# BLIND ADAPTIVE EQUALIZATION OF MIMO IIR CHANNELS

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## ABSTRACT

**Abstract-** An adaptive filtering method is proposed for blind deconvolution of multiple input multiple output (MIMO) IIR channels. This method consists of two algorithms. The adaptive blind identification algorithm estimates the MIMO system impulse response. These estimates are used in an adaptive Weiner type filter to extract the instantaneous mixture of input sources. Such a mixture can be further processed by a blind source separation algorithm to obtain the individual sources. Only second order (SOS) statistics is used, and precise knowledge of the system order is not required as long as it is over-modeled. The developed algorithms are globally convergent.

## 1. INTRODUCTION

The objective of blind equalization of MIMO systems is to recover input (source) signals from noisy output observations without the knowledge of system impulse response, and where no training sequence is available or used. Such problems arise in applications related to communications, speech and image processing, biomedical signal analysis, etc. For example, in wireless communications, sensors may receive superposition of several signals via different channels from several mobile sources. During transmission, a source signal undergoes a convolutive distortion between its symbols and the channel impulse response and a mixture distortion from other source signals. These distortions are referred to as intersymbol interference (ISI) and interuser interference (IUI), respectively. Generally, equalization is a two phase process. The first phase involves removal of the convolutive effect of the system to produce an instantaneous mixture of source signals. Normally this task relies on second order statistics and it involves identification of the system impulse response (see for example [1]-[15]), and references therein. Complete source recovery is achieved by applying an appropriate blind source separation (BSS) algorithm. BSS methods exploit higher order statistics (see for example [18], [19]), and the references therein). Most of the above work assumes that the system is FIR, irreducible, and in MIMO case column reduced. In [12], IIR systems with common minimum phase factors are considered, and the MIMO system does not have to be column reduced. The basis of the proposed solution is multistep linear prediction.

Subspace methods for channel estimation have been studied in [1]-[3]. Linear prediction methods were considered in [9], [13], and [11]. Direct design of equalizers based on whitening approach is presented in [4], [12], and [14]. In [15] blind identification and equalization of SIMO systems is considered by using multiple zero-forcing equalizers that whiten the noise-free data at multiple delays. Prediction error methods and whitening approaches do not require knowledge of the system order, as long as one over fits. In [20], a nonlinear adaptive whitening method is proposed for blind deconvolution of MIMO systems by whitening the received data in both time and space. Blind deconvolution of MIMO linear convolutive mixtures by using a set of hierarchical maximum entropy type criteria is discussed in [16]. A system theoretic foundation for blind equalization of FIR MIMO systems is investigated in [17]. It is shown that every channel has an FIR irreducible paraunitary factorization, and can be reduced to a paraunitary FIR system by decorrelation and using only SOS.

In this paper we propose a new recursive algorithm for blind adaptive equalization of IIR MIMO systems. The paper is organized as follows. In section II, we present the system model with underlying assumptions. Section III presents algorithms for blind system identification, and temporal whitening. Section IV gives illustrative simulation experiments.

Notation: In this paper $(.)^T$ denotes the usual transpose operation , $(.)^*$ stands for complex conjugate transpose, $I$ is the identity matrix of dimension $m \times m$, while $O$ $m \times n$ is the zero matrix of dimension $m \times n$. Whenever it is clear from the context, the dimensions of $I$ or $O$ will be omitted.

## 2. PROBLEM STATEMENT

Consider a discrete-time Multiple Input Multi Output (MIMO) system with $l$ inputs and $m$ outputs

$$y(i) = x(i) + w(i), \quad \mathrm{i} = 0,1,2,... \tag{1}$$
$$x(i) = H(z^{-1})s(i)$$

where $y(i)$ is the $m$ dimensional output vector, $s(i)$ is an $l$ dimensional source vector, $w(i)$ is an $m$ dimensional noise vector, while $H(z^{-1})$ is the transfer function operator given by

$$H(z^{-1}) = \sum_{k=0}^{\infty} H_k z^{-k} \tag{2}$$

with $\{H_k\}, k \geq 0$ being an $m \times l$ matrix sequence called the system impulse response, and $z^{-1}$ is unit delay operation. All of the above variables can be complex valued. The objective is to recover the inputs $s(i)$ based on the noisy observations $y(i)$, without the knowledge of the transfer operator $H(z^{-1})$. This task is generally a two stage process. First, the convolutive effect of the channel is removed using second order statistics. As a result an instantaneous mixture of the input is obtained. The second part performs source separation by means of Higher Order Statistics (HOS). Equation (1) approximately models a situation when an antenna array is used, or when a single receiver of multiple sources is fractionally sampled.

We make the following assumptions regarding the system model (1).

*Assumption A1:* $H(z^{-1}) = D(z^{-1})^{-1}N(z^{-1})$, *where* $D(z^{-1}) = I_m + \sum_{k=1}^{n_D} D_k z^{-k}$ *is* $m \times m$ *polynomial matrix, and* $N(z^{-1}) = \sum_{k=0}^{n_N} N_k z^{-k}$ *is of dimension* $m \times l$, *where* $m > l$.

*Assumption A2:* $rank[N(z^{-1})] = l$ *for all complex* $z \neq 0$, *i.e.* $N(z^{-1})$ *is irreducible.*

*Assumption A3:* $\det[D(z^{-1})] \neq 0$ *for* $|z| \geq 1$.

*Assumption A4:* $\{s(k)\}, k \geq 0$ *is a* $l$ *dimensional martingale difference sequence satisfying*

$$E\{s(k+1) \mid F_1(k)\} = 0,$$

$$E\{s(k+1)s(k+1)^* \mid F_1(k)\} = I_l,$$

*and*

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n} s(k)s(k)^* = I_l$$

*where* $F_1(k) = \{s(0), \cdots, s(k)\}$. *Components of the vector* $s(k)$ *are independent, with nonzero fourth order cumulants.*

*Assumption A5:* $\{w(k)\}, k \geq 0$ *is a* $m$ *dimensional martingale difference sequence satisfying*

$$E\{w(k+1) \mid F_2(k)\} = 0,$$

$$E\{w(k+1)w(k+1)^* \mid F_2(k)\} = \sigma_w^2 I_m,$$

*and*

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n} w(k)w(k)^* = \sigma_w^2 I_m$$

where $F_2(k) = \{w(0), \cdots, w(k)\}$. Also $s(k)$ and $w(k)$ are independent sequence.

$$E[s(k)s(k+i)^*] = \begin{cases} I_l, & i = 0 \\ O, & i \neq 0 \end{cases}, \tag{3}$$

and

$$E[w(k)w(k+i)^*] = \begin{cases} \sigma_w^2 I_m, & i = 0 \\ O, & i \neq 0 \end{cases} \tag{4}$$

where $O$ in (3) and (4) is zero matrix of appropriate dimension.

It can be easily shown that $x(i)$ defined by (1) can be represented as a finite order autoregressive process driven by $\{N_0 s(i)\}, i > 0$. Following the well-known Bezout identity of polynomial matrices will be useful for this development.

Lemma1 [21]: Let assumption A2 hold and $m > l$. Then there exists $l \times m$ polynomial matrix $F(z^{-1}) = \sum_{k=0}^{n_F} F_K z^{-k}$ with finite order $n_F$, such that $F(z^{-1})N(z^{-1}) = I_l$.

By assumption A1, $x(i)$ from (1) can be written as follows

$$D(z^{-1})x(i) = N(z^{-1})s(i). \tag{5}$$

Application of Lemma 1 in (5) gives

$$F(z^{-1})D(z^{-1})x(i) = s(i)\,. \quad (6)$$

On the other hand (5) can be transformed into

$$D(z^{-1})x(i) = N_0 s(i) + N^1(z^{-1})s(i-1)\,, \quad (7)$$

where $N^1(z^{-1}) = 2[N(z^{-1}) - N_0]$. Substituting $s(i)$ from (6) in the second term, the RHS of (7) yields

$$D(z^{-1})x(i) = N_0 s(i) + N^1(z^{-1})F(z^{-1})D(z^{-1})x(i-1)\,. \quad (8)$$

Therefore one obtains

$$A(z^{-1})x(i) = N_0 s(i)\,, \quad (9)$$

with

$$A(z^{-1}) = [I_m - z^{-1}N^1(z^{-1})F(z^{-1})]D(z^{-1})\,. \quad (10)$$

For the future reference we write the matrix polynomial $A(z^{-1})$ in the form

$$A(z^{-1}) = I_m + \sum_{k=1}^{n_A} A_k z^{-k}\,. \quad (11)$$

It can be shown that $n_A \geq n_D + (2l)n_N - 1$.

Observe that from (1) and (10) we can arrive at the following signal model:

$$A(z^{-1})[y(i) - w(i)] = N_0 s(i)\,. \quad (12)$$

## 3. BLIND ADAPTIVE EQUALIZATION

In this section, we develop an adaptive algorithm for obtaining an estimate of the instantaneous mixture $N_0 s(i), i \geq 0$. The algorithm consists of two parts. Part one estimates the polynomial matrix $A(z^{-1})$. Part two uses this knowledge in an adaptive Wiener type filter to remove the $A(z^{-1})w(i)$ component from $A(z^{-1})y(i)$ in (12).

Define

$$\theta^* = [A_1, \ldots, A_{n_A}]\,. \quad (13)$$

The following algorithm is proposed to adaptively estimate $\theta^*$ for $i \geq 1$.

$$\begin{aligned}\hat{\theta}(i) = \hat{\theta}(i-1) + p(i)\varphi(i-1)[y(i)^* - \varphi(i-1)^*\hat{\theta}(i-1)] \\ + p(i)\hat{\sigma}_w^2[(i-1)\hat{\theta}(i-1) - (i-2)\hat{\theta}(i-2)],\end{aligned} \quad (14)$$

where

$$\varphi(i-1)^T = [-y(i-1)^T, \ldots, -y(i-n_A)^T]\,, \quad (15)$$

$$p(i) = p(i-1) - \frac{p(i-1)\varphi(i-1)\varphi(i-1)^* p(i-1)}{1 + \varphi(i-1)^* p(i-1)\varphi(i-1)}\,, \quad (16)$$

and $\hat{\sigma}_w$ is an estimate of $\sigma_w$ from assumption A5. Initial $\hat{\theta}(0)$ is an arbitrary vector of finite norm, $p(0)$ is an arbitrary positive definite matrix. Typical choice is $p(0) = p_0 I$, where $p_0$ is a positive scalar. In the following, we give the heuristics behind the algorithm (14)-(16). Note that (10) can be written in the form

$$x(i) = \theta^* \varphi_x(i-1) + N_0 s(i)\,, \quad (17)$$

where $\theta^*$ is defined by (13), while

$$\varphi_x(i-1)^T = [-x(i-1)^T, \ \ldots, \ -x(i-n_A)^T]. \quad (18)$$

Minimum mean-square estimate of $\theta$ is obtained by minimizing the following cost function

$$J_1 = E[(x(i) - \hat{\theta}^*\varphi_x(i-1))(x(i)^* - \varphi_x(i-1)^*\hat{\theta})]\,. \quad (19)$$

Setting to zero the gradient of $J_1$ with respect to $\hat{\theta}^*$ gives

$$E[\varphi_x(i-1)x(i)^*] = E[\varphi_x(i-1)\varphi_x(i-1)^*]\hat{\theta}\,. \quad (20)$$

Define

$$\varphi_w(i-1)^T = [-w(i-1)^T, \ \cdots, \ -w(i-n_A)^T]. \quad (21)$$

Then we have

$$\varphi(i) = \varphi_x(i) + \varphi_w(i)\,, \quad (22)$$

where we have used the fact that by (1) $y(i) = x(i) + w(i)$. By using (22) and assumption A5, one can derive

$$E[\varphi_x(i-1)\varphi_x(i-1)^*] = E[\varphi(i-1)\varphi(i-1)^*] - \sigma_w^2 I_{mn_A}\,. \quad (23)$$

Similarly we can obtain

$$\begin{aligned}&E[\varphi_x(i-1)x(i)^*] = E[(\varphi_x(i-1) + \varphi_w(i-1))x(i)^*] \\ &= E[\varphi(i-1)x(i)^*] = E[\varphi(i-1)(x(i) + w(i))^*] \\ &= E[\varphi(i-1)y(i)^*],\end{aligned} \quad (24)$$

where we have used the fact that by assumption A5, $E[\psi_w(i-1)x(i)^*] = 0$ and by (4) $E[\varphi(i-1)w(i)^*] = 0$. By substituting (23) and (24) in (20) we get

$$E[\varphi(i-1)y(i)^*] = E[\varphi(i-1)\varphi(i-1)^* - \sigma_w^2 I_{mn_A}]\hat{\theta}\,. \quad (25)$$

Replacing expectations in the previous equation with sample averages, one can obtain

$$\frac{1}{i}\sum_{k=1}^{i}\varphi(k-1)y(k)^* = \frac{1}{i}\sum_{k=1}^{i}\varphi(k-1)\varphi(k-1)^*\hat{\theta}(i) - \sigma_w^2\hat{\theta}(i),$$

or

$$\sum_{k=1}^{i}\varphi(k-1)y(k)^* = \sum_{k=1}^{i}\varphi(k-1)\varphi(k-1)^*\hat{\theta}(i) - \sigma_w^2 i\hat{\theta}(i), \quad (26)$$

where $\hat{\theta}$ in (25) is replaced with $\hat{\theta}(i)$ to signify the fact that it is the estimate derived based on the observations up to sample time $i$. If in (26) $i$ is replaced with $i-1$, we have

$$\sum_{k=1}^{i-1}\varphi(k-1)y(k)^* = \sum_{k=1}^{i-1}\varphi(k-1)\varphi(k-1)^*\hat{\theta}(i-1) - (i-1)\sigma_w^2\hat{\theta}(i-1). \quad (27)$$

Subtracting (27) from (26) yields

$$\varphi(i-1)y(i)^* = p(i)^{-1}\hat{\theta}(i) - p(i-1)^{-1}\hat{\theta}(i-1) - \\ -\sigma_w^2[i\hat{\theta}(i) - (i-1)\hat{\theta}(i-1)], \quad (28)$$

where

$$p(i)^{-1} := \sum_{k=1}^{i}\varphi(k-1)\varphi(k-1)^*. \quad (29)$$

Clearly

$$p(i)^{-1} = p(i-1)^{-1} + \varphi(i-1)\varphi(i-1)^*. \quad (30)$$

Then substituting $p(i-1)^{-1} = p(i)^{-1} + \varphi(i-1)\varphi(i-1)^*$ in (28) yields

$$\varphi(i-1)y(i)^* = p(i)^{-1}[\hat{\theta}(i) - \hat{\theta}(i-1)] \\ + \varphi(i-1)\varphi(i-1)^*\hat{\theta}(i-1) - \sigma_w^2[i\hat{\theta}(i) - (i-1)\hat{\theta}(i-1)]. \quad (31)$$

At this point of algorithm construction we assume that asymptotically $\hat{\theta}(i) \cong \hat{\theta}(i-1)$, and in the last term on the RHS of (31), $\hat{\theta}(i)$ is replaced with $\hat{\theta}(i-1)$. We thus obtain

$$\varphi(i-1)y(i)^* = p(i)^{-1}[\hat{\theta}(i) - \hat{\theta}(i-1)] \\ + \varphi(i-1)\varphi(i-1)^*\hat{\theta}(i-1) \\ - \sigma_w^2[(i-1)\hat{\theta}(i-1) - (i-2)\hat{\theta}(i-2)]. \quad (32)$$

From the above, (14) directly follows by replacing the unknown $\sigma_w$ with its a-priori estimate $\hat{\sigma}_w$. Equation (16) is obtained by using matrix inversion lemma in (30).

Let

$$W_x := \lim_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n}\psi_x(i)\psi_x(i)^*. \quad (33)$$

Note that the above limit exists by virtue of assumptions A1 and A4.

The following proposition qualifies the convergence properties of algorithm (14)-(16).

**Theorem 1:** Let assumptions A1 to A5 hold, and $\hat{\sigma}_w \le \sigma_w$. Then

$$\lim_{i\to\infty}\hat{\theta}(i) = \theta + [W_x + (\sigma_w^2 - \hat{\sigma}_w^2)I]^{-1}(\hat{\sigma}_w^2 - \sigma_w^2)\theta, \quad (34)$$

where $\theta$ is defined by (13), and $W_x$ is given by (33).

Proof: The proof of this result is omitted due to space limitations, and it follows the same lines as the proof of Theorem 2 in [6].

Define

$$\Im(i) = A(z^{-1})y(i), \quad (35)$$

and

$$u(i) = A(z^{-1})w(i). \quad (36)$$

Then (12) becomes

$$\Im(i) = N_0 s(i) + u(i). \quad (37)$$

Next we develop the Wiener type adaptive filter to extract $N_0 s(i)$ from (37). This filter is used to estimate $u(i)$, and then from (37) we obtain $N_0 s(i)$. In this process we assume that $\Im(i)$ is available. Note that algorithm (14)-(16) estimates $A(z^{-1})$.

Let the filter be defined by

$$v(i) = \sum_{k=0}^{M} G_k \Im(i-k), \quad (38)$$

where $M$ is the parameter chosen by the designer, and $\{G_k\}$ is the impulse response sequence determined by minimizing the following cost function

$$J_2 = E[(u(i) - v(i))(u(i) - v(i))^*]. \quad (39)$$

Observe that (38) can written in the form

$$v(i) = \alpha^*\phi(i), \quad (40)$$

where

$$\phi(i)^T = [\Im(i)^T, \Im(i-1)^T, \cdots, \Im(i-M)^T], \quad (41)$$

and

$$\alpha^* = [G_0, G_1, \cdots, G_M]. \quad (42)$$

Setting to zero the gradient of $J_2$ with respect to $\alpha^*$, we can derive

$$E[\phi(i)(u(i)-v(i))^*]=0\,, \tag{43}$$

wherefrom by using (40) we have

$$E[\phi(i)u(i)^*]=E[\phi(i)\phi(i)^*]\alpha\,. \tag{44}$$

Substituting $u(i)$ from (37) in (44) yields

$$E[\phi(i)\Im(i)^*-\phi(i)(N_0 s(i))^*]=E[\phi(i)\phi(i)^*]\alpha\,. \tag{45}$$

Let us evaluate $E[\phi(i)s(i)^* N_0^*]$. Components of $\phi(i)$ are $\Im(i-k)=u(i-k)+N_0 s(i-k), 0\le k\le M$. Then by assumption A4 and A5,

$$E[\Im(i)s(i)^* N_0^*]=N_0 N_0^*\,, \tag{46}$$

and $E[\Im(i-k)s(i)^* N_0^*]=O_{m\times m}, k\ge 1$.
Hence

$$E[\phi(i)s(i)^* N_0^*]=e_0 N_0 N_0^*\,, \tag{47}$$

where

$$e_0^T=[I_{m\times m},O_{m\times m},\cdots,O_{m\times m}]\,. \tag{48}$$

Matrix $e_0$ contains $M$ blocks of zero matrices. Next we express the RHS of (47) in terms of available quantities. Since $\{s(i)\}$ and $\{w(i)\}$ are independent sequences, from (37) we have

$$E[\Im(i)\Im(i)^*]=E[u(i)u(i)^*]+N_0 N_0^*\,. \tag{49}$$

From where by the assumption A4 and (35) one obtains

$$E[\Im(i)\Im(i)^*]=\sigma_w^2 I_m+\sum_{k=1}^{n_A} A_k A_k^* \sigma_w^2+N_0 N_0^*\,. \tag{50}$$

Substituting $N_0 N_0^*$ from (50) in (47), we can derive

$$E[\phi(i)s(i)^* N_0^*]=e_0[E(\Im(i)\Im(i)^*)-\sigma_w^2 R]\,, \tag{51}$$

with

$$R=I_m+\sum_{k=1}^{n_A} A_k A_k^*\,. \tag{52}$$

By using (51) in (45) it follows that

$$\begin{aligned}E[\phi(i)\Im(i)^*]=e_0[E(\Im(i)\Im(i)^*)-\sigma_w^2 R]=\\ E[\phi(i)\phi(i)^*]\alpha,\end{aligned} \tag{53}$$

which is the equation defining optimal $\alpha$.

We propose the following adaptive algorithm for estimation of $\alpha$ defined by the previous equation:

$$\begin{aligned}\hat{\alpha}(i)=\hat{\alpha}(i-1)+Q(i)\hat{\phi}(i)[\hat{\Im}(i)^*-\hat{\phi}(i)^*\hat{\alpha}(i-1)]-\\ -Q(i)e_0[\hat{\Im}(i)\hat{\Im}(i)^*-\hat{\sigma}_w^2\hat{R}(i)],\end{aligned} \tag{54}$$

$$\hat{\phi}(i)^T=[\hat{\Im}(i)^T,\hat{\Im}(i-1)^T,\cdots,\hat{\Im}(i-M)^T], \tag{55}$$

$$\hat{\Im}(i)=y(i)+\sum_{k=1}^{n_A}\hat{A}_k(i)y(i-k)\,, \tag{56}$$

$$\hat{R}(i)=I_{m\times m}+\sum_{k=1}^{n_A}\hat{A}_k(i)\hat{A}_k(i)^*\,, \tag{57}$$

$$\hat{Q}(i)=\hat{Q}(i-1)-\frac{\hat{Q}(i-1)\hat{\phi}(i)\hat{\phi}(i)^*\hat{Q}(i-1)}{1+\hat{\phi}(i)^*\hat{Q}(i-1)\hat{\phi}(i)}\,,$$

(58)

$$\hat{u}(i)=\hat{\alpha}(i)^*\hat{\phi}(i) \tag{59}$$

$$\overline{N_0 s(i)}=\hat{\Im}(i)-\hat{u}(i)\ \text{(estimate of } N_0 s(i)\text{)}. \tag{60}$$

Initial $\hat{\alpha}(0)$ is an arbitrary vector with finite norm, $\hat{Q}(0)$ is an arbitrary positive definite matrix and $\hat{\sigma}_w$ is the a-priori estimate of $\sigma_w$, and it is the same as in (14). For every $i\ge 0$, $\hat{A}_k(i), 1\le k\le n_A$ is generated by the algorithm (14)-(16). We show that $\hat{\alpha}(i)$ converges (a.s.), and when $\hat{\sigma}_w=\sigma_w$, $\hat{\alpha}(i)$ converges toward optimal $\alpha$ defined by (53). At every time instant $i=1,2,\cdots$, estimate of $N_0 s(i)$ is obtained by using (60). Note that $\hat{u}(i)$ in (59) is the output of the adaptive filter, and it is the estimate of $v(i)$ given by (40). Actually $\hat{u}(i)$ is obtained from (40) by replacing unmeasurable $\phi(i)$ with $\hat{\phi}(i)$, and $\alpha$ with $\hat{\alpha}(i)$. A suitable blind source separation algorithm can be adaptively applied on $\overline{N_0 s(i)}$ to estimate the source vector $s(i)$ [18, 19].

In the following, we give a heuristic justification of this algorithm. After replacing expectations in (53) with respective sample averages, we obtain

$$\begin{aligned}\frac{1}{i}\sum_{k=0}^{i}[\phi(k)\Im(k)^*]-e_0\frac{1}{i}\sum_{k=0}^{i}[\Im(k)\Im(k)^*]+e_0\sigma_w^2 R=\\ =\frac{1}{i}\sum_{k=0}^{i}[\phi(k)\phi(k)^*]\alpha(i),\end{aligned} \tag{61}$$

where instead of $\alpha$ we write $\alpha(i)$ to emphasize the fact that this $\alpha$ is obtained based on the data up to time $i$.
Let

$$\overline{Q}^{-1}(i) := \sum_{k=0}^{i}[\phi(k)\phi(k)^*].$$

(62)

Then (61) becomes

$$\sum_{k=0}^{i}[\phi(k)\Im(k)^*] + e_0 i\sigma_w^2 R = e_0\sum_{k=0}^{i}[\Im(k)\Im(k)^*] + \\ + \overline{Q}(i)^{-1}\alpha(i).$$

(63)

The following equation is same as (63), where $i$ is replaced with $i-1$, i.e.

$$\sum_{k=0}^{i-1}[\phi(k)\Im(k)^*] + e_0(i-1)\sigma_w^2 R = e_0\sum_{k=0}^{i-1}[\Im(k)\Im(k)^*] + \\ + \overline{Q}(i-1)^{-1}\alpha(i-1). \quad (64)$$

Substituting (64) and (63) gives

$$\phi(i)\Im(i)^* + e_0\sigma_w^2 R = e_0\Im(k)\Im(k)^* + \overline{Q}(i)^{-1}\alpha(i) \\ - \overline{Q}(i-1)^{-1}\alpha(i-1). \quad (65)$$

Since from (62),

$$\overline{Q}^{-1}(i) = \overline{Q}^{-1}(i-1) + \phi(i)\phi(i)^*, \quad (66)$$

we can derive from (65)

$$\phi(i)\Im(i)^* + e_0\sigma_w^2 R = e_0\Im(k)\Im(k)^* + \\ + \overline{Q}(i)^{-1}(\alpha(i) - \alpha(i-1)) + \phi(i)\phi(i)^*\alpha(i-1), \quad (67)$$

or

$$\alpha(i) = \alpha(i-1) + \overline{Q}(i)\phi(i)[\Im(i)^* - \phi(i)^*\alpha(i-1)] - \\ - \overline{Q}(i)e_0[\Im(i)\Im(i)^* - \sigma_w^2 R]. \quad (68)$$

Algorithm (54) is obtained from (68) by replacing unavailable data with available ones, i.e., $\Im(i)$ with $\hat{\Im}(i)$, $\phi(i)$ with $\hat{\phi}(i)$, $R(i)$ with $\hat{R}(i)$, and $\overline{Q}(i)$ with $Q(i)$. Observe that if in (66) we replace $\overline{Q}(i)$ with $Q(i)$ and $\phi(i)$ with $\hat{\phi}(i)$, and apply matrix inversion lemma, (65) will follow.

**Theorem 2:** Let assumptions A1-A5 hold, and $\hat{\sigma}_w = \sigma_w$. Then the algorithm (54)-(60) provides

$\lim_{i\to\infty}\hat{\alpha}(i) = \alpha$ [a.s]

where $\alpha$ is optimal parameter matrix satisfying (53).

Proof: The proof of this theorem is omitted due to space limitations.

## 4. SIMULATION

For this simulation, a 2-input 3-output MIMO FIR channel is considered and the channel impulse response is given by

$$H[0] = \begin{bmatrix} -1.8 & -0.5 \\ -0.5 & 0.4 \\ -1.3 & 0.7 \end{bmatrix},$$

and

$$H[1] = \begin{bmatrix} 1 & -1.5 \\ -0.8 & 0.5 \\ 0.2 & -0.6 \end{bmatrix}.$$

Two QPSK sequences are considered as input. AWGN of 5dB is added. In order to estimate $A(z^{-1})$ $n_A = 3$ is chosen. Therefore $\theta^*$ in (13) is a $3\times 6$ matrix. For the Wiener filter $M = 5$ is chosen. Fig. 1 shows one of the un-equalized received symbols. Fig.2 shows the scatter plot of the equalized signal before Blind Source Separation (BSS).

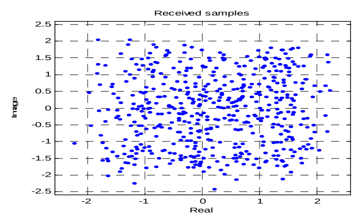


Fig. 1 Un-equalized received symbols

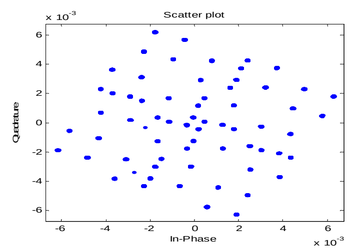


Fig. 2 Equalized symbols before ICA

For BSS, Independent Component Analysis (ICA) is used. Fast ICA developed in [18] is used in this simulation. More on ICA can be found in [18]. Fig. 3 shows the scatter plot of one of the received signals after BSS. It is clear that effective equalization has been accomplished.

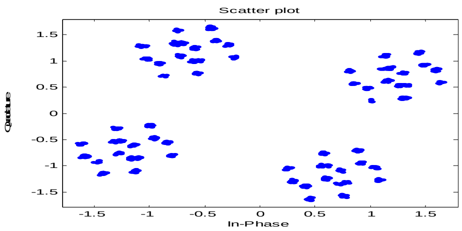


Fig. 3 Equalized symbols after ICA

## 5. CONCLUSION

The paper proposes a recursive blind adaptive filtering method for MIMO channels to temporally whiten the input signals. The algorithm consists of two parts: a recursive blind identification scheme and a Weiner type adaptive filter used to extract the instantaneous mixture of the source signals. Both algorithms are globally convergent.

## 6. REFERENCES


[1] L. Tong, G.Xu, and T. Kailath, " A new approach to blind identification and equalization of multipath channels", IEEE Trans. Inform.Theory, vol.40, pp.340-349, Mar1994.

[2] H. Liu, and G. Xu, "Closed form blind symbol estimation in digital communications", IEEE Trans. Signal Processing, vol.43, pp.2714-2723, Nov.1995.

[3] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, "subspace methods for blind identification of multichannel FIR filters", IEEETrans. Signal Processing, vol.43, pp.516-525, Feb. 1995.

[4] G. Giannakis and S. Halford, "Blind fractionally spaced equalization of noisy FIR cannels: Direct and adaptive solutions", IEEE Trans. Signal Processing, vol.45, pp.2277-2292, Sept.1997

[5] Special Issue on Signal Processing for Advanced Communications, IEEE Trans. Signal Processing, vol.45, Jan.1997.

[6] M. Radenkovic, and T. Bose, "A recursive blind adaptive identification algorithm and its almost sure convergence", IEEE Trans. Circuits and Systems-I: Regular papers", vol. 54, June 2007.

[7] A. J. van der Veen, S. Talwar, and A. Paulraj, "A subspace approach to blind space time signal processing for wireless communication systems", IEEE Trans. Signal Processing, vol. 45, pp.173-190, Jan.1997.

[8] K. Abed-Meriam, P. Loubaton, and E. Moulines,"A subspace algorithm for certain blind identification problems", IEEE Trans.Inform., Theory, vol. 43, pp.499-511, Mar. 1997.

[9] K. Abed-Meriam, E. Moulines, and P. loubaton, " Prediction error method for second order blind identification", IEEE Trans. Signal Processing, vol. SP-45, pp.694-705, Mar.1997.

[10] K. Abed-Meriam et al., "On subspace methods for blind identification of single input multiple output FIR systems", IEEE Trans. Signal Processing, Vol.45, pp.42-55, Jan. 1997.

[11] J. Tugnait, and B. Huang, "second order statistics based blind equalization of IIR single input multiple output channels with common zeros", IEEE Trans. Signal Processing, vol. 47, pp.147-157, Jan. 1999.

[12] J. Tugnait, and B. Huang, "Multistep linear predictor based identification and equalization of multiple input multiple output channels", IEEE Trans. Signal Processing, vol. 48, Jan. 2000.

[13] D. Gesbert and P. Duhamel, "Unbiased blind adaptive channel identification and equalization", IEEE Trans. Signal Processing, vol. 48, pp.148-158, Jan. 2000.

[14] R. Liu, and G. Dong, "A fundamental theorem for multiple channel blind equalization", IEEE Trans. Cir. Syst. I: Fundamental Theory and Appl., vol. 44, pp.472-473, May 1997.

[15] J. Tuhnait, "A multidelay whitening approach to blind identification and equalization of SIMO channels", IEEE Trans. Wireless Com., vol. 1, July 2002.

[16] A. Touzni, I. Fijalkow, M. Larimore, and J. Treichler, "A globally convergent approach for blind MIMO adaptive deconvolution", IEEE Trans. Signal Processing, vol. 49, pp. 1166-1177, June 2001.

[17] Y. Inouye, and R. Liu, "A system theoretic foundation for blind equalization of an FIR MIMO channel system", IEEE Trans. Circuits and Syst_I: Fundamental Theory and Appl., vol.4, April 2002.

[18] A. Hyvarinen, J. Karhunen, E. Oja, "Independent Component analysis", John Wiley & Sons, 2001.

[19] A. Cichocki, S. Amari, "Adaptive blind signal and image processing", John Wiley &Sons, 2002.

[20] H. Chen, X. Cao, H. Fang, and J. Zhu, "Nonlinear adaptive blind whitening for MIMO channels", IEEE Trans. Signal Processing, vol. 53, pp. 2635-2647, Aug. 2005.

[21] T. Kailath, Linear systems, Englewood Clifs, NJ: Prentice-Hall, 1980.