only the first N filters have active role.

The same filter set is used for the analysis. Thus,

$$\begin{bmatrix} S_0^{\mathrm{c}}(z) \\ S_1^{\mathrm{c}}(z) \\ \vdots \\ S_{N-1}^{\mathrm{c}}(z) \\ * \\ \vdots \\ * \end{bmatrix} = \begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{2L-1}(z) \end{bmatrix} X(z)$$

where stars indicate the redundant output and the superscripts 'c' are added to emphasize that the generated output sequences are complex-valued. Also, note that these output sequences are not decimated. Decimations (equivalent to samplers) is performed at the next step.

Implementation of MCFB: CMT (analysis/receiver)

From the previous results, one finds that

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{2L-1}(z) \end{bmatrix} = \mathrm{diag}(1, j, \cdots, j^{N-1}, 0, \cdots, 0)\mathcal{F}$$

$$\times \begin{bmatrix} E_0(-z^{2L}) & 0 & \cdots & 0 \\ 0 & E_1(-z^{2L}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{2L-1}(-z^{2L}) \end{bmatrix}$$

$$\times \begin{bmatrix} 1 \\ W_{2L}^{-\frac{1}{2}} z^{-1} \\ \vdots \\ W_{2L}^{-\frac{2L-1}{2}} z^{-(2L-1)} \end{bmatrix}.$$

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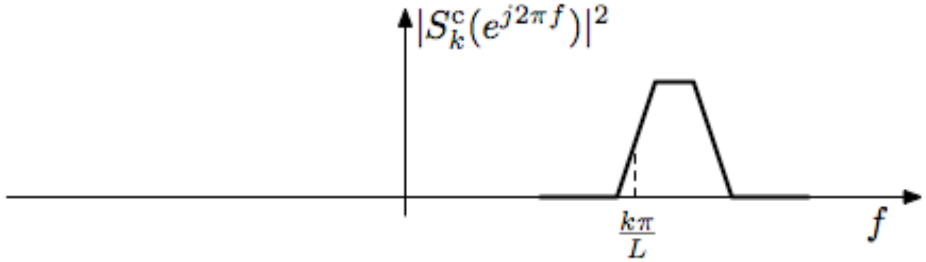
Implementation of MCFB: CMT (analysis/receiver)

Signal spectra pertinent to demodulation process in CMT. The spectra are at the output of the kth analysis filter:

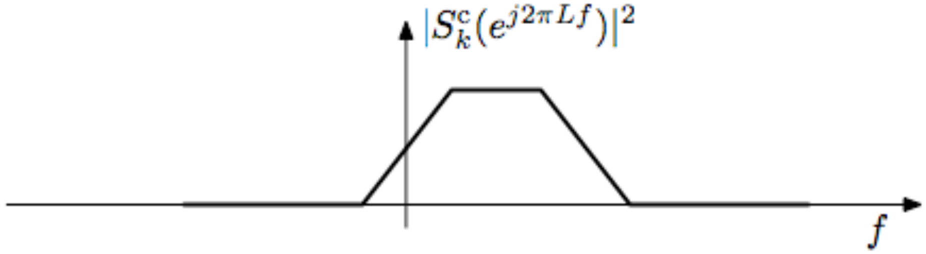
(a) before decimation,

(b) (b) after decimation for k even,

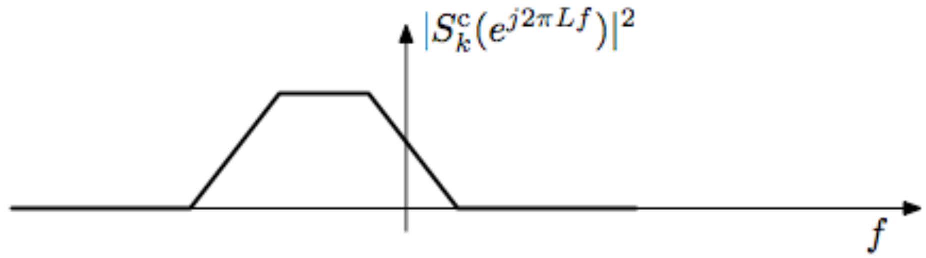
(c) (c) after decimation for k odd.



(a)



(b)



(c)

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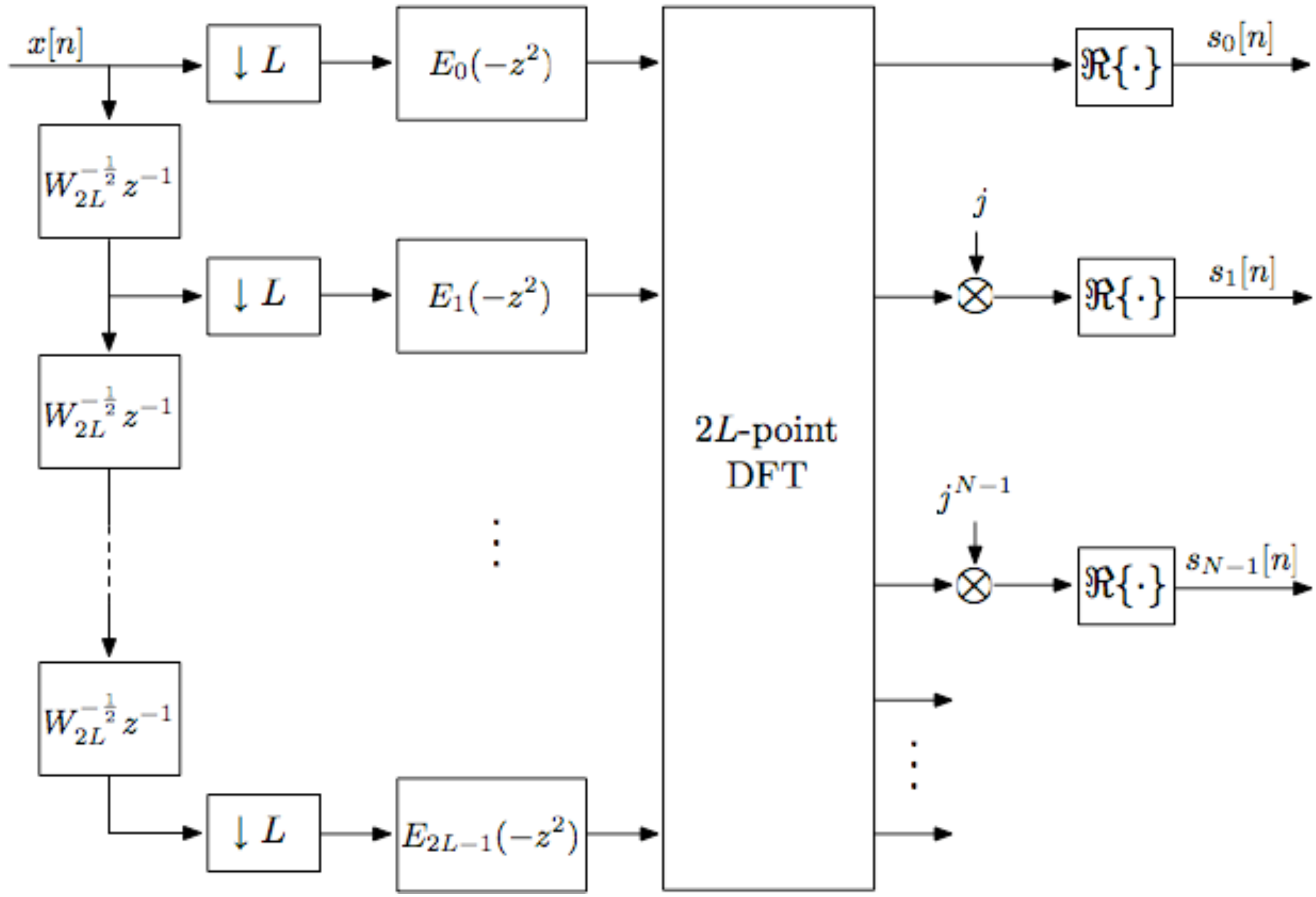
Implementation of MCFB: CMT (analysis/receiver)

Finally, from the results of the previous slide, the analysis structure is obtained as



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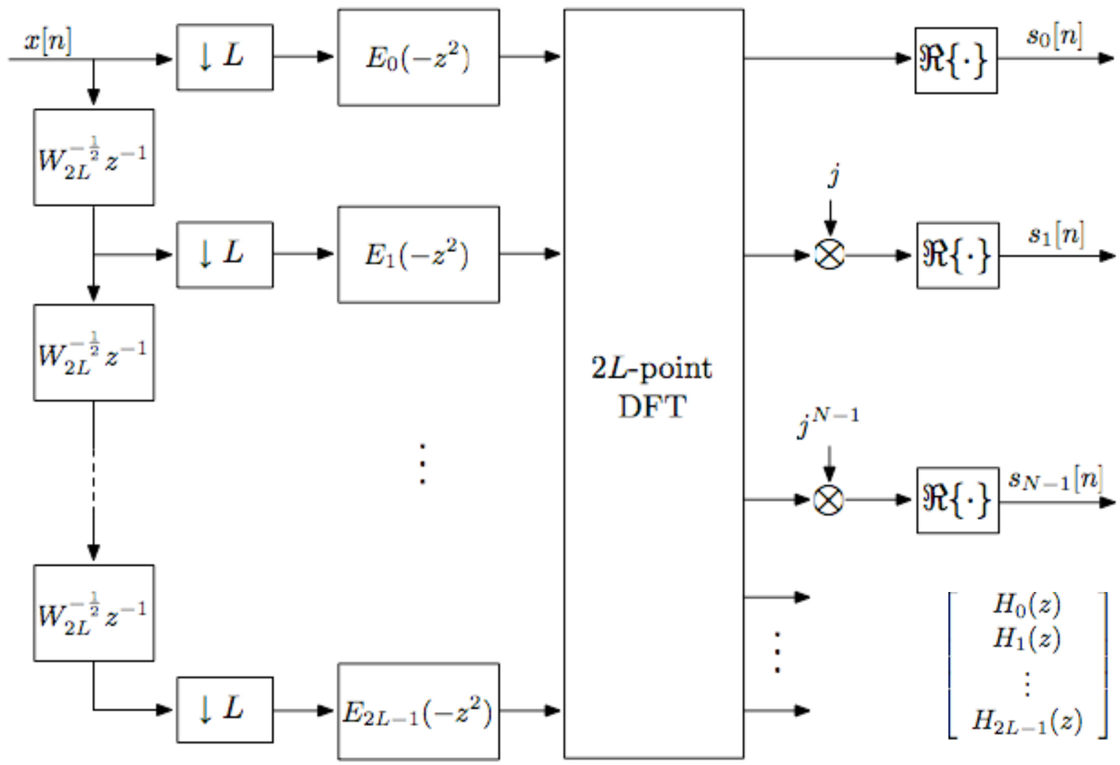
Implementation of MCFB: CMT (analysis/receiver)

Finally, from the results of the previous slide, the analysis structure is obtained as



$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{2L-1}(z) \end{bmatrix} = \mathrm{diag}(1, j, \cdots, j^{N-1}, 0, \cdots, 0)\mathcal{F}$$

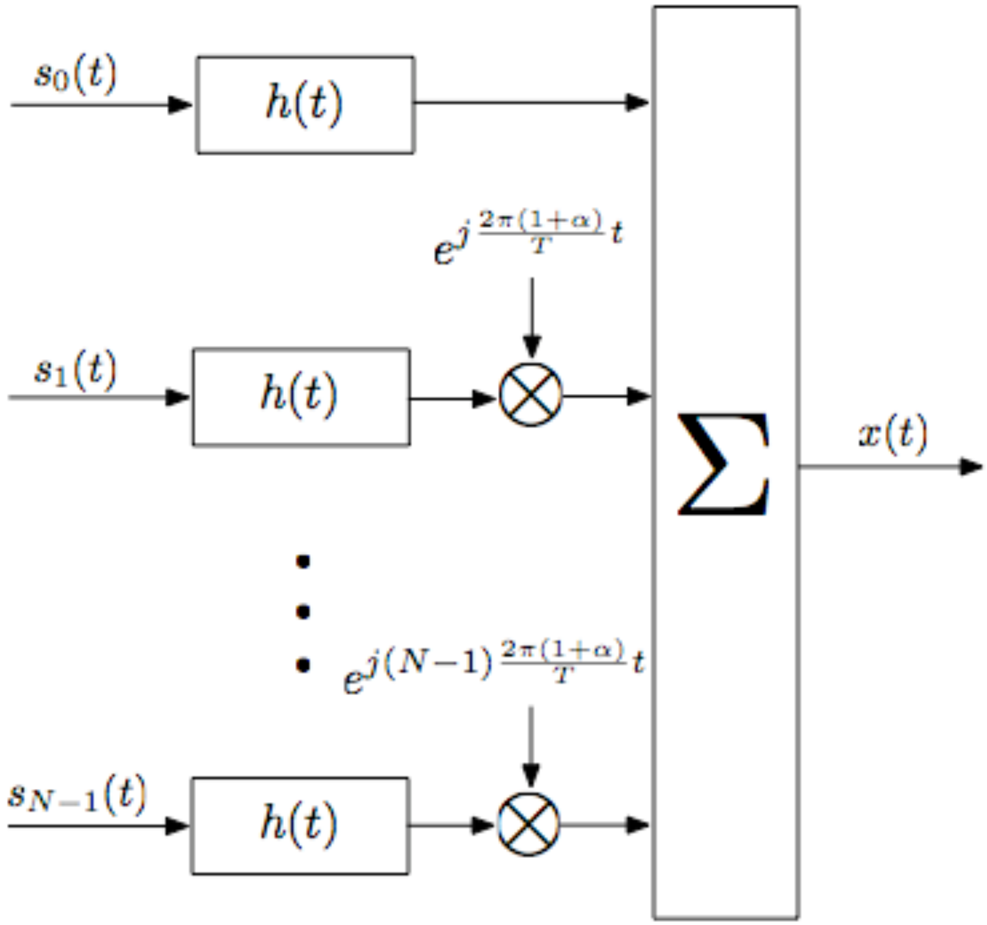
$$\times \begin{bmatrix} E_0(-z^{2L}) & 0 & \cdots & 0 \\ 0 & E_1(-z^{2L}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{2L-1}(-z^{2L}) \end{bmatrix}$$

$$\times \begin{bmatrix} 1 \\ W_{2L}^{-\frac{1}{2}} z^{-1} \\ \vdots \\ W_{2L}^{-\frac{2L-1}{2}} z^{-(2L-1)} \end{bmatrix}.$$

Filtered Multitone (FMT)

Implementation of MCFB: FMT (Synthesis/transmitter)

Transmitter:



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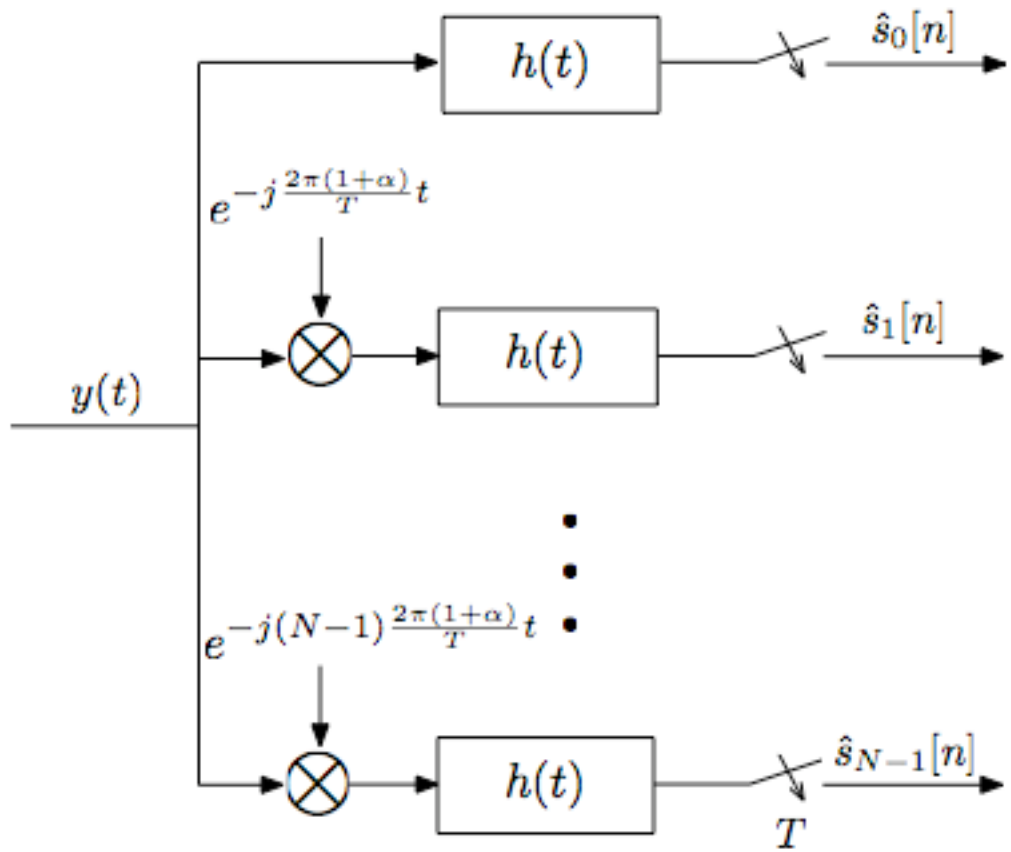
Implementation of MCFB: FMT (Analysis/receiver)

Receiver:



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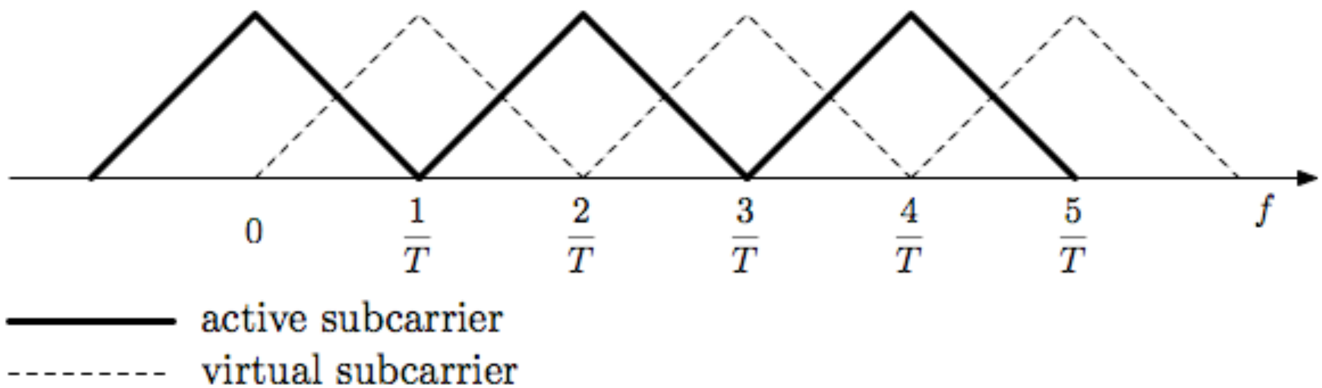
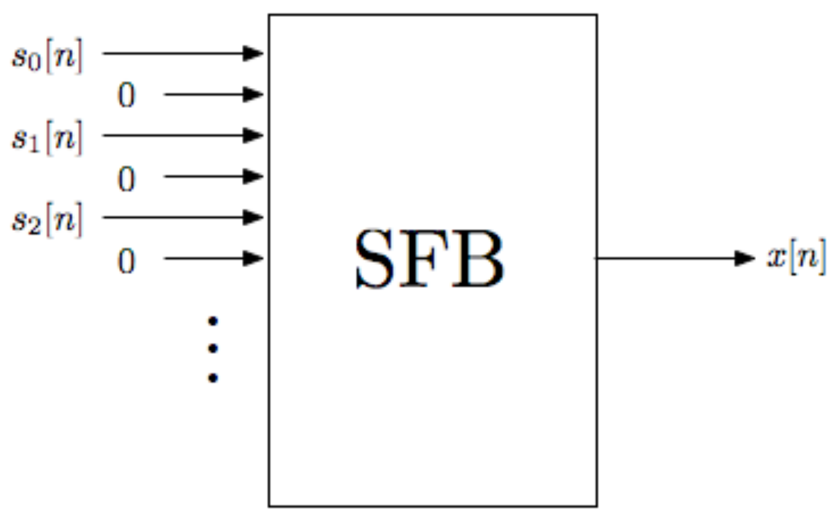
Implementation of MCFB: FMT (Synthesis/transmitter)

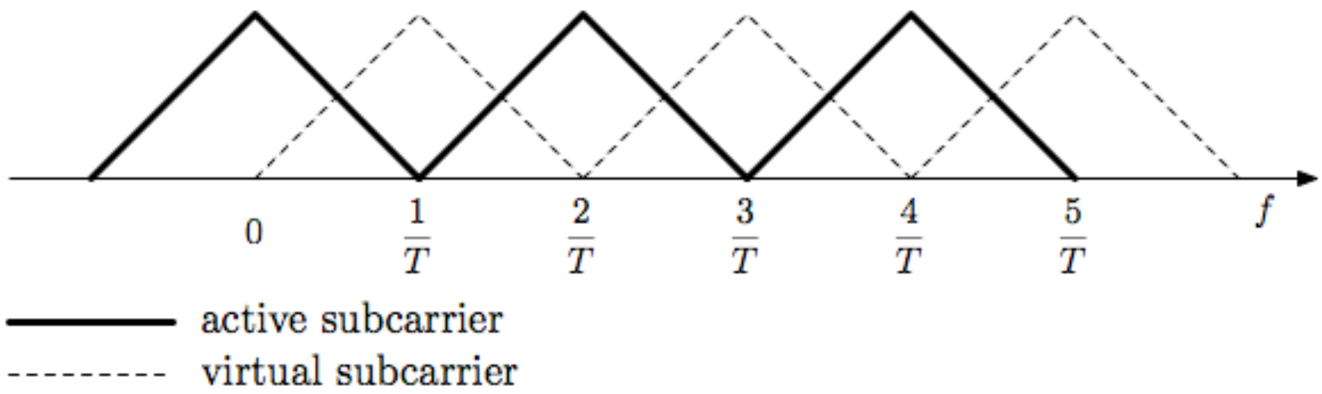
Case of α = 1:

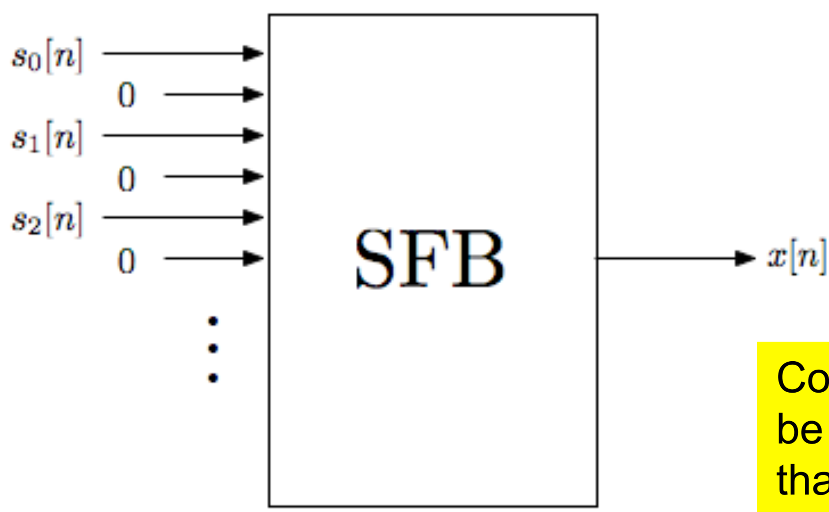




Implementation of MCFB: FMT (Synthesis/transmitter)

Case of α = 1:

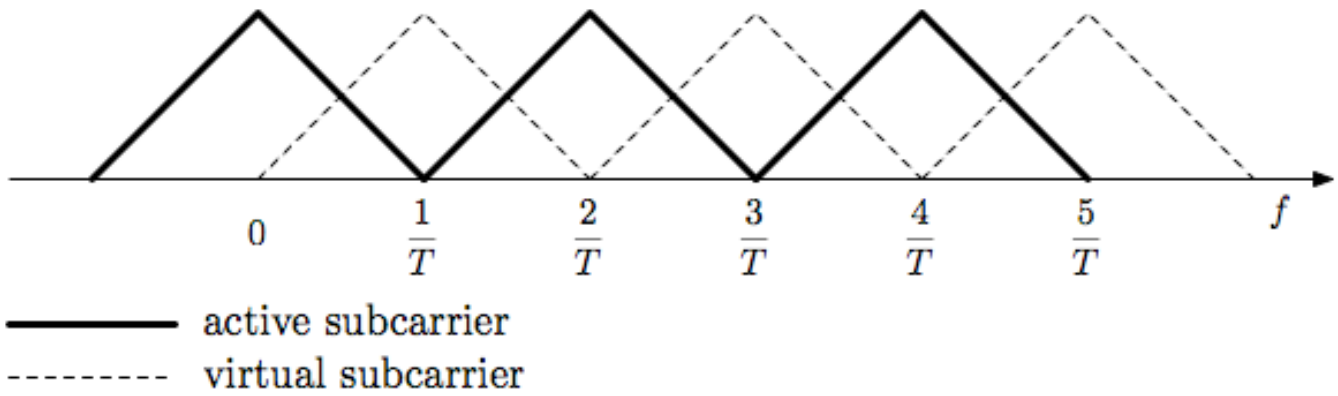


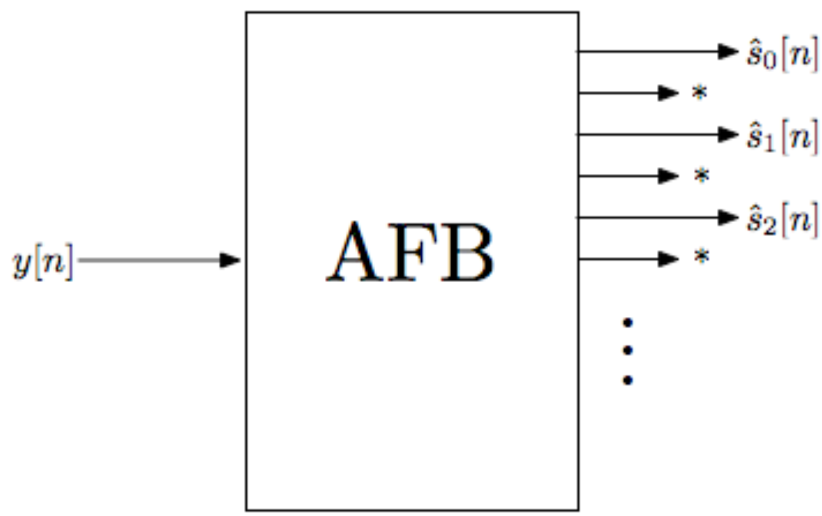


Computational saving can be made, using the fact that alternate inputs are zero.

Implementation of MCFB: FMT (Analysis/receiver)

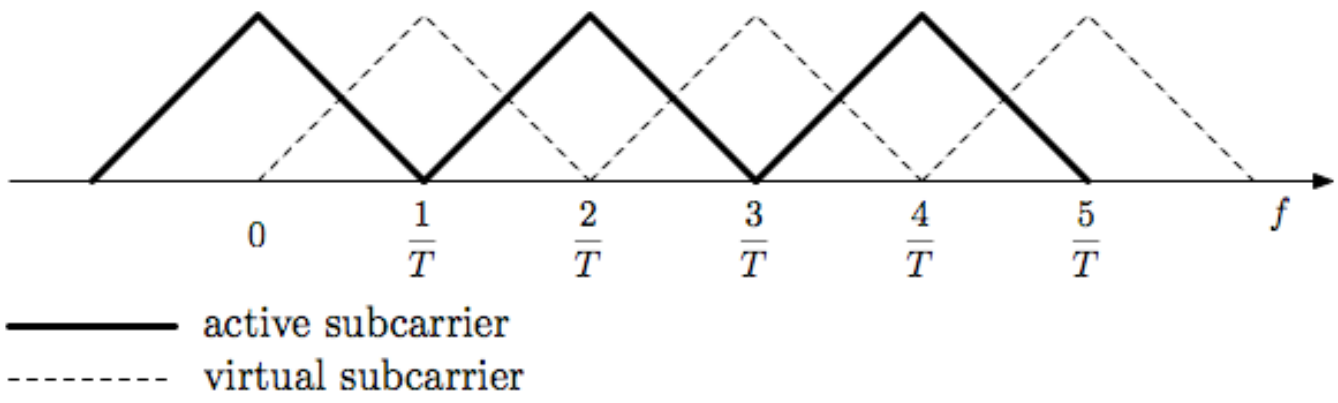
Case of α = 1:

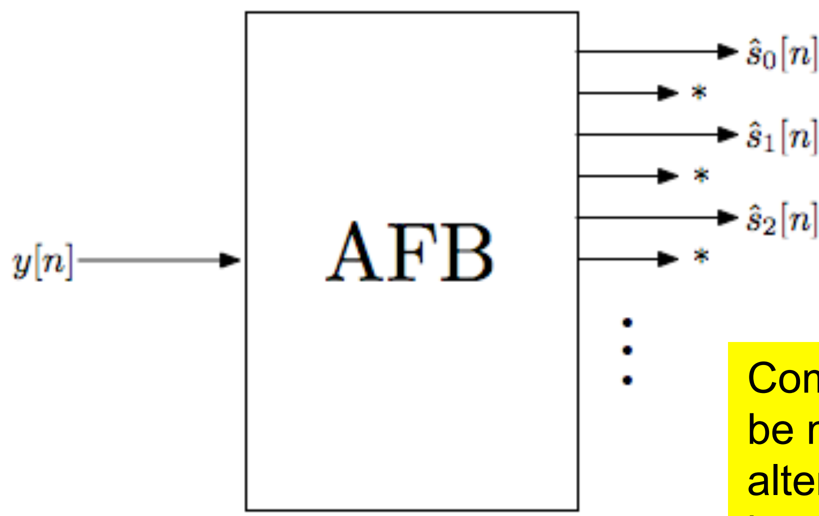




Implementation of MCFB: FMT (Analysis/receiver)

Case of $\alpha = 1$:





Computational saving can be made, noting that only alternate outputs should be calculated.

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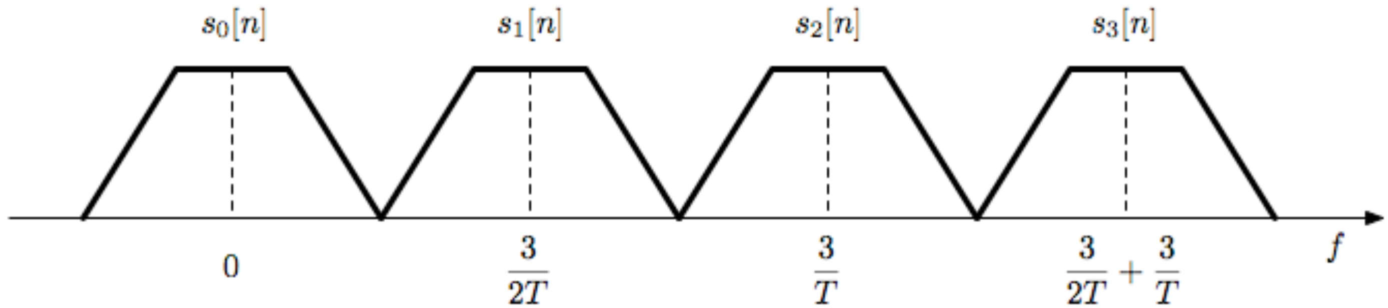
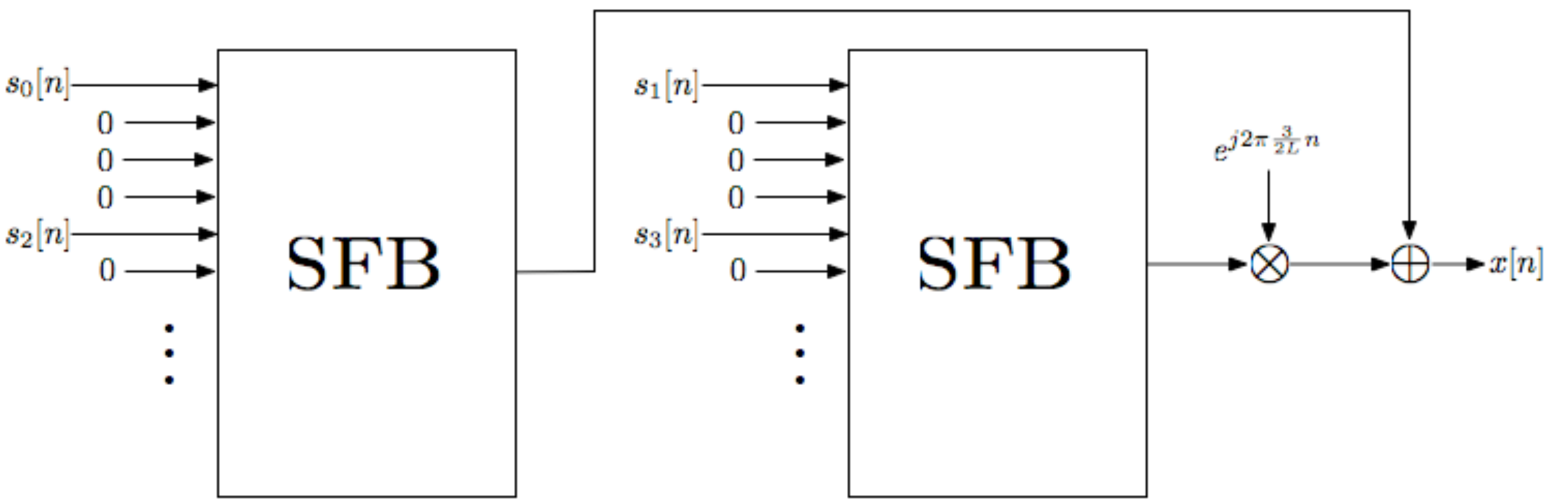
Implementation of MCFB: FMT (Synthesis/transmitter)

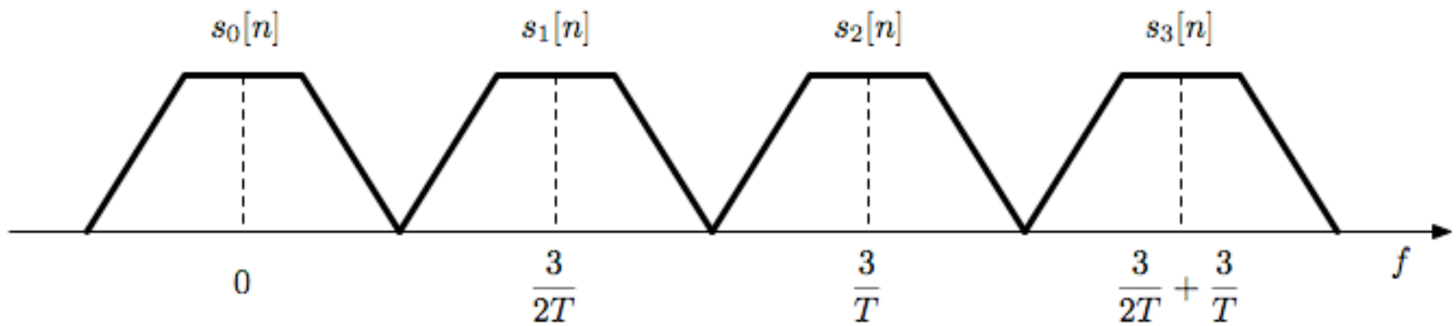
Case of α = 0.5:

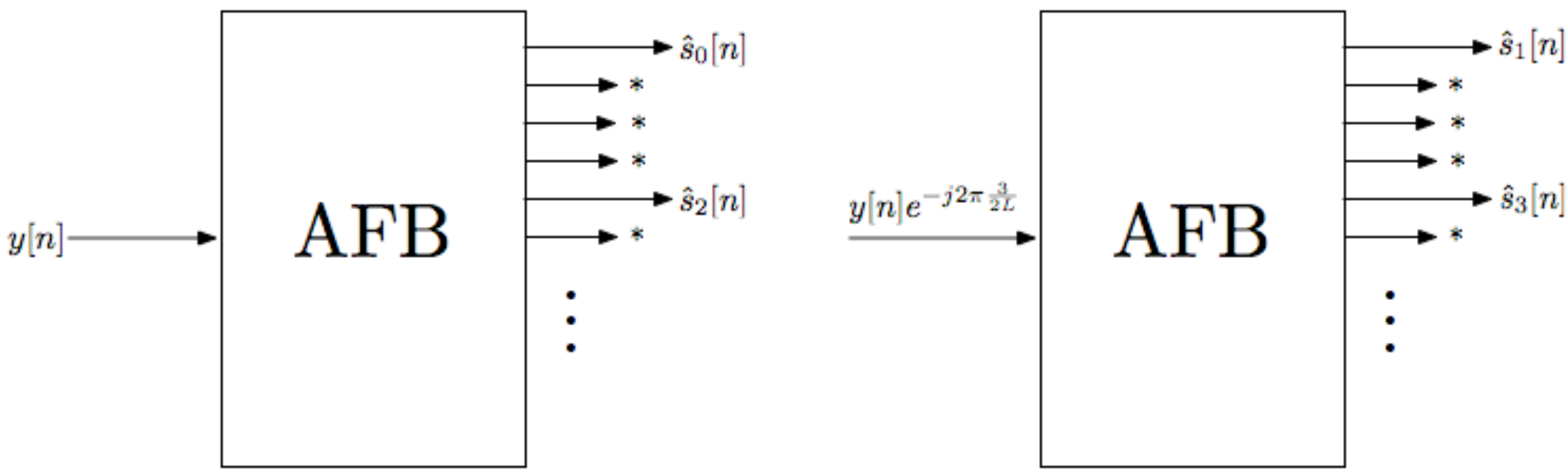




Implementation of MCFB: FMT (Analysis/Receiver)

Case of α = 0.5:





Conclusions

- We presented the general concept of multicarrier communication and channel sensing for CR networks.
  - Conventional OFDM needs many costly fixes before it can become applicable in CR networks
- Three methods of multicarrier communication (FMT, SMT, and CMT) were introduced and studied in detail.
  - These methods offer a number of advantages over OFDM: higher bandwidth efficiency, reduced intercarrier interference.
- Multitaper method (MTM), a near optimum spectral estimation method, was introduced and evaluated.
- We proposed the use of filter banks as a spectral estimation tool.
  - We compared the proposed filter bank spectral estimator with MTM and found that they behave very similarly

Other Relevant Works

In addition to the methods discussed in this tutorial, there is a body of the works that take advantage of the cyclostationarity of digital communication signals to detect such signals when they at level below the channel noise. Some relevant works are:

Cabric, D.; Mishra, S.M.; Brodersen, R.W., "Implementation issues in spectrum sensing for cognitive radios," in Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers, Volume 1, 7-10 Nov. 2004, pp. 772 - 776
AbstractPlus | Full Text: PDF(744 KB) IEEE CNF

Tkachenko, Artem; Cabric, Danijela; Brodersen, Robert W., "Cognitive Radio Experiments using Reconfigurable BEE2," Fortieth Asilomar Conference on Signals, Systems and Computers, Oct.-Nov. 2006,pp. 2041 - 2045.

Kim, Kyouwoong; Akbar, Ihsan A.; Bae, Kyung K.; Um, Jung-Sun; Spooner, Chad M.; Reed, Jeffrey H., "Cyclostationary Approaches to Signal Detection and Classification in Cognitive Radio," 2nd International Symposium on IEEE New Frontiers in Dynamic Spectrum Access Networks, DySPAN, 17-20 April 2007, pp. 212 - 215.

Ghozzi, Mohamed; Marx, Francois; Dohler, Mischa; Palicot, Jacques, "Cyclostatilonarilty-Based Test for Detection of Vacant Frequency Bands," 1st International Conference on Cognitive Radio Oriented Wireless Networks and Communications, 8-10 June 2006, pp. 1 - 5.

Ning Han; SungHwan Shon; Jae Hak Chung; Jae Moung Kim, "Spectral correlation based signal detection method for spectrum sensing,in IEEE 802.22 WRAN systems," The 8th International Conference onAdvanced Communication Technology, ICACT 2006, Volume 3, 20-22 Feb. 2006, 6 pp.

Thank You!

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