ORTHOGONAL-CODED SELECTIVE MAPPING (OCSM) FOR OFDM PEAK-TO-AVERAGE POWER REDUCTION WITHOUT SIDE INFORMATION

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ABSTRACT

This paper elaborates on a new technique for Peak to Average Power (PAP) Control in Orthogonal Frequency Division Multiplexing (OFDM) Systems. The PAP levels are particularly problematic for Software Defined Radio (SDR) platforms, which must reproduce a large number of waveforms (possibly including OFDM) with a finite set of RF electronics, amplifiers, and Analog-to-Digital converters (ADCs). On an SDR radio, it is extremely desirable to utilize a software approach (i.e. as proposed here) to level PAP over multiple waveforms.

Various techniques have been devised to reduce the PAP of OFDM waveforms; including coding [1], symbol limiting/clipping, tone reservation/peak cancellation [2], and diversity methods such as selective mapping [3,4] and partial transmit sequence generation [5]. The diversity methods are attractive in their simplicity, but may require that side information be sent in order to decode the OFDM symbol at the receiver, reducing overall throughput. This paper continues discussion on a new technique for selective mapping, which utilizes orthogonal coding for the diversity sequences so that the OFDM symbols can be detected and decoded at the receiver without the need for sending this "side information". This methodology is similar to other coding schemes in that the code sequence can be detected at the receiver, introducing a residual BER floor. However this technique utilizes Hadamard or pseudo-orthogonal codes across the subcarriers as opposed to employing either convolutional coding [4], or complementary sequences [1].

The technique proposed is a modified form of Selective Mapping, which utilizes orthogonal coding for the diversity sequences so that the OFDM symbols can be detected and decoded at the receiver without the need for sending "side information". In doing so, it allows the designer to trade off an increase in power for increased throughput - at the same time allowing higher output of the power amplifiers by allowing operation at a higher operating point. It has been shown in [6] and [7] that a 1.4 dB of increased transmitter power could allow for more than a 7.0 dB decrease in peak to average power, with an overall gain in transmitter power efficiency of greater than 5 dB. Here we will review those results and extend them from Hadamard orthogonal codes to pseudo-random codes. Theoretical results are presented, corroborated by numerical simulations, and compared to existing approaches. Methods to reduce the power requirements and/or further increase the detection probability of this method are also introduced. The paper will conclude with a discussion on aspects of hardware implementation.

1. INTRODUCTION

Various techniques have been devised to reduce the peak-to-average-power ratio (PAP) of Orthogonal Frequency Division Multiplex (OFDM) waveforms; including coding [1,8,9], symbol limiting/clipping, tone reservation/peak cancellation [2], and diversity methods such as Selected Mapping [3,4] and Partial Transmit Sequence Generation [5]. The diversity methods are attractive in their simplicity, but they may require that side information be sent in order to decode the OFDM symbol at the receiver, thereby reducing overall throughput.

This paper continues work on a new technique for Selected Mapping, termed as Orthogonal-Coded Selected Mapping (OCSM) [6,7]. OCSM utilizes orthogonal coding for the diversity sequences so that the OFDM symbols can be detected and decoded at the receiver without the need for sending this "side information". This is achieved by introducing a small offset in the symbol constellations at the transmitter. In doing so, it allows the designer to trade off an increase in transmitted power for increased throughput - at the same time allowing higher efficiency and output power from the power amplifiers by allowing operation at a higher operating point. This methodology is similar to other coded schemes in that the code sequence can be detected at the receiver, introducing a residual BER floor [4]. However this technique utilizes Hadamard codes across the subcarriers as opposed to employing either convolutional coding [4], or complementary sequences [1,8,9].

The paper is organized as follows: Section 2 discusses the PAP problem associated with OFDM and presents the statistical distribution of PAP. Section 3 reviews previously proposed PAP reduction techniques, including the Selected Mapping methods that this proposal is based upon. Section 4 presents the concept of OCSM, discusses its performance and detection properties, and compares simulation results with theoretical predictions. Section 5 discusses ongoing work to increase the performance and decrease the power requirements of this method. Finally, Section 6 examines the FFT/IFFT processing requirements that OCSM may require.

2. OFDM PEAK TO AVERAGE POWER

Consider an OFDM system with N subcarriers. The frequency domain OFDM symbol consists of N complex numbers, $\vec{d}$, corresponding to the values of the subcarrier constellations. The time domain complex baseband OFDM symbol vector is the inverse discrete Fourier transform (IDFT) of the frequency domain symbol, $\bar{s} = \underline{F}^H \cdot \vec{d}$, where $\underline{F}$ is the discrete Fourier transform (DFT) matrix. Under the assumption that the subcarriers are uncorrelated (i.e., no over-sampling and no null subcarriers), the PAP distribution for N-subcarrier OFDM is approximately (for large N):

$$P(PAP > Z) \approx 1 - \left(1 - e^{-Z}\right)^N$$

One of the most serious effects that a large PAP has is that it requires that the transceiver system remain linear over a very wide dynamic range. Because the amplifiers, D/A, and A/D converters can only remain linear over a finite range, there can be instances where signal clipping occurs. This waveform clipping leads to spectral re-growth and introduces non-linear distortion.

3. PAP REDUCTION TECHNIQUES

PAP reduction in OFDM systems has been the focus of much research. This section introduces some of the common approaches to reducing PAP in OFDM systems.

CODING

A common use of coding to reduce PAP in an OFDM system is to employ a reduced set of OFDM symbols, so that the PAP can be controlled though selection of possible subcarrier combinations. This implies that an exhaustive search must be performed over a very large symbol set. For example, an OFDM system using 128 subcarriers, each using 64-QAM modulation, has a search space with 64^{128} values. Several methods have been proposed that employ joint PAP reduction and block forward error correction (FEC) utilizing Reed-Muller or Golay Complementary codes in order to facilitate implementation of the algorithms [1,8,9].

WAVEFORM SHAPING

The main approach to using waveform shaping is to either assign or reserve subcarriers so that their amplitude and phase can be adjusted to reduce peaks within the OFDM time domain signal. A well-regarded example of this approach was proposed by Tellado and Cioffi [2] in which tone reservation techniques were used for wireline ADSL.

SCRAMBLING

Several methods have been proposed by Muller, et al. [3-5], that are based upon generating multiple representations of each OFDM symbol, each representing the same information. These representations are generated through a scrambling process, which is either implemented through Selected Mapping (SM), or Partial Transmit Sequence (PTS) generation. For this paper, we will focus only on SM, as the method proposed herein is a modified form of SM.

In SM, U scrambling codes are applied to each ODFM symbol, resulting in U uncorrelated parallel representations of the same symbol information. Parallel IDFTs are then performed, and the time domain symbol with the lowest PAP is selected for transmission over the channel. In order for the receiver to de-scramble the symbol, side information identifying the code selected at the transmitter needs to be sent along with the original symbol information, resulting in throughput loss. Selected mapping with N subcarriers and U branches results in a maximum PAP reduction as follows:

$$P(U, PAP > Z) \approx \left[1 - \left(1 - e^{-Z}\right)^N\right]^U$$

For low clipping probability levels and large code sets, it is seen that PAP reduction on the order of 8 dB is possible utilizing this method (ex. $N = 64$, $U = 64$, $P_{clip} < 10^{-9}$).

4. SELECTED MAPPING WITH ORTHOGONAL CODING

While SM is a powerful tool for reducing PAP in OFDM systems, its main drawback is that it leads to a reduction in data throughput due to the transmission of side information. Data throughput reduction is on the order of $log2(N)/\sum M_i$, where M_i is the number of bits/symbol corresponding to the i^{th} subcarrier constellation.

In order to eliminate the need for this side information, the proposed OCSM method uses orthogonal subcarrier coding in conjunction with two-stage detection at the receiver. To facilitate detection at the receiver, the subcarrier constellations must be offset by a fractional amount ε before code scrambling. For the remainder of this paper, this offset is implemented along the real axis of the subcarrier constellations. Although this implies that the constellations are no longer zero-mean over time, the antipodal scrambling process conditions them to again be zero-mean before transmission. The transmission and detection processing for OCSM is shown in Figure 1.

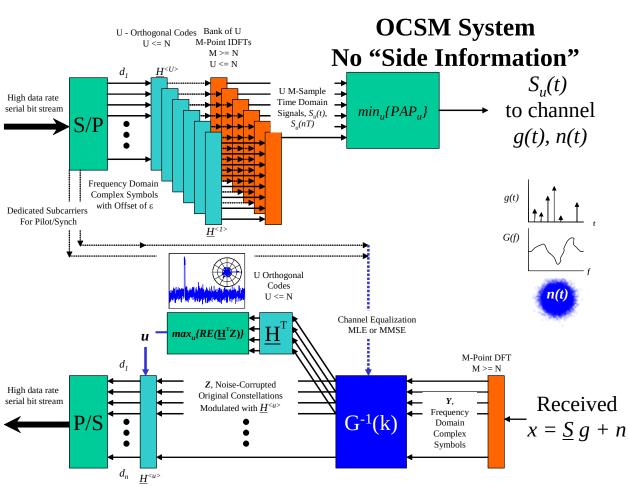

Figure 1, OCSM Transmitter and Receiver Processing

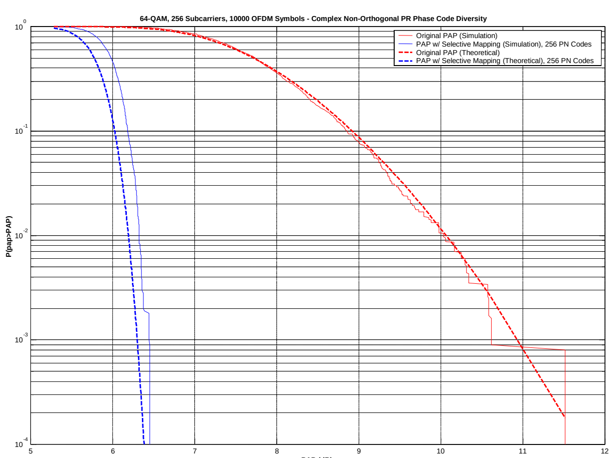

Figure 2, PAP Reduction from Selected Mapping (Random Phase Codes)

In this paper, the orthogonal codes employed are either pseudo-random phase or binary antipodal Hadamard codes, which are formed from columns of a Hadamard matrix. However, any real or complex orthogonal or pseudo-orthogonal codes may be employed using this technique. It should be noted that the Hadamard codes are not optimal in terms of generating independent and uncorrelated realizations of the original OFDM symbol. The reason for this is that in SM there may be many phase states in the scrambling codes, while the antipodal Hadamard codes only have two possible states: either no phase change, or a 180 degree phase reversal. Because of this, the Hadamard codes are less effective when compared to more complex codes used in traditional SM, on the order of 0.4 dB. This is shown in Figure 2 and Figure 3 for 256 subcarriers with 64-QAM.

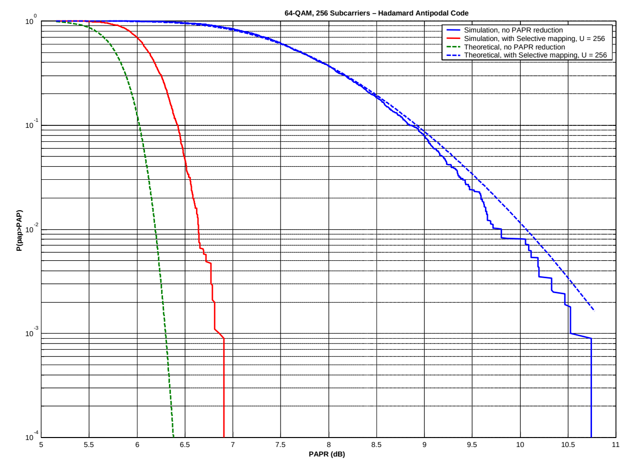

Figure 3, PAP Reduction from Selected Mapping (Hadamard Orthogonal Binary Codes)

In the proposed system, at the receiver, the scrambled symbol is returned to the frequency domain via DFT, and then equalized to remove channel distortions, and to correct for timing and carrier frequency offsets. The symbol then goes though a correlation detector implemented as a matrix multiplication by the full Hadamard matrix, $\underline{H}$. In order to select the correct code, the maximum value of the real part of the resulting vector is chosen. Once the correct code, u, has been detected, it is applied to the received scrambled signal, de-rotating all subcarrier values to their correct positions. From there on, normal constellation detection of the noise-corrupted symbol is performed.

DETECTION STATISTICS

After channel equalization and application of the full antipodal Hadamard matrix, we have the following expression from which to make a determination of the orthogonal code selected at the transmitter:

$$\vec{R} = \underline{H} \cdot diag\left(\vec{d}\right) \cdot \underline{H}^{<u>} + \varepsilon \cdot \underline{H} \cdot \underline{H}^{<u>} + \underline{H} \cdot \vec{Z}$$

Where $\underline{H}^{<u>}$ is the u^{th} column of $\underline{H}$, which contains the code used to scramble the OFDM symbol. For large N, the first and third terms are distributed according to zero-mean complex Gaussian distributions, with variance $N\sigma_c^2$ and $N\sigma_n^2$, respectively. Here σ_c^2 is the variance of the subcarrier constellation modulation, and σ_n^2 is the residual complex Gaussian component after the receiver DFT. These can be combined into a composite zero-mean complex Gaussian distribution of variance:

$$\sigma^2 = \tfrac{1}{2}\left(\sigma_c^{\ 2} + \sigma_n^{\ 2}\right)$$

Assuming that the offset ε is real (note that ε is not constrained to be real – it was shown in [6] that it can be complex without affecting the results), we can select the channel i which has the largest value of $real(R_i)$ associated with it, then select the appropriate corresponding code to de-scramble the OFDM symbol before final constellation-level detections are performed. The probability of an incorrect detection is then:

$$P_e = P(X > R_u) = \int_{-\infty}^{\infty}\int_{-\infty}^{x} f(x,r)\,dr\,dx$$

where X is defined as:

$$X = \max\left[real\left(R_i\right), i \neq u\right]$$

with probability density function (pdf):

$$f_X(x) = \frac{d}{dx}\left\{Q\left(\frac{x}{\sigma}\right)^{N-1}\right\}$$

And the pdf of R_u is $\sim N(\varepsilon N, \sigma)$, i.e.:

$$f_{R_u}\left(r_u\right) = \frac{1}{\sqrt{2\cdot\pi}\cdot\sigma}\cdot e^{-\frac{1}{2}\left[\frac{r_u - \varepsilon\cdot N}{\sigma}\right]^2}$$

The integral does not yield a closed form solution, but can be reduced to:

$$P_e = \frac{N-1}{\sqrt{2\cdot\pi}\cdot\sigma}\int_{-\infty}^{\infty} Q\left(\frac{x}{\sigma}\right)^{N-2} Q\left(\frac{x-\varepsilon\cdot N}{\sigma}\right)\cdot e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$$

This can be numerically integrated to provide the detection error probability P_e, which is clearly a function of N, σ, and ε. Since this constellation offset ε results in transmitted power that does not increase detection at the constellation level, it could be thought of as either S/N or E_b/N_o loss, with a value of $10\log(1+\varepsilon^2/\sigma_c^2)$ dB.

For σ_c^2 = 0.44 (corresponding to 64-QAM and σ_n^2 = 0.01), theoretical results are illustrated in Figure 4 for N = 64, 128, 256, 512, and 4096. As expected, the performance is highly dependent upon the constellation offset ε. Note that even for 64 codes, a 0.7 dB budget for PAP coding (see Figure 4) translates to a about a 6 dB decrease in amplifier back off, and/or link gain (see Figure 2).

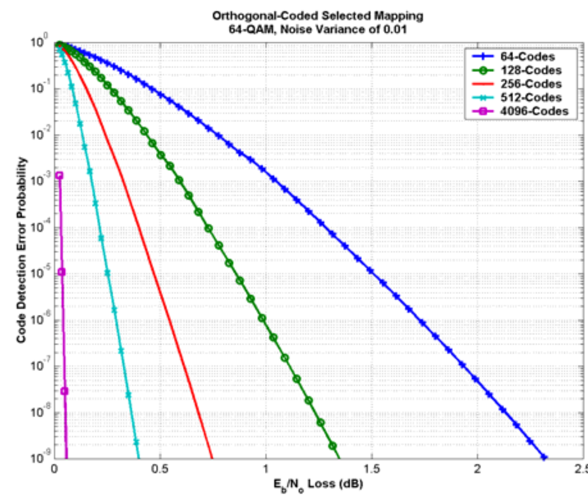

Figure 4, Theoretical Error Probability of Orthogonal Code Detection

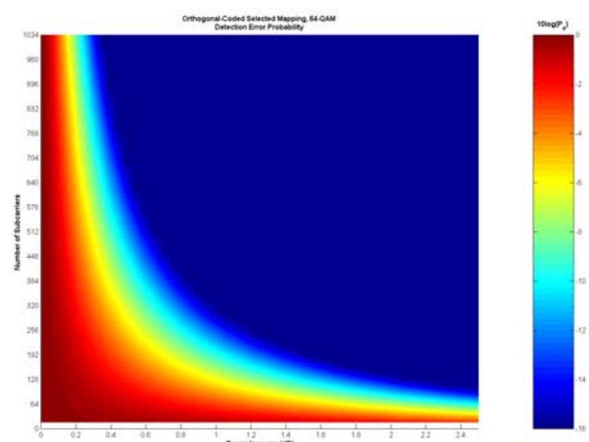

Figure 5, Theoretical Error Probability of Code Detection

Figure 5 also shows theoretical results for various values of ε and N, using σ_c^2 corresponding go 64-QAM. Notice that the detection error probability diminished rapidly in both N and σ_c^2. The reader should also note that the residual error floor set by the code detection error probability need only be on the order of the block error rate for ARQ systems retransmitting at the block level. For a 512 bit block, and a corrected BER rate of $10^{-4,}$ the code detection probability can be on the order of the block error rate, or about 5%, assuming that errored blocks are retransmitted.

SIMULATION AND RESULTS

Figure 3 shows several simulation results for code detection probability, along with the (numerically-integrated) theoretical predictions. As is seen, these agree well, with residual error floors of 10^{-5} achievable with between 0.45 and 1.4 dB E_b/N_o loss for 256 and 64 subcarriers, respectively. Simulation results obtained using random phase codes gave identical detection results to the Orthogonal Codes, but offered much better PAP reduction (0.4 dB for 256 codes).

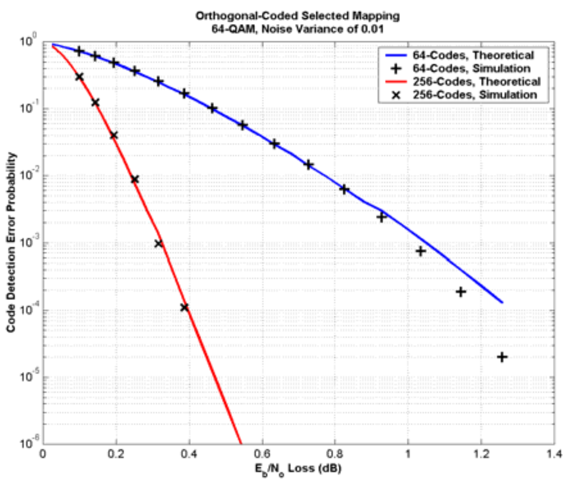

Figure 6, Code Detection Error Probability

5. ENHANCEMENTS OF PROPOSED METHODOLOGY

There is an opportunity to improve the detection probability and/or decrease the E_b/N_o loss through a feedback loop to the post-constellation-detection FEC results, and by carrying multiple detection possibilities to that point in the receiver. This will offset nearly all of the drawbacks of OCSM over methods that send side information. Instead of picking the scrambling code that corresponds to the maximum $real(R_i)$ after the Hadamard correlation step, carry the top q peaks all the way through to the actual subcarrier constellation detection point. At that point, all q possibilities could then be compared after detection and decoding, and the CRC and FEC could be leveraged to obtain the best fit to the q-carried possibilities. This would reduce the orthogonal code detection error from P_e to $\sim P_e^q$. This is obviously a better detection process, one that jointly utilizes information from each stage of the two-stage process.

6. FFT/IFFT PROCESSING CONSIDERATIONS

Processing requirements, especially the IDFT/DFT computations are a significant issue. In order to assess the processing power required to implement the proposed method, an examination of the number of floating point operations that need to be performed was undertaken. These results are presented in Table 1, for typical symbol period scenarios (4800 sym/sec-TIA Public Safety SAM and IEEE 802.11a).

FFT Size	Code Depth	Total Adds*	Total Multiplies*	Total Operations (FLOP)	MFLOPS at SAM Rate	MFLOPS at 802.11a rate
4	4	96	32	128	6.14E-01	4.00E+01
8	8	576	192	768	3.69E+00	2.40E+02
16	16	3,072	1,024	4,096	1.97E+01	1.28E+03
32	32	15,360	5,120	20,480	9.83E+01	6.40E+03
64	64	73,728	24,576	98,304	4.72E+02	3.07E+04
128	64	172,032	57,344	229,376	1.10E+03	7.17E+04
512	128	1,769,472	589,824	2,359,296	1.13E+04	7.37E+05
1,024	128	3,932,160	1,310,720	5,242,880	2.52E+04	1.64E+06
2,056	256	17,369,088	5,789,696	23,158,784	1.11E+05	7.24E+06
4,096	256	37,748,736	12,582,912	50,331,648	2.42E+05	1.57E+07

***Taking into account repeated operations**

Table 1, Processing Power for Several OFDM Technologies

From Table 1, it is clear that a large amount of processing power is needed to realize the proposed method. Note that reconfigurable logic is not that helpful in implementing the proposed method, as dedicated power is needed for the parallel IDFT's. However, the processing power for the most likely scenarios is not extreme, and is available with many commercial-off-the-shelf (COTS) products today [10, 11, 12]. In general, this method would require a K-times increase in FFT processing power over a typical OFDM system, where K is number of orthogonal codes employed. This would seem to point to a somewhat less than a K-fold increase in silicon area.

In order to implement the feedback to the FEC for BER floor reduction, parallel Viterbi decoders would also be required. This would lead to approximately an M-times increase over baseline in Viterbi/turbo decoding processing power, where M is the number of parallel constellation detection channels utilized. This would seem to point to increased silicon area also. Three-channel/trellis power appears to be available now with state-of-the-art products (decoding at up to OC-3 total rates for IEEE802.11a), see [10, 11, 12].

7. SUMMARY AND CONCLUSION

As illustrated in this paper, the OCSM method for PAP reduction has promise. OCSM is simple in concept, easy to understand, and amenable to analysis. Simulation results presented agree well with theoretical predictions.
Although OCSM may require an increase in transmit power, the power tradeoff is more than offset by the resulting PAP reduction, which allows a higher operating point for the system amplifiers. It is shown that a few dB in power can allow for high power amplifier (HPA) operating point increases of up to 8 dB, allowing for an increase in link power of up to 6 dB. Although a residual BER floor is introduced, it can cause less overall throughput reduction than sending side information. Furthermore, this BER floor can either be reduced, or eliminated altogether through the use of a feedback loop from the FEC detection point in the receiver.

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