

# **CONTINUOUS PHASE MODULATION (CPM)**

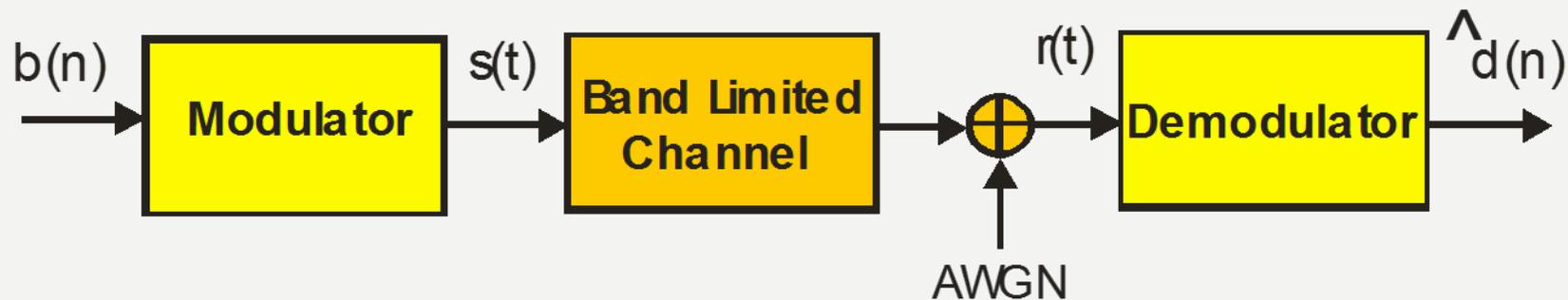
## **PREAMBLES, DOPPLER ESTIMATES AND OPTIMAL RECEIVERS**

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PACIFIC)**

# DIGITAL MODULATION



$$\begin{aligned} s(t) &= R(t) \cdot \cos(\omega_c t + \phi(t)) : \text{ Amplitude and Phase Modulated Carrier} \\ &= R(t) \cdot \cos(\phi(t)) \cdot \cos(\omega_c t) - R(t) \cdot \sin(\phi(t)) \cdot \sin(\omega_c t) \\ &= I(t) \cdot \cos(\omega_c t) - Q(t) \sin(\omega_c t) \end{aligned}$$

$$s(t) = \text{Re}\{R(t) e^{j\phi(t)} e^{j\omega_c t}\} : \text{ Polar Complex form with Baseband Modulation}$$

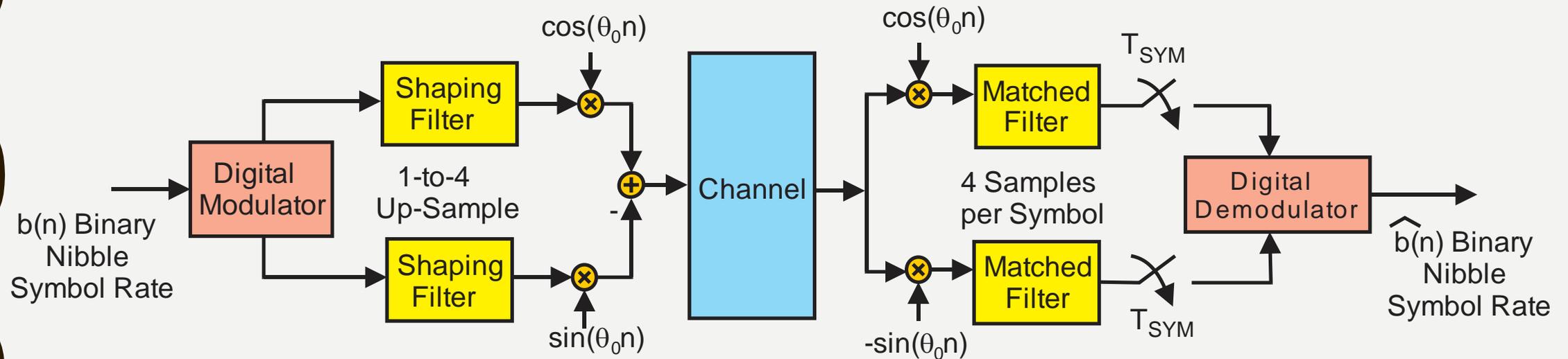
$$R(t) e^{j\phi(t)} = \sum_k I_k g_k(t - kT) : \text{ Digital Baseband Modulation, Sum of Scaled Offset Waveshapes}$$

$I_k$  : one of  $2^M$  Complex Amplitudes from Finite Look-up Table

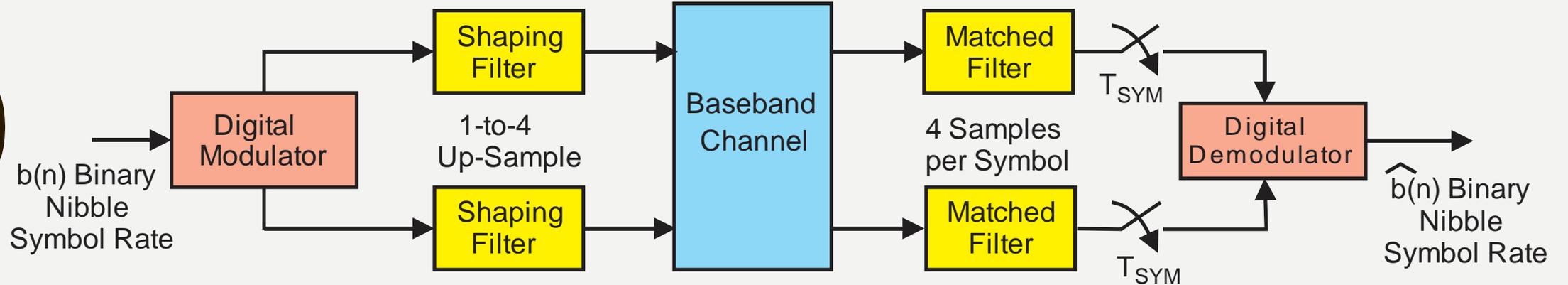
$g_k$  : one of  $2^Q$  Waveshapes from Finite List

# SHAPING FILTER AND MATCHED FILTER IN MODULATOR AND DEMODULATOR

# REAL, CARRIER CENTERED MODEL OF MODULATOR AND DEMODULATOR



# Complex, Baseband Model of Modulator and Demodulator



# VARIOUS QAM MODULATION CLASSES

- Linear Memoryless Modulation
  - Pulse Amplitude Modulation (PAM)
  - Phase Shift Keying (PSK)
  - Quadrature Phase Shift Keying (QPSK)
  - Quadrature Amplitude Modulation (QAM)
- Linear Modulation with Memory
  - Offset QPSK (O-QPSK)
  - $\pi/4$ -QPSK (pi/4-QPSK)
- Non Linear Modulation with Memory
  - Continuous Phase FSK (CPFSK)
  - Minimum-Shift Keying (MSK)
  - Gaussian MSK (GMSK)
  - Continuous-Phase Modulation (CPM)
  - Shaped Binary Phase Shift Keying (SBPSK)

# “Practical Digital Wireless Signals, Cambridge University Press, Earl McCune

## 6.1 Signals and characteristics

The newest fundamental signal type to join the DWC pantheon is **phase-shift keying (PSK)**. The reason for PSK’s relatively tardy entry is easy to understand, however. As will be seen in detail later in the signal quality required for successful generation is quite high, and the amount of information required for successful reception of a PSK signal is the greatest of these fundamental signal types. In the early decades of DWC this type of signal quality was beyond the state of the art, so ASK and FSK were used. As time and technology progressed, achieving the required signal quality became possible – and PSK began to be used.

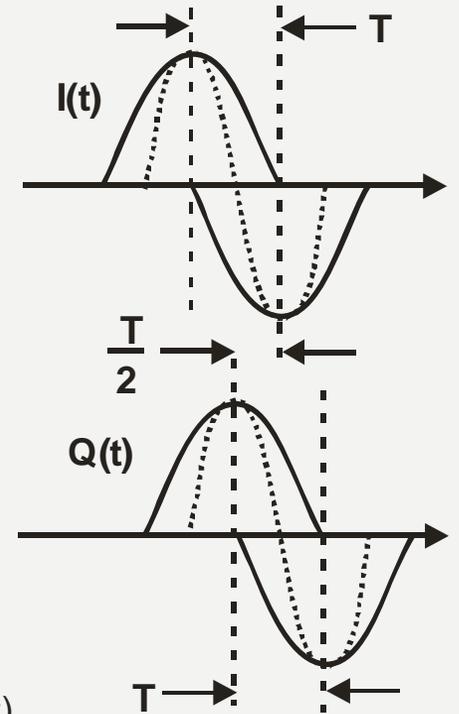
### Author’s dilemma

In the logical signal progression being followed so far, one might expect that this discussion on PSK will focus on a Digital Wireless Communication signal type that strictly modulates the signal phase parameter, just as ASK only modulates the amplitude parameter and FSK only modulates the frequency parameter. Indeed, this type of signal is possible, practical, and useful. Yet for a number of reasons this form of PSK, which we shall refer to here as “pure-PSK”, is almost never used. What the great majority of signal designers call PSK is actually a special case of QAM (quadrature amplitude modulation).

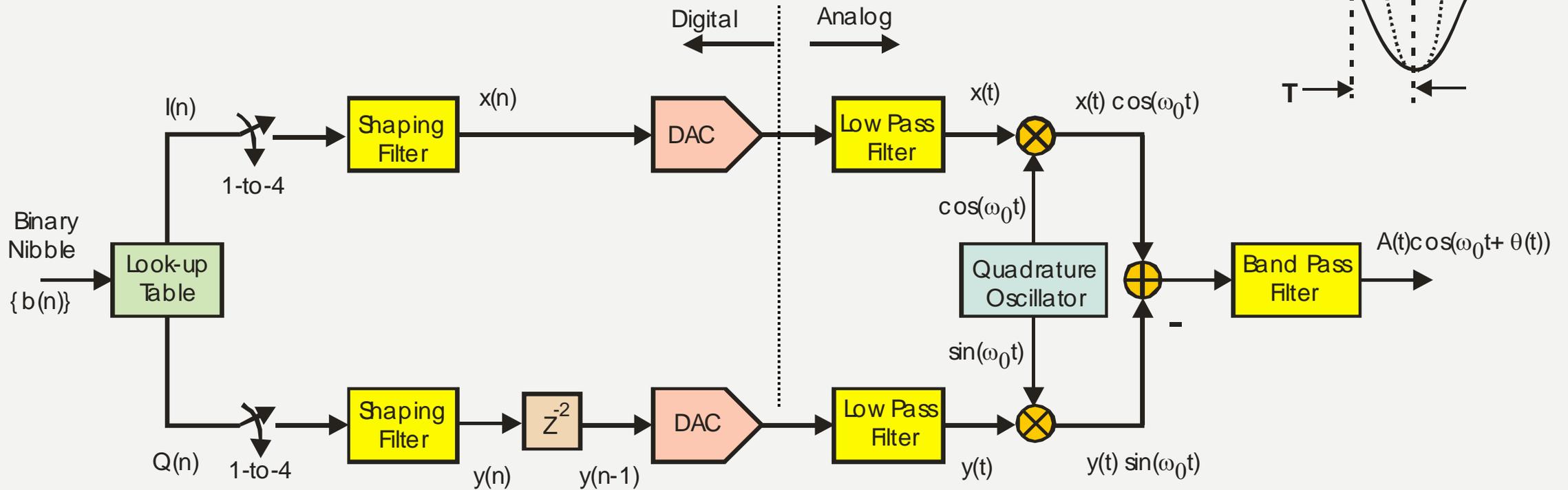
In the remainder of this text we shall refer to this form as “conventional-PSK”. Therefore, the main difference between pure- and conventional PSK is that pure-PSK is restricted to being a constant envelope signal by operating a one-dimensional phase-only modulation process directly from the PSK states, while conventional-PSK is not restricted in this way. Both forms of PSK are discussed in this chapter. Interleaved discussions address each type of PSK as appropriate in the context.

# OFFSET-QPSK MODULATOR

QPSK Signals:  $T$  Second Spacing Of Successive Symbols

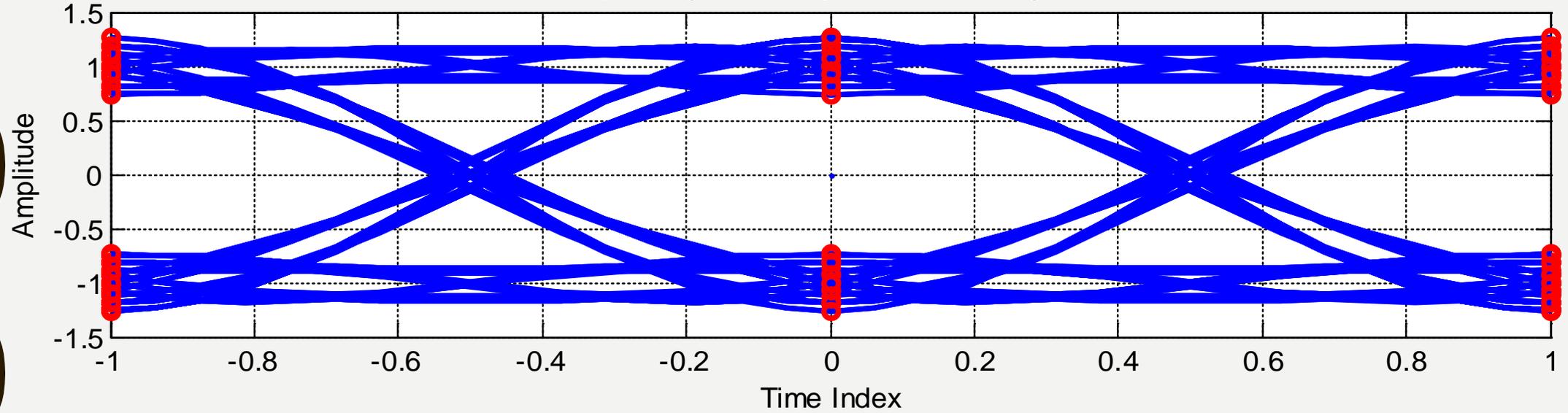


O-QPSK Signals:  $T/2$  Second Spacing Of In-Phase & Quadrature Symbols

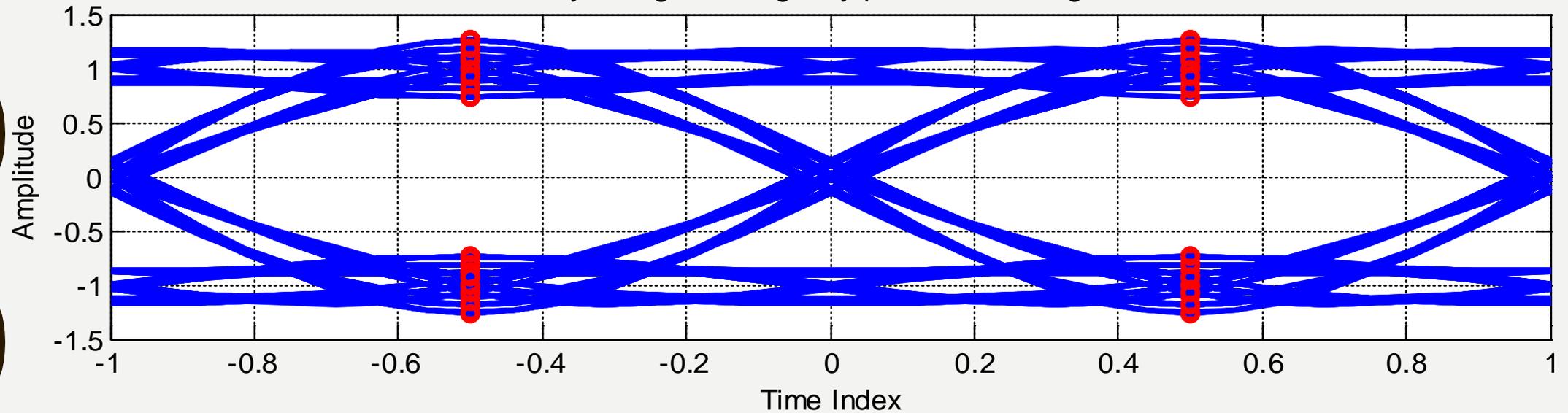


# Modulator Eye Diagrams O-QPSK

Eye Diagram Real part O-QPSK Signal

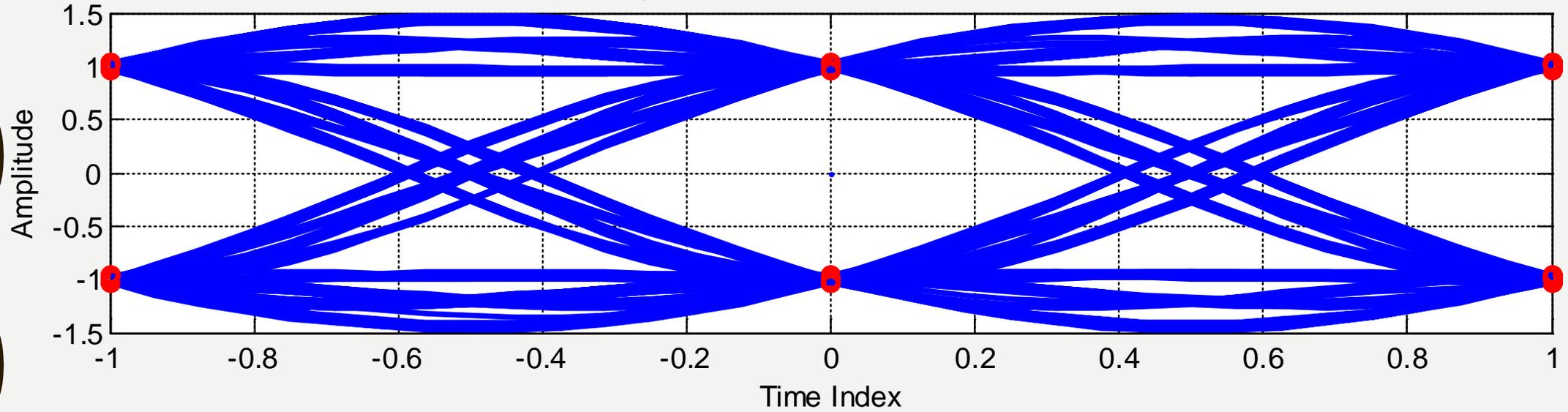


Eye Diagram Imaginary part O-QPSK Signal

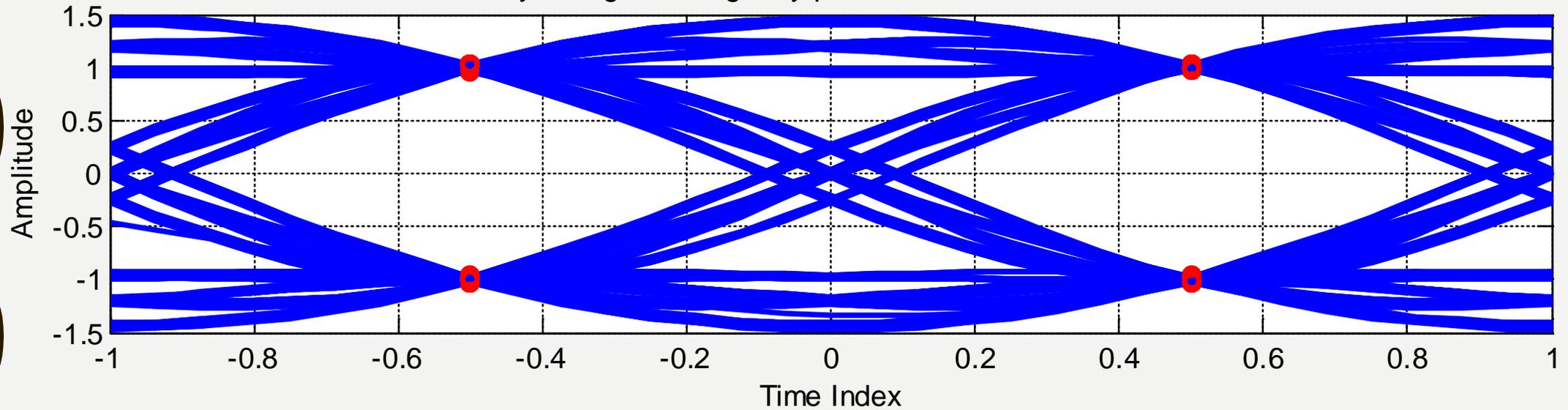


# Demodulator Eye Diagrams O-QPSK

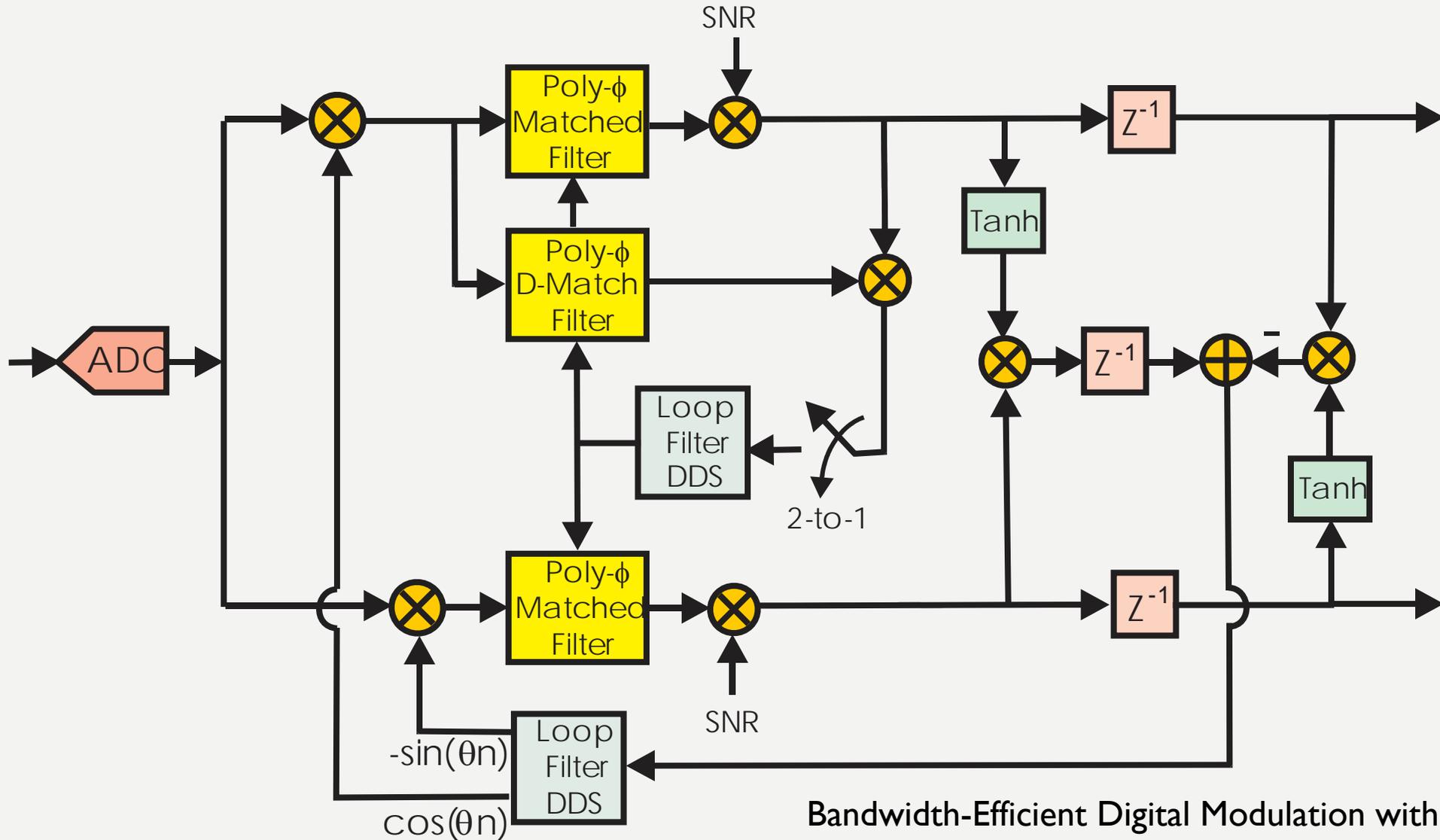
Eye Diagram Real part Matched Filter O-QPSK



Eye Diagram Imaginary part Matched Filter O-QPSK

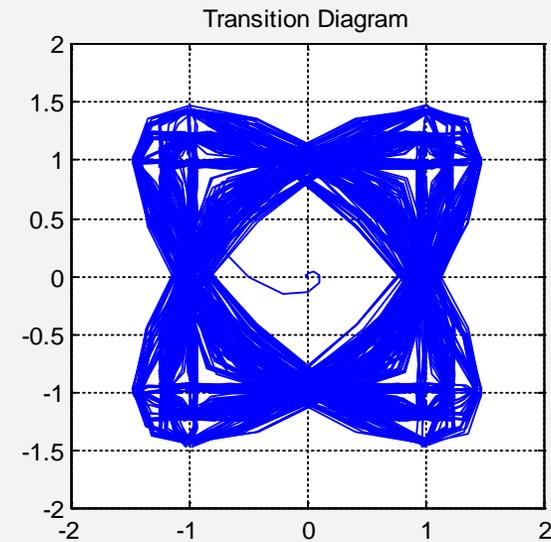
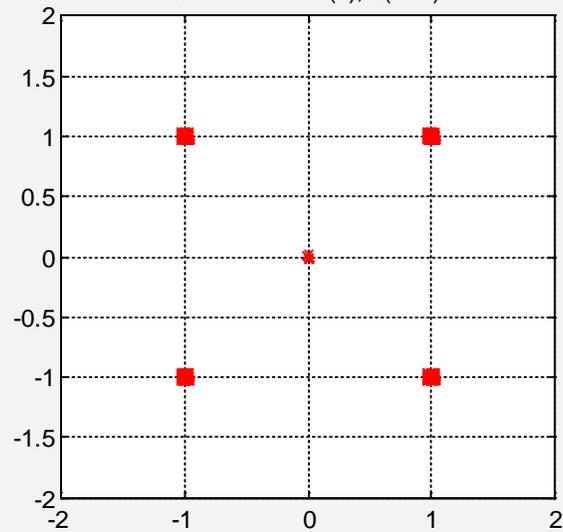
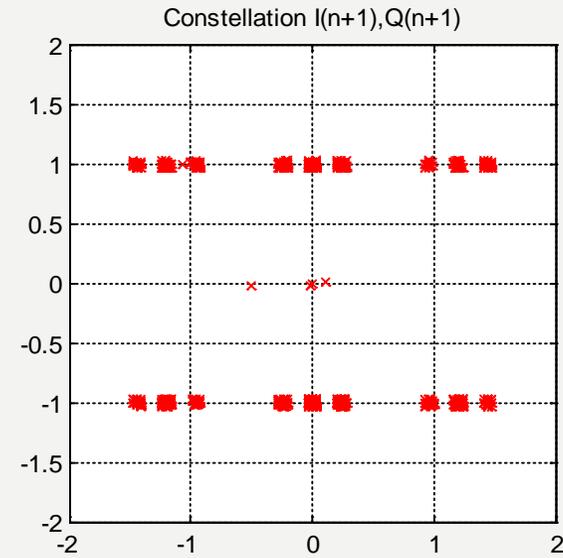
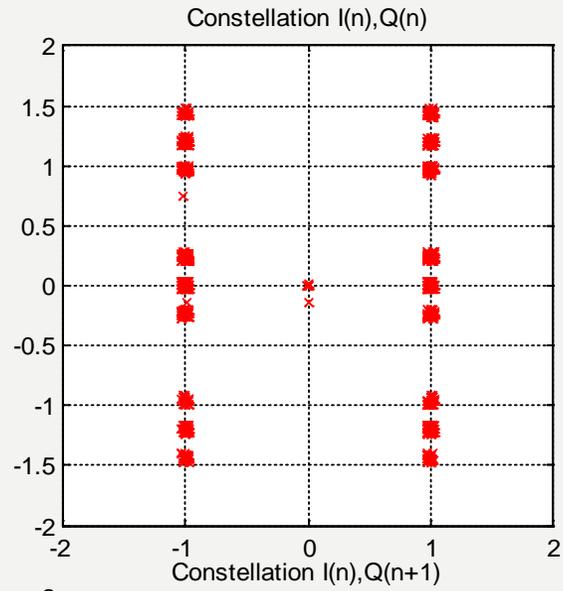


# OQPSK COUPLED TIMING-CARRIER LOOPS

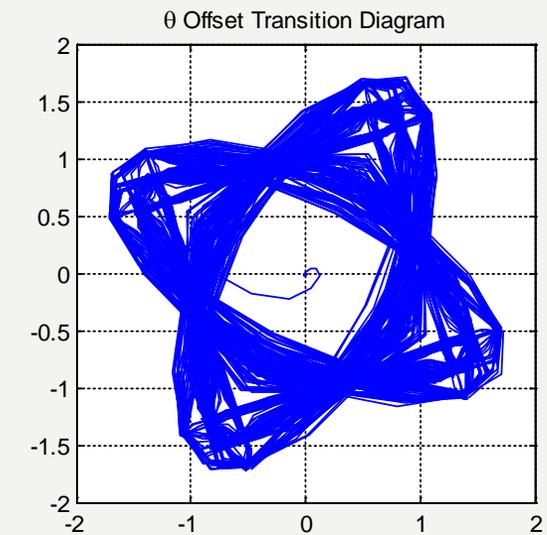
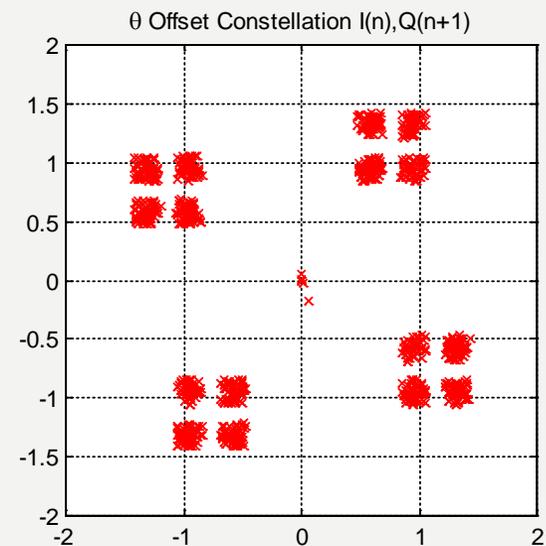
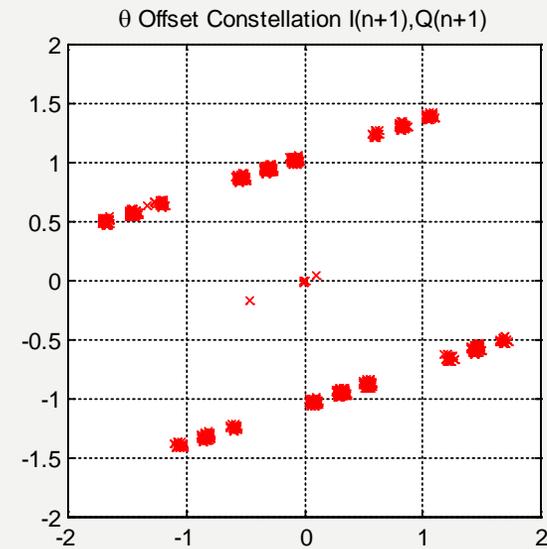
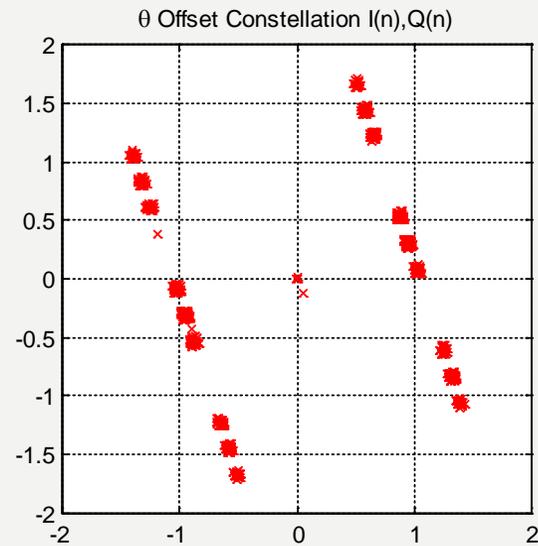


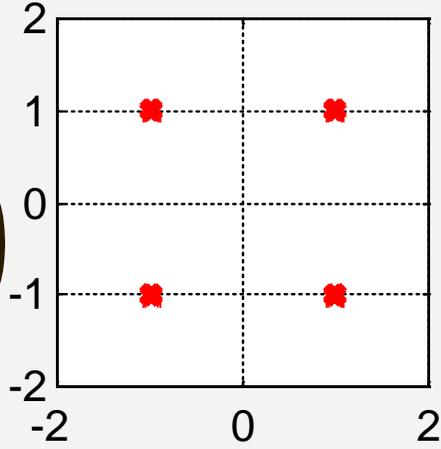
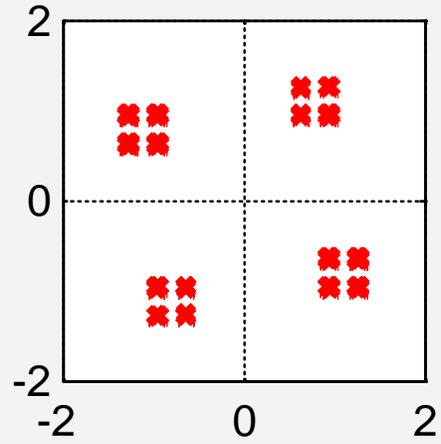
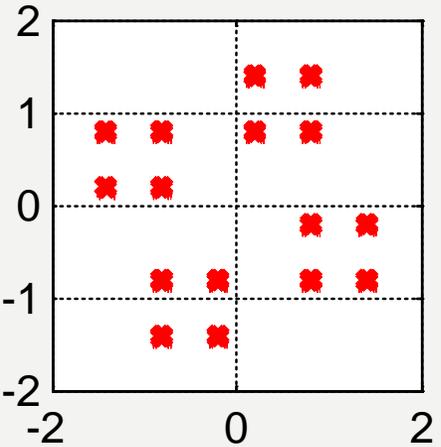
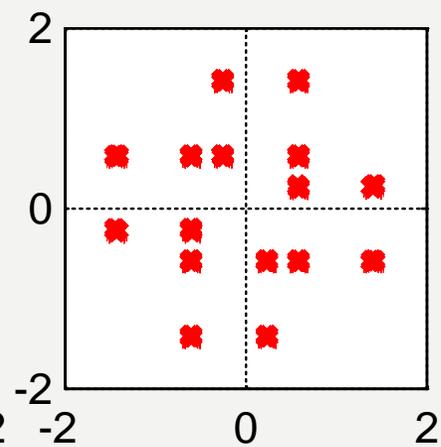
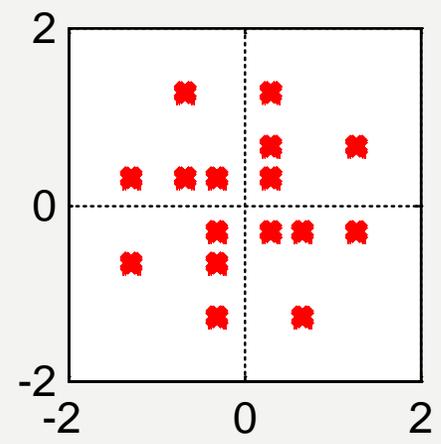
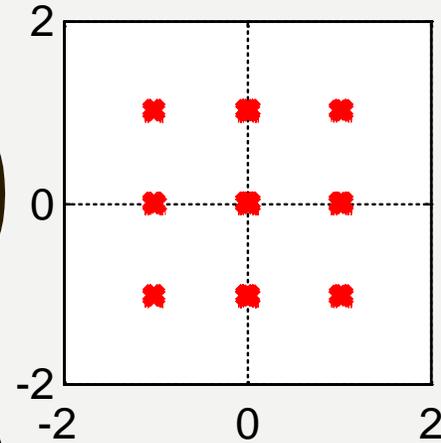
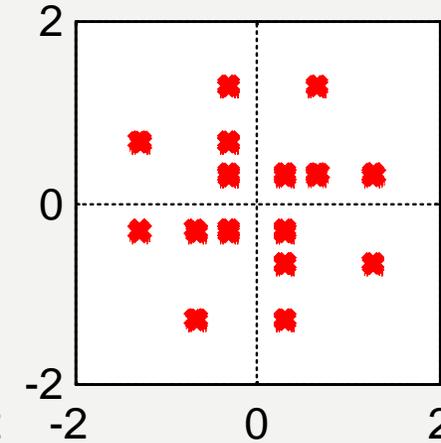
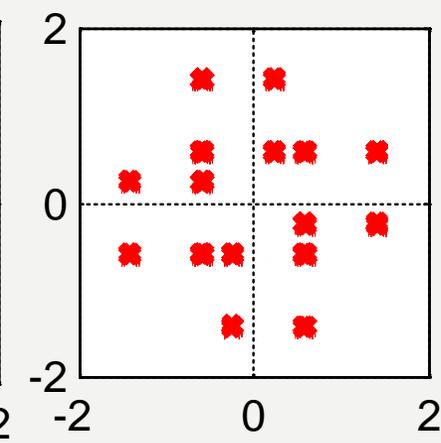
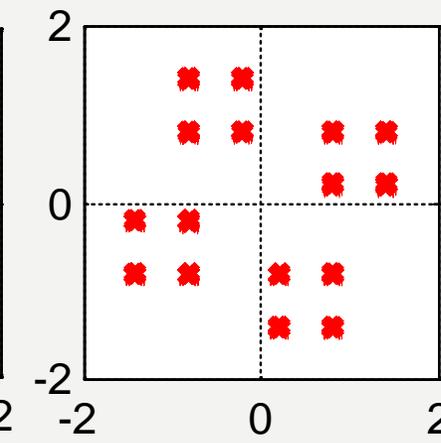
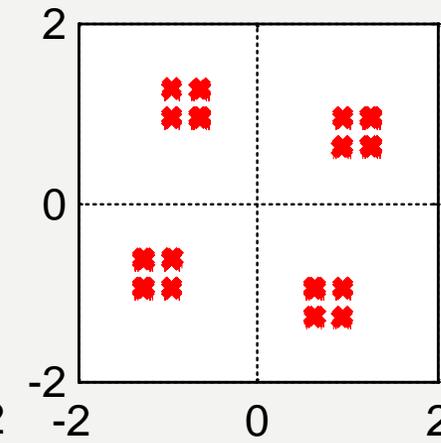
Bandwidth-Efficient Digital Modulation with Application to Deep Space Communications, John Wiley, 2003

# OQPSK CONSTELLATIONS



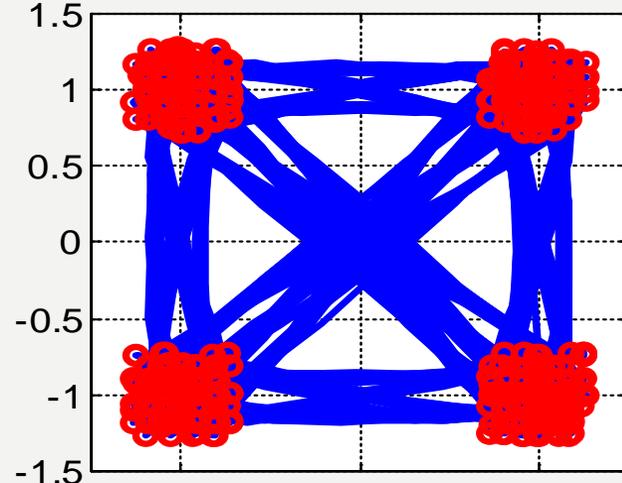
# PHASE OFFSET OQPSK CONSTELLATIONS



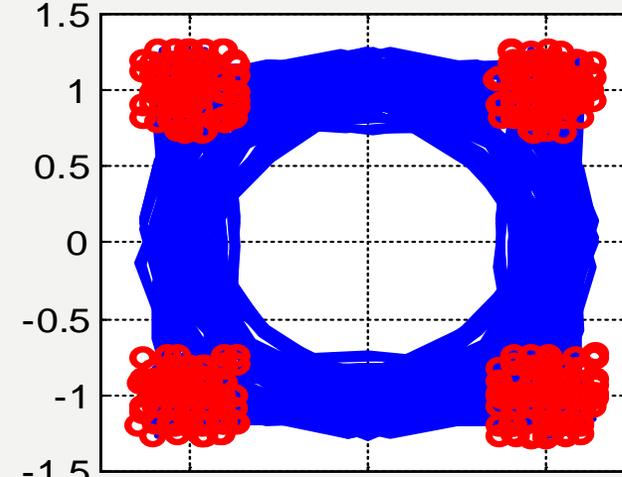
Phase Shift  $2\pi 0/20$ Phase Shift  $2\pi 1/20$ Phase Shift  $2\pi 2/20$ Phase Shift  $2\pi 3/20$ Phase Shift  $2\pi 4/20$ Phase Shift  $2\pi 5/20$ Phase Shift  $2\pi 6/20$ Phase Shift  $2\pi 7/20$ Phase Shift  $2\pi 8/20$ Phase Shift  $2\pi 9/20$ 

# TRANSITIONS: QPSK, OFFSET QPSK, CPM

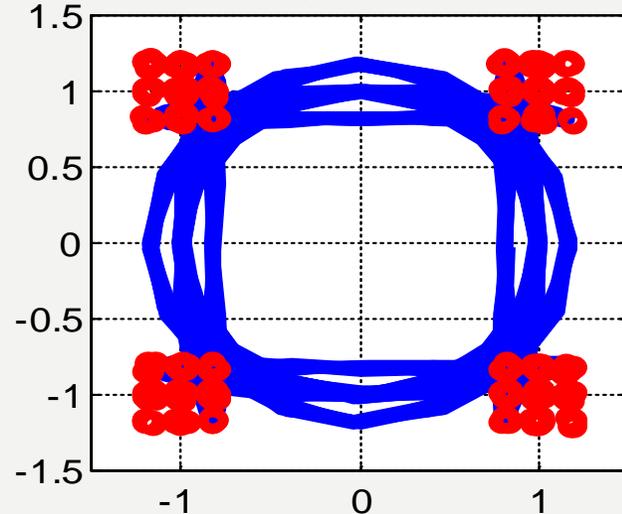
Trans. Diag. QPSK,  $\alpha=0.5$



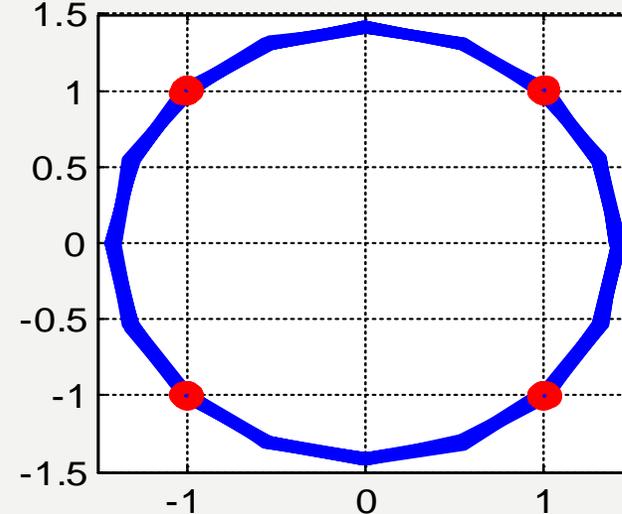
Trans. Diag. OQPSK,  $\alpha=0.5$



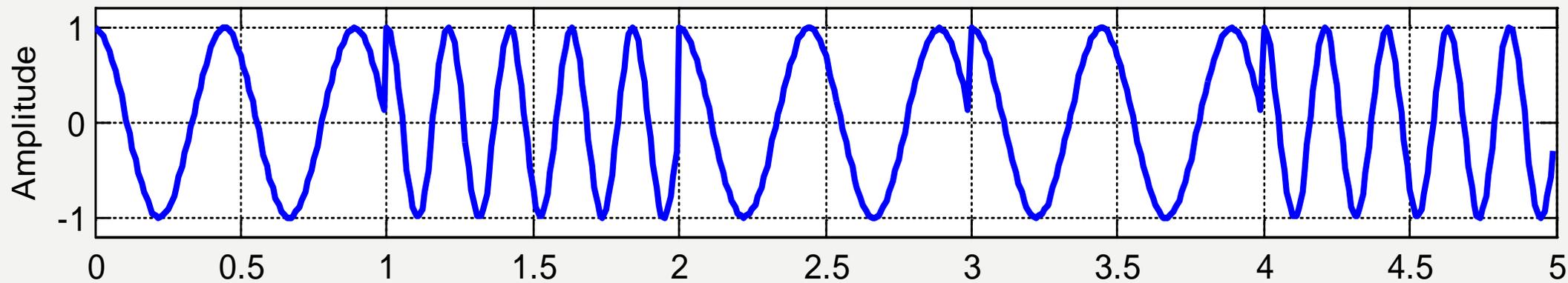
Trans. Diag. QPSK,  $\alpha=0.8$



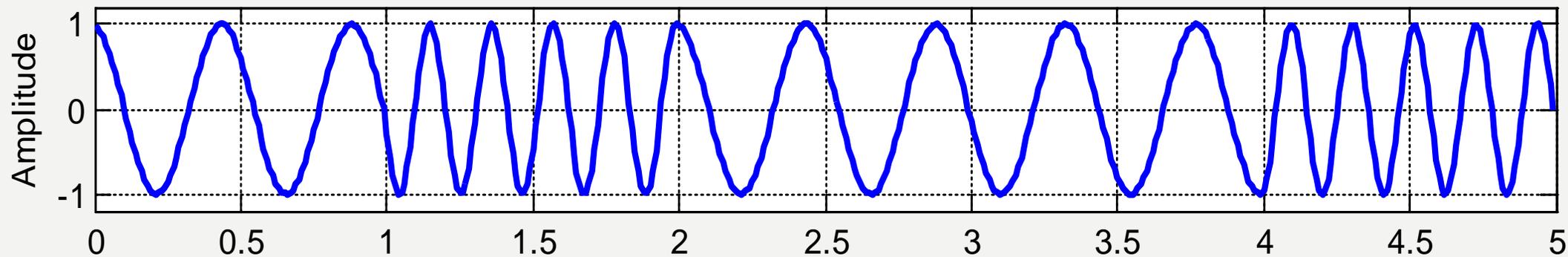
Trans. Diag. CPM,  $h=0.5$



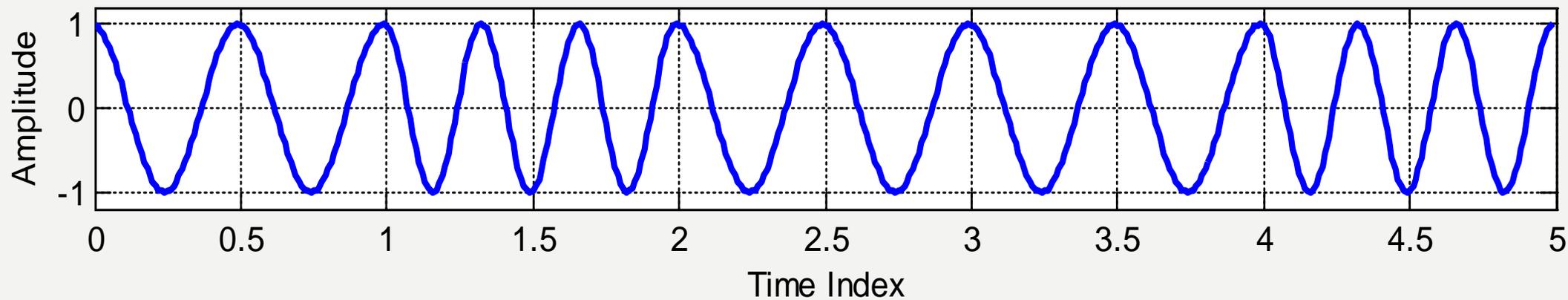
Discontinuous FSK



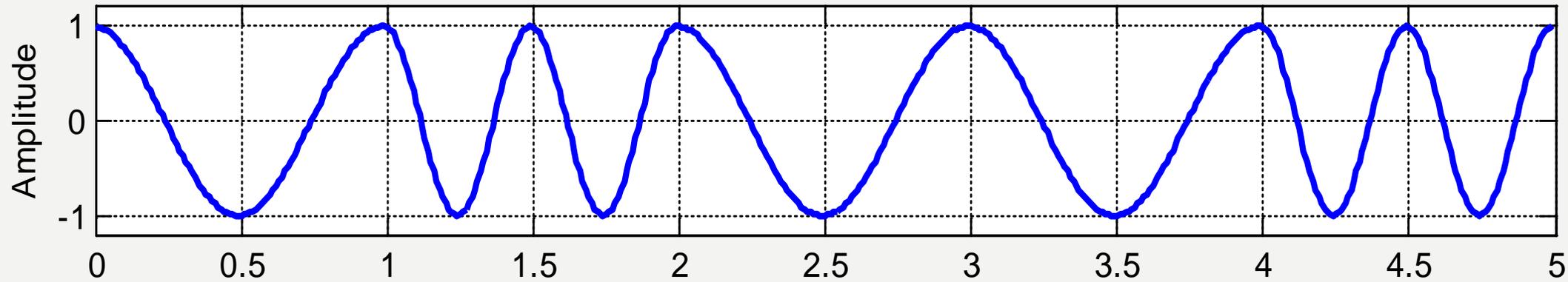
Continuous FSK



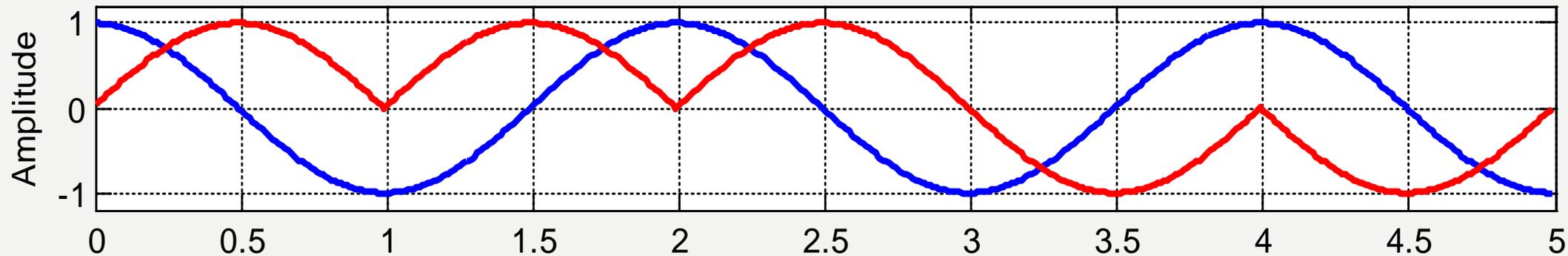
Integer Frequency Difference for Orthogonal Continuous FSK



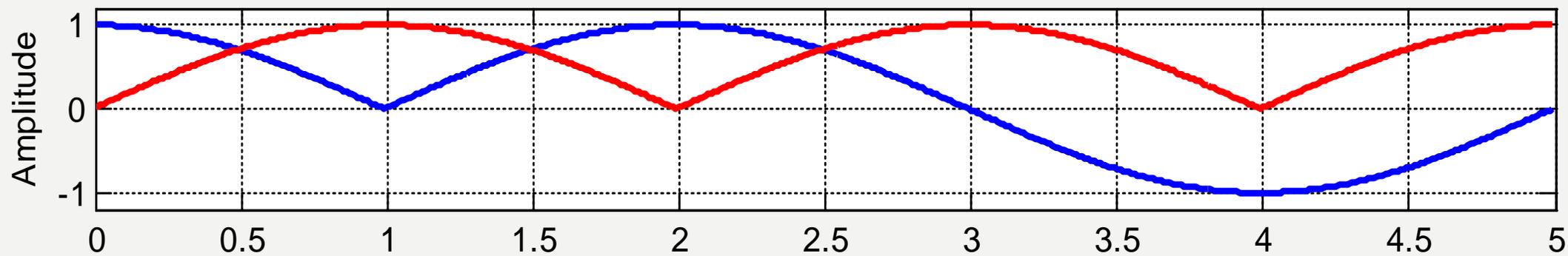
# Minimum Integer Frequency for Orthogonal Continuous FSK



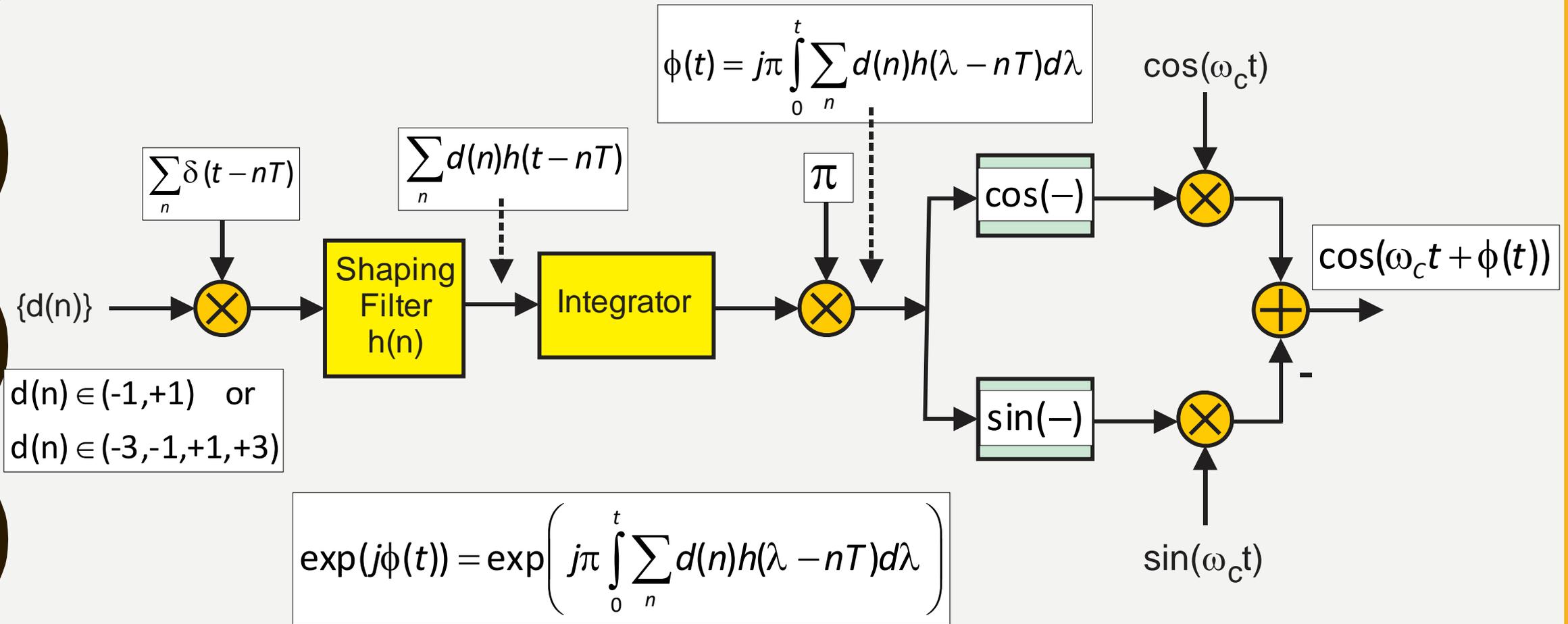
Plus or Minus Half Circle Rotation CPM



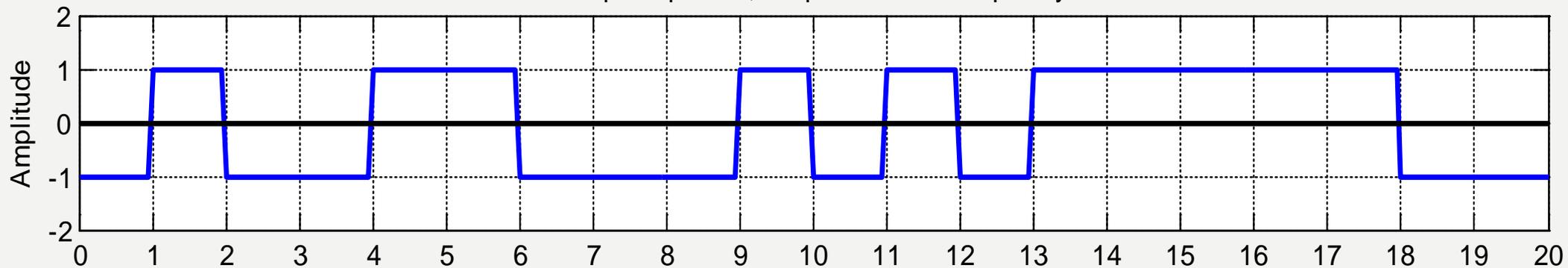
Plus or Minus Quarter Circle Rotation CPM, (MSK)



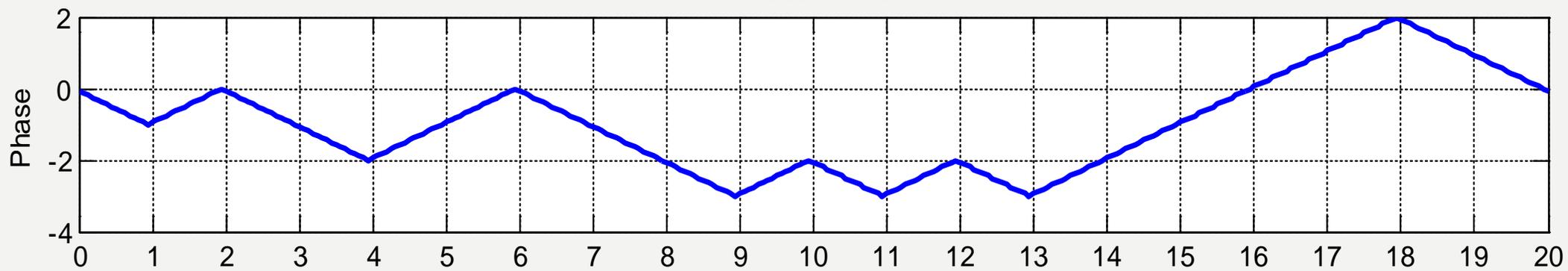
Time Index



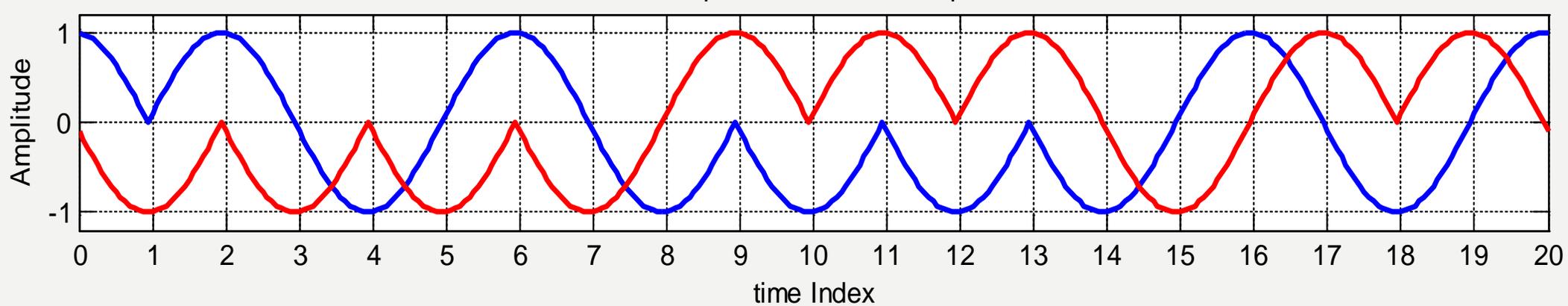
2-Level Input Alphabet, Proportional to Frequency Profile



Phase Profile

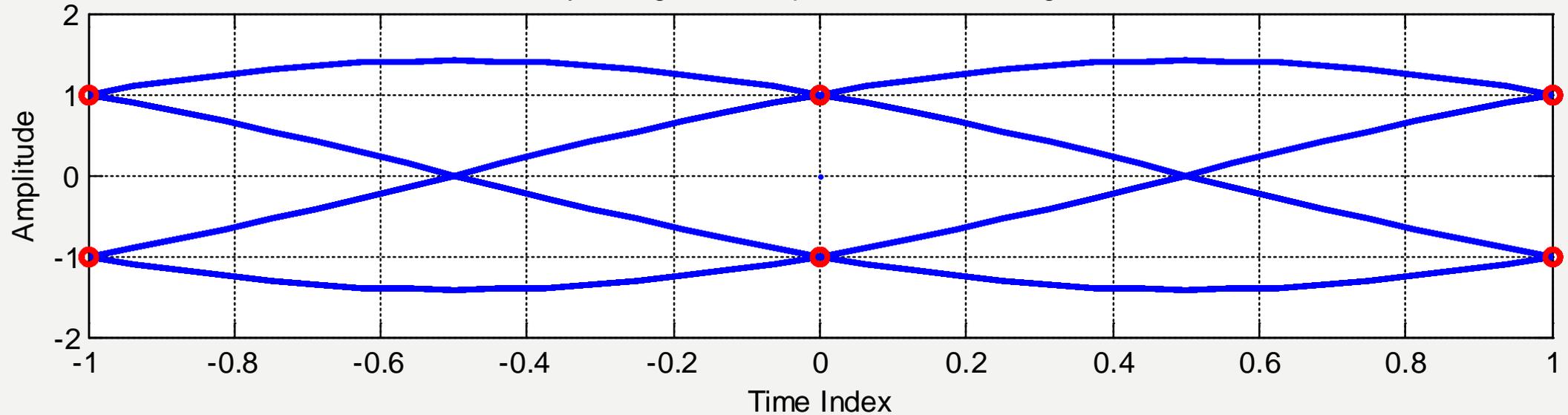


CPM Complex Quadrature Components

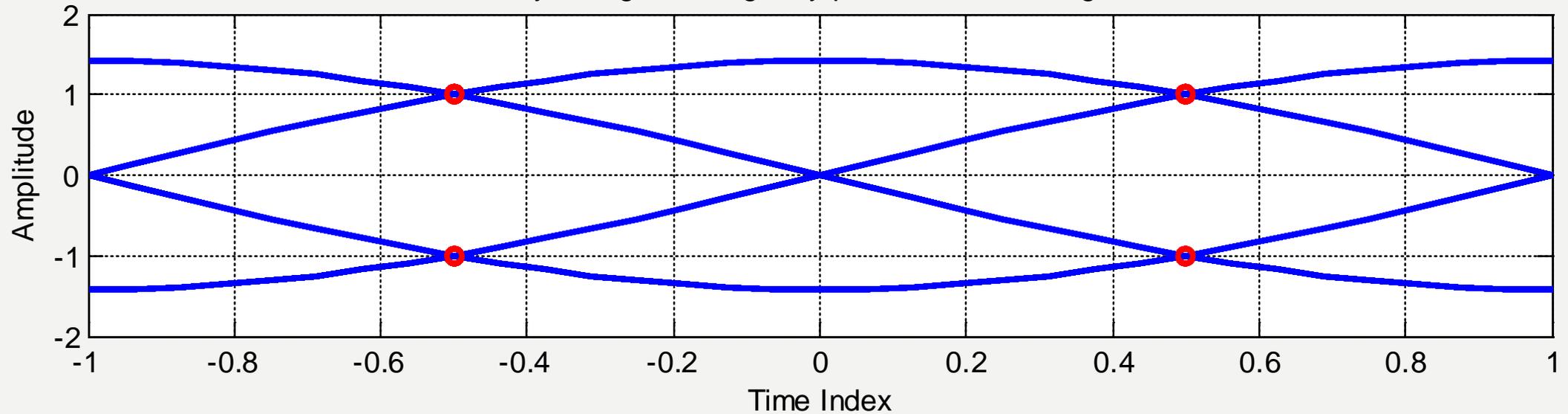


# Modulator Eye Diagrams CPM

Eye Diagram Real part QPSK CPM Signal

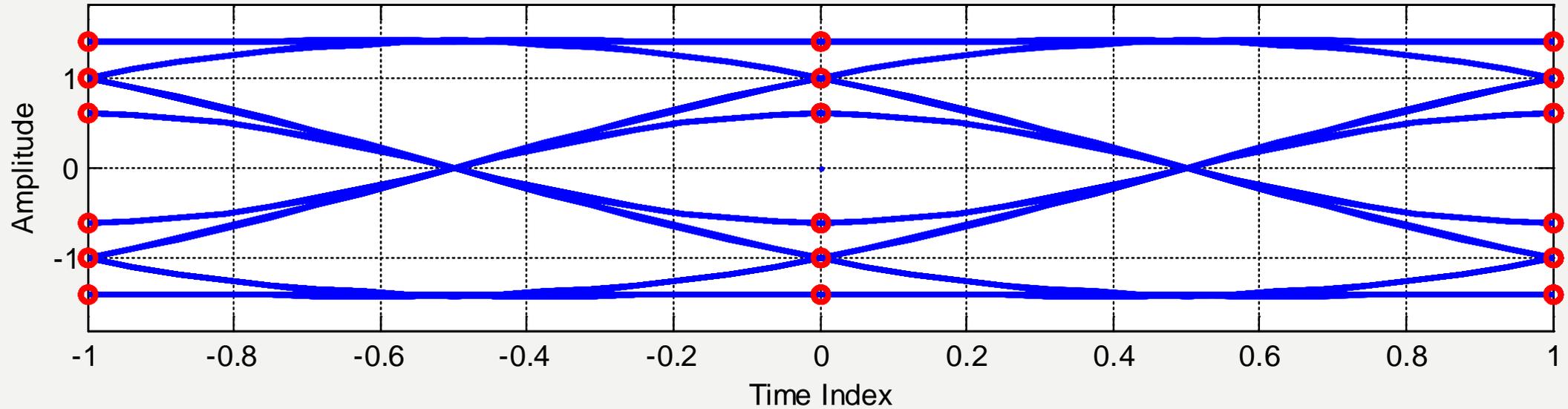


Eye Diagram Imaginary part QPSK CPM Signal

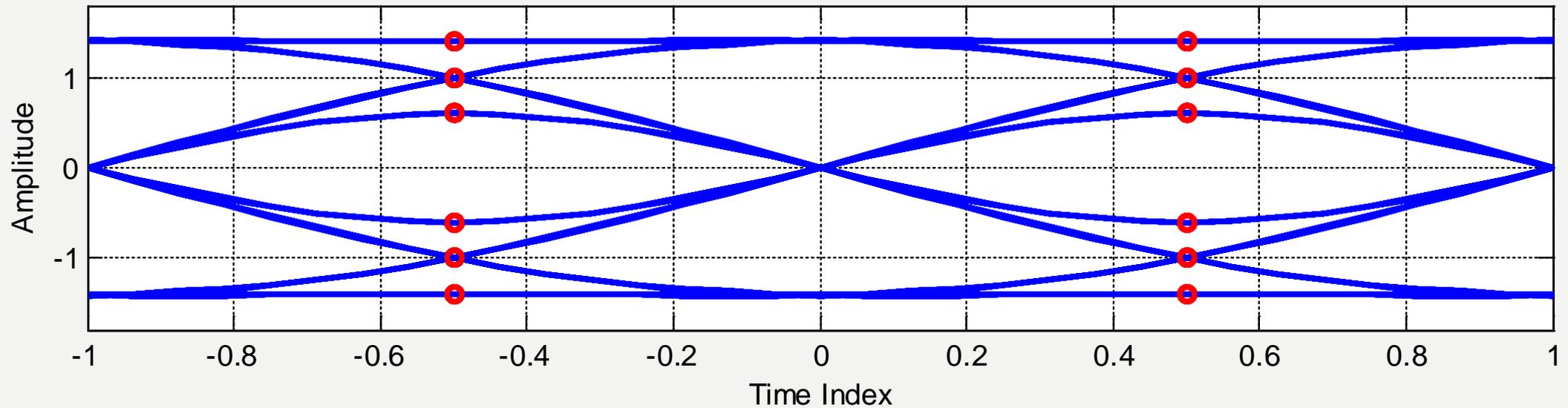


# Demodulator Eye Diagrams CPM

Eye Diagram Real part Matched Filter QPSK CPM



Eye Diagram Imaginary part Matched Filter QPSK CPM

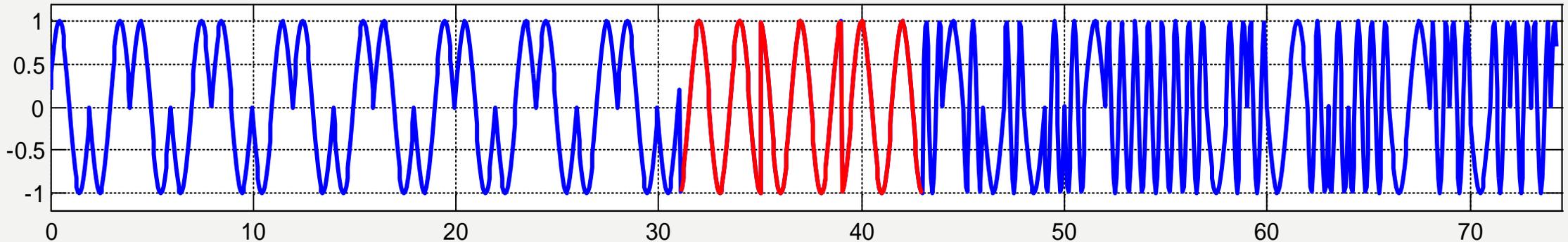


# Typical (stylized) CPM Signal Structure

Preamble

Start of Message

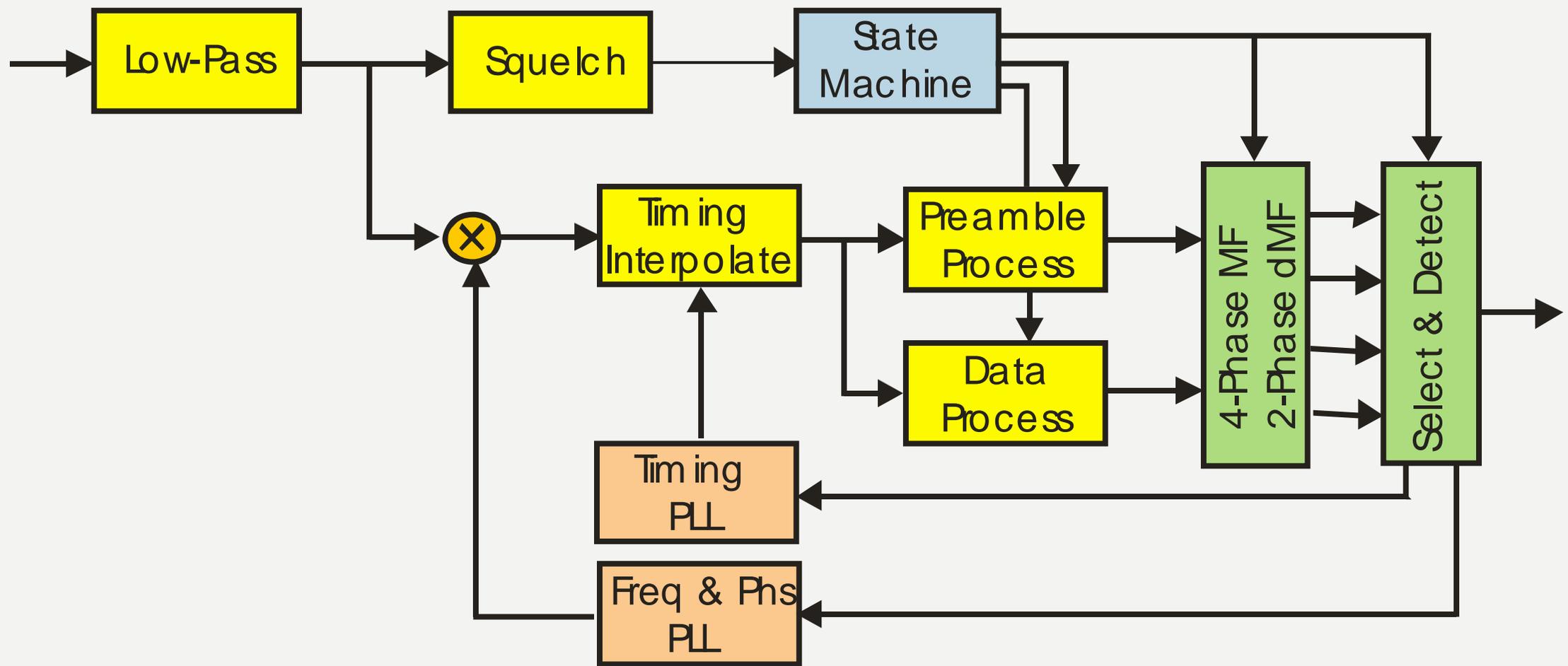
Random Data Payload



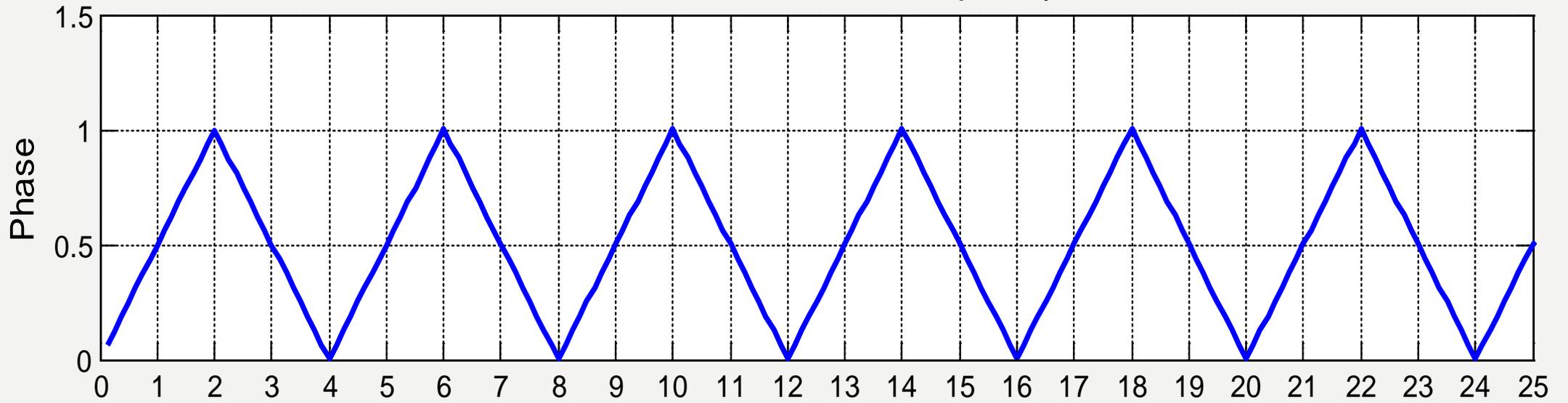
Perform Squelch Detection  
Detect and Remove Doppler  
Perform AGC  
Matched Filter Preamble  
Perform Timing Synch

Detect SOM  
Inform State Machine  
Enable All Matched Filters  
Enable Comparisons of  
Matched Filter Outputs  
Engage Phase PLL

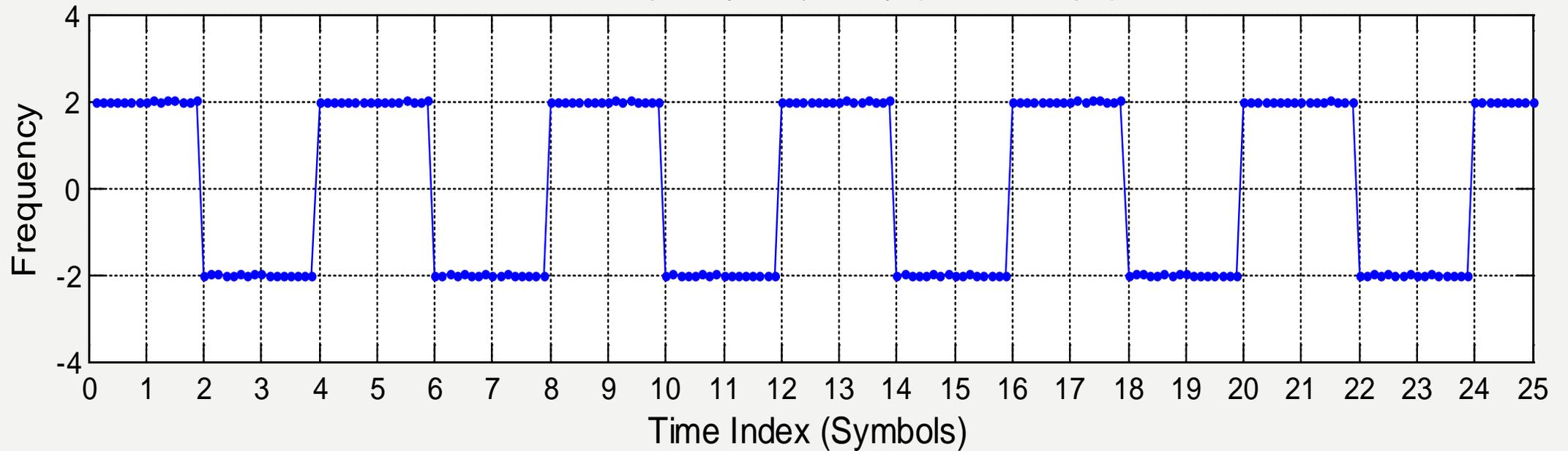
Operate Timing PLL  
Operate Phase PLL  
Operate AGC Loop  
Operate all Matched Filters  
Compare MF Outputs and  
Select Largest Output  
Detect Signal of Selected MF



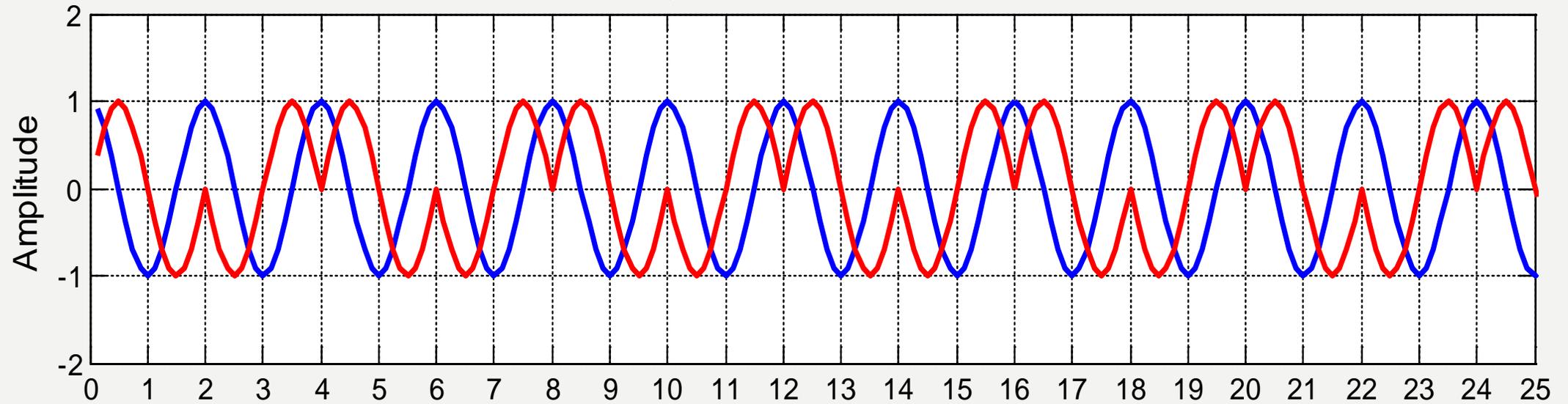
### CPM Preamble Phase Trajectory



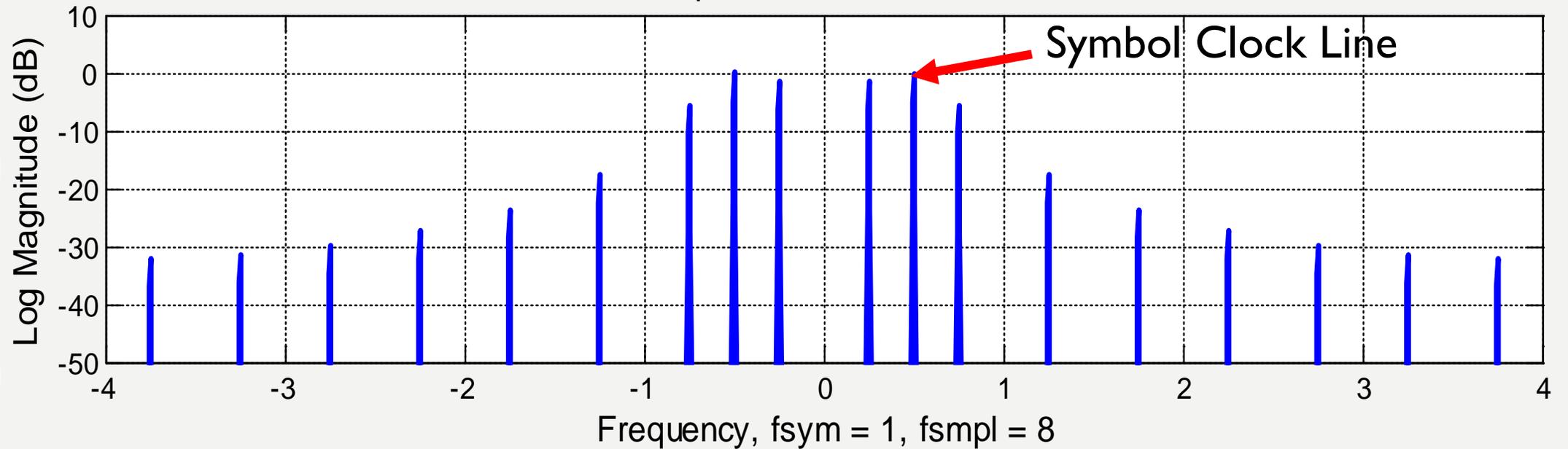
### CPM Frequency Trajectory (Phase Slope)



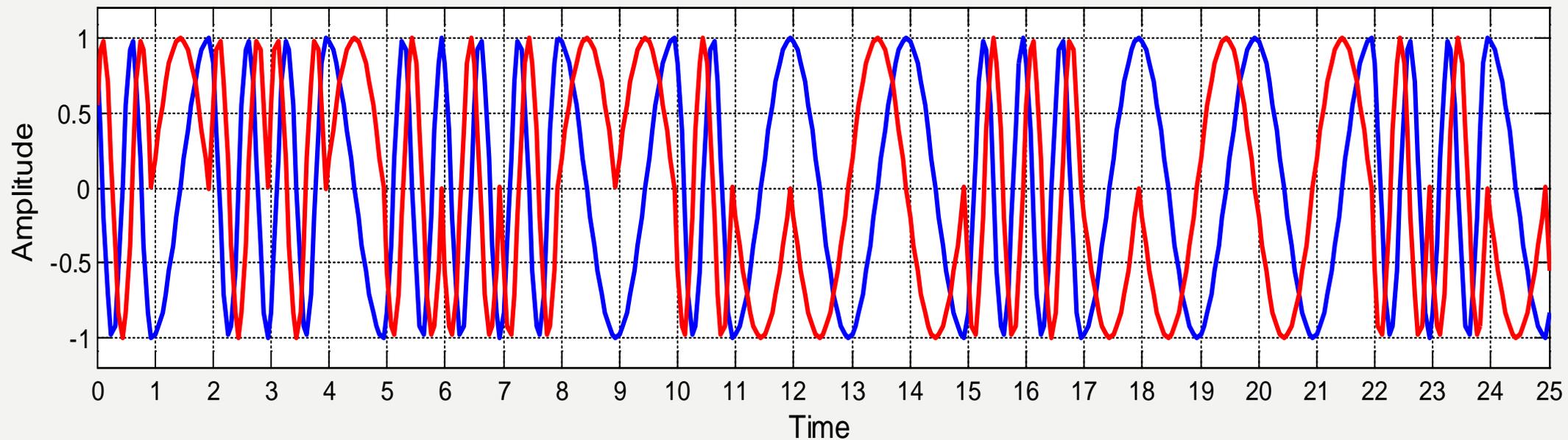
Binary CPM Preamble,  $h = 0.5$



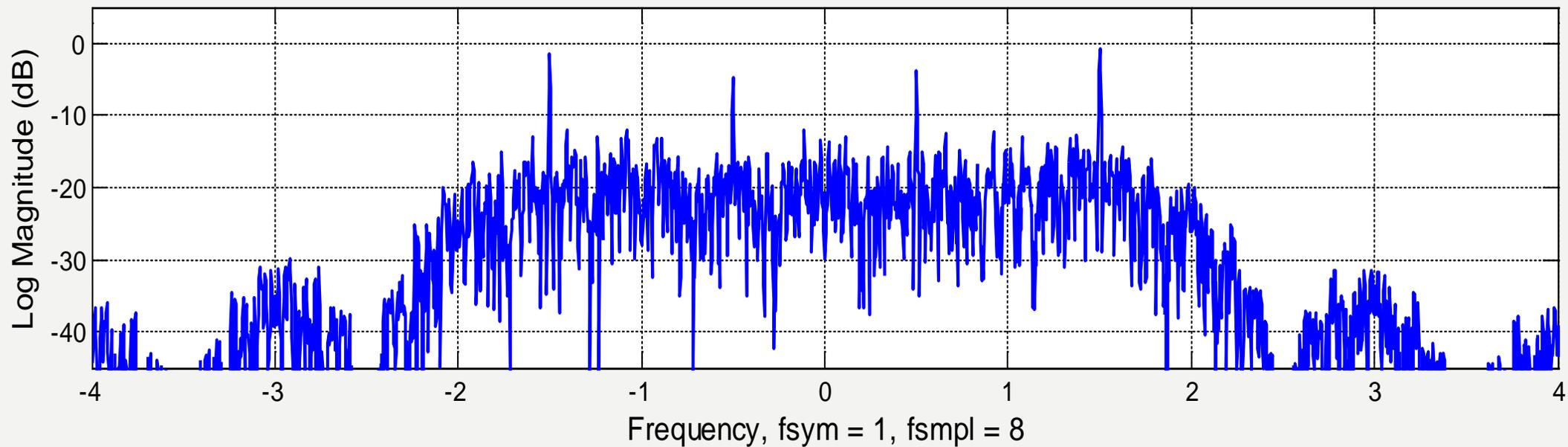
Spectrum, Preamble



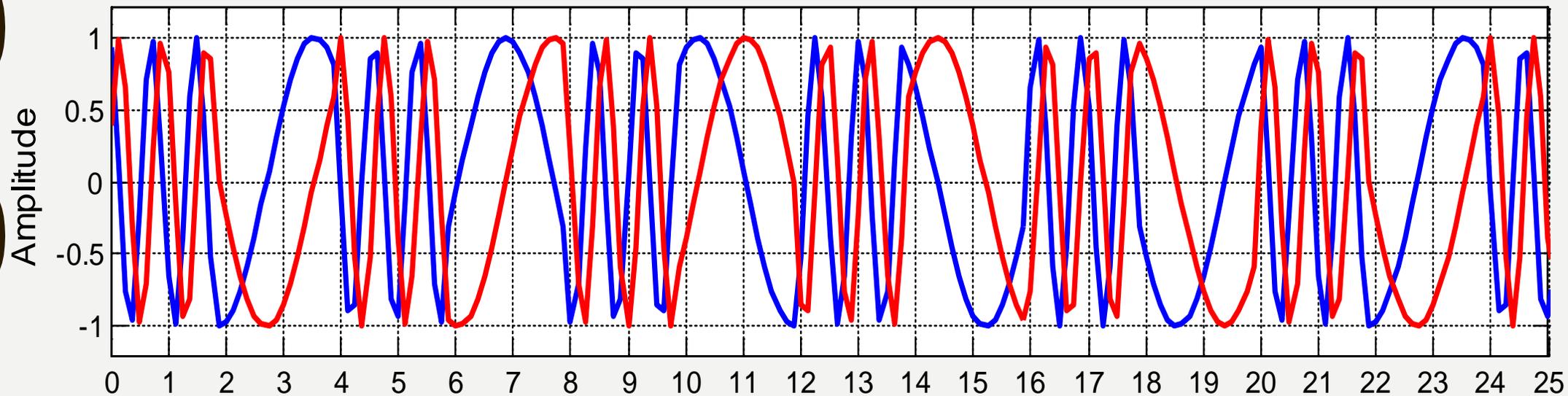
Quartenary CPM Modulated Signal,  $h=0.5$



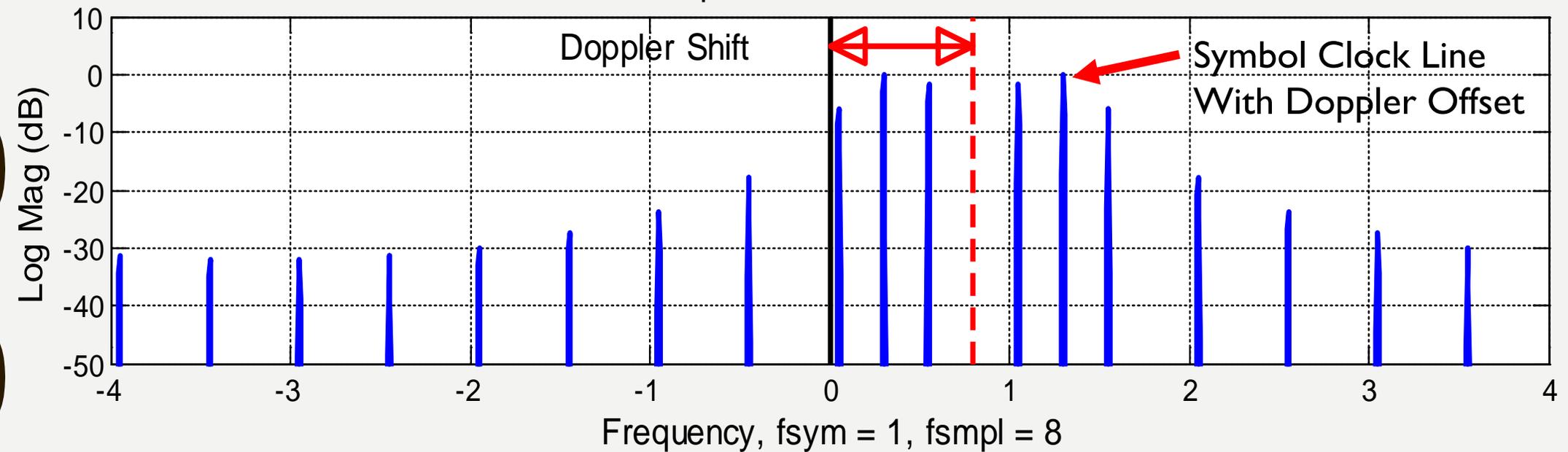
Spectrum Random Modulated Signal



Binary CPM Preamble with Doppler)

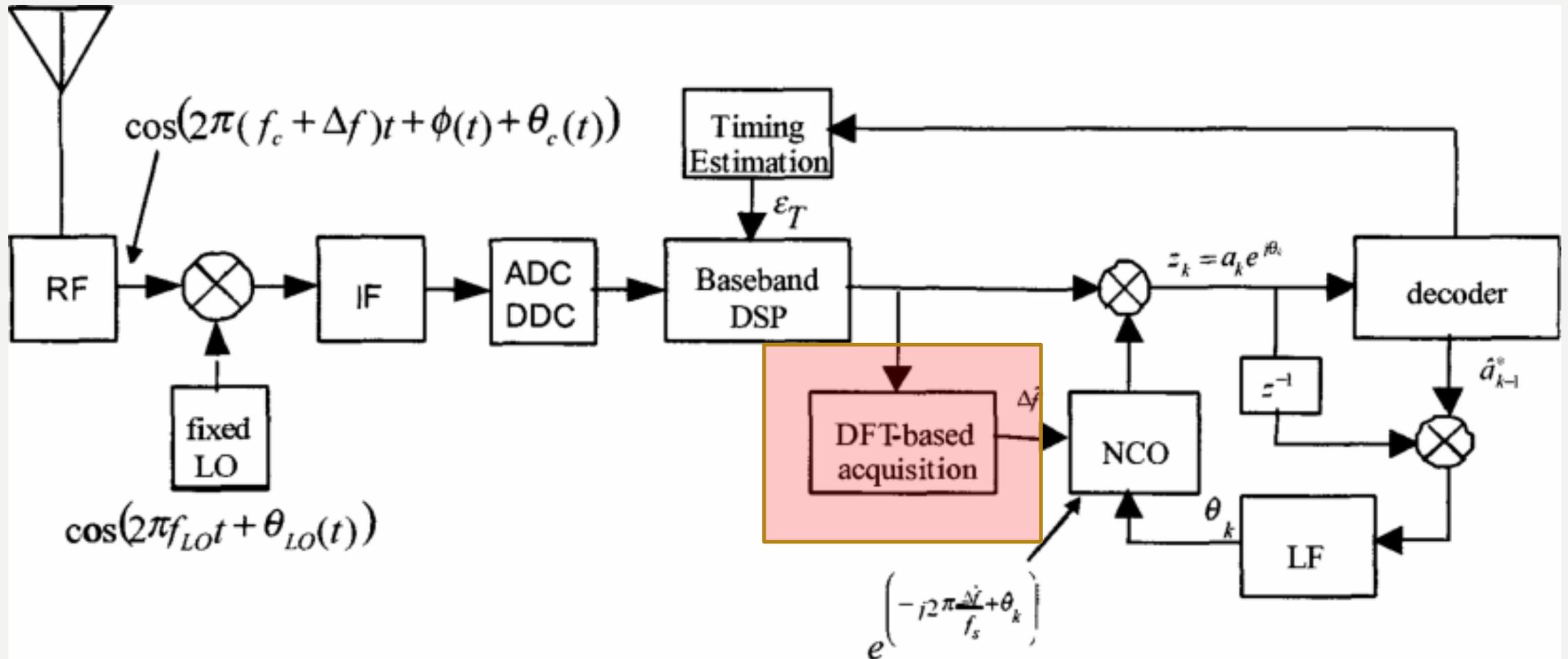


Spectrum, Preamble

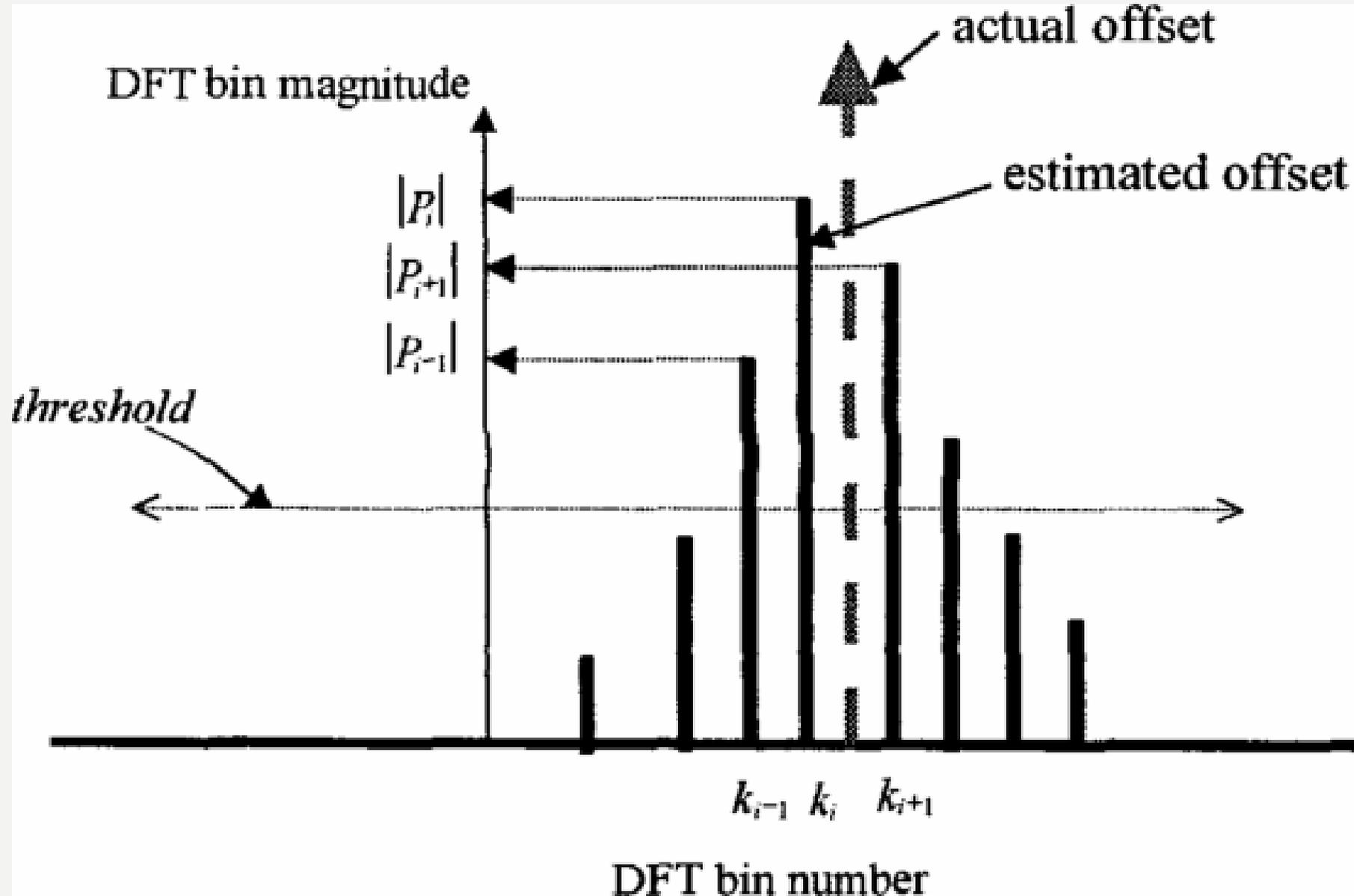


# Techniques for Acquiring and Tracking MIL-STD-181B Signals

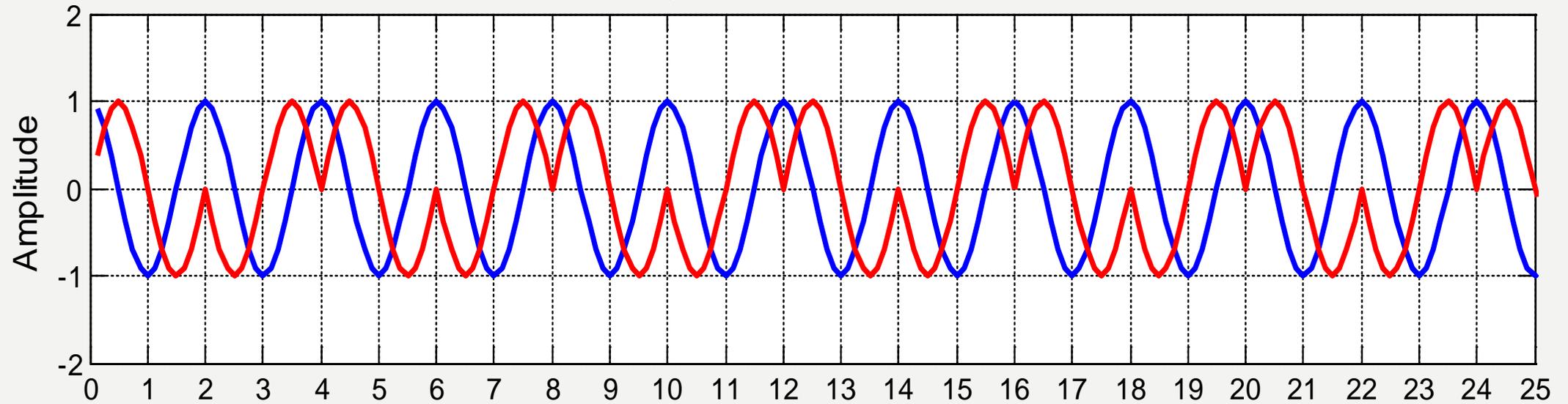
Milcom 7-10 October 2002, Anaheim CA., Mohammed K. Nezami, PhD, Raytheon, Saint Petersburg, FL.



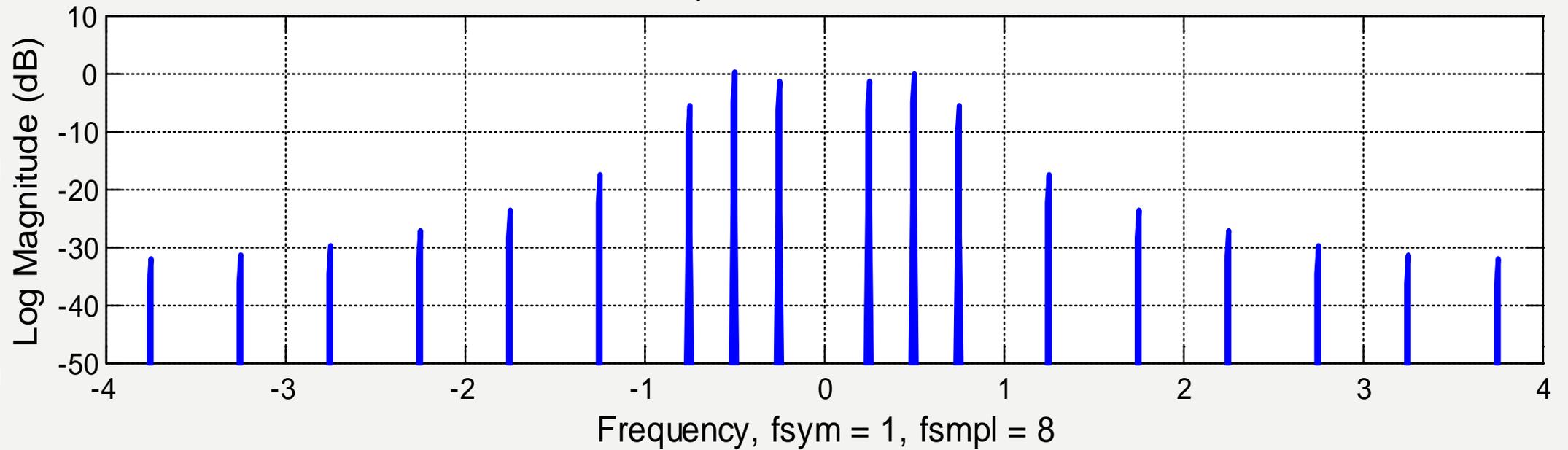
Estimate Spectral Center of Mass, and  
Direct DDS to shift Spectrum by the estimated Offset.



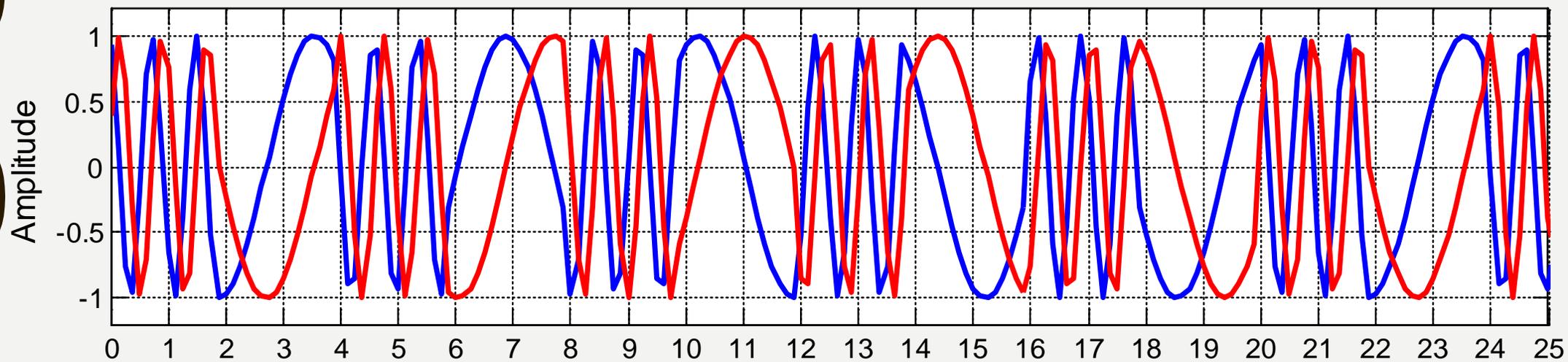
Binary CPM Preamble,  $h = 0.5$



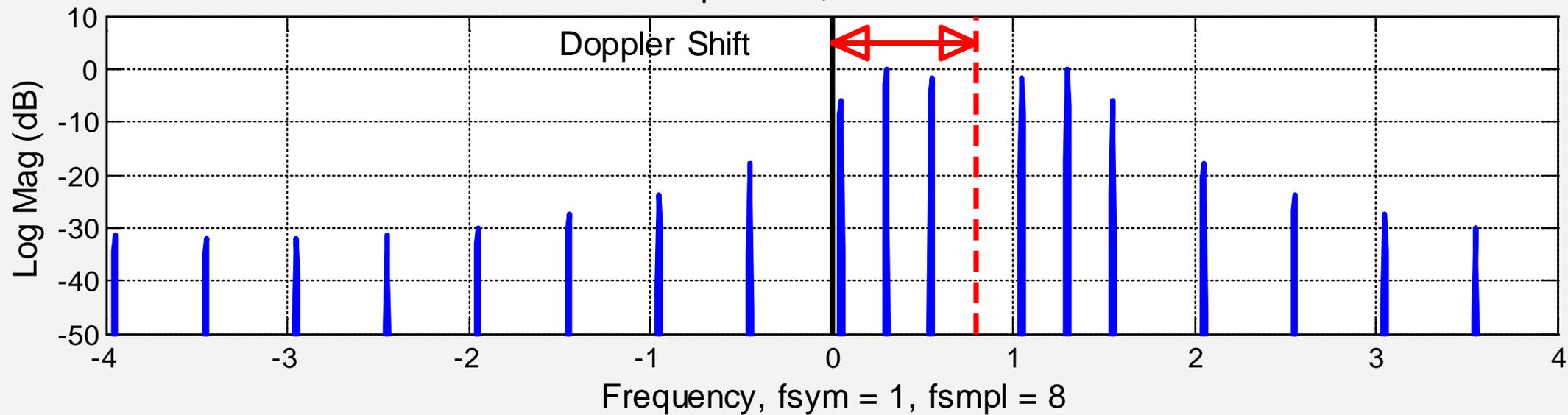
Spectrum, Preamble



Binary CPM Preamble with Doppler)



Spectrum, Preamble



$$s(n) = e^{j\theta_{prmb}n} e^{j\theta_{dpl}n} :$$

$$n = 0 : N-1$$

$$s(n+N) = e^{-j\theta_{prmb}n} e^{j\theta_{dpl}n} :$$

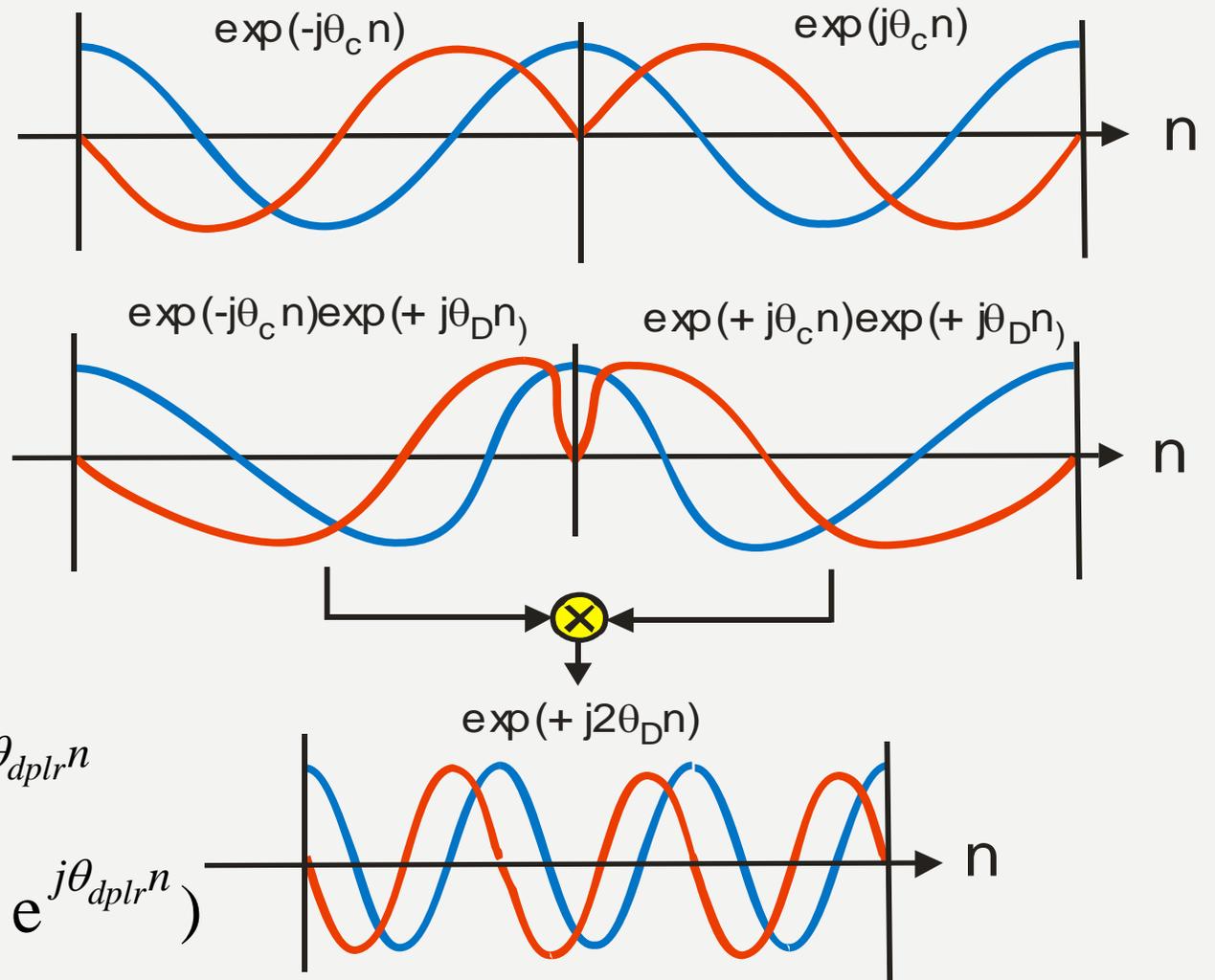
$$n = 0 : N-1$$

$$p(n) = s(n) \cdot s(n+N)$$

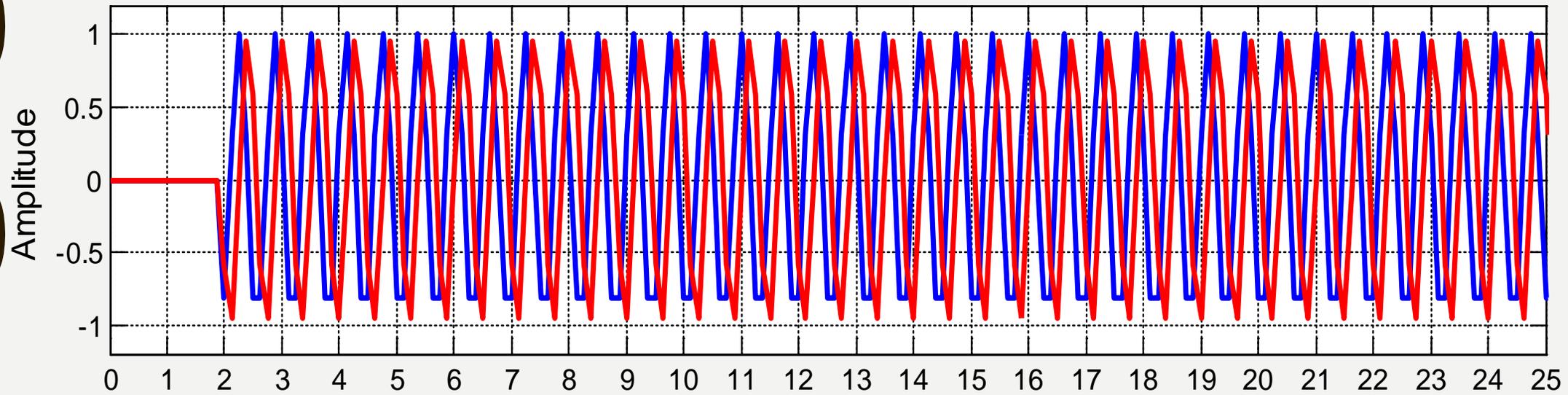
$$= e^{+j\theta_{prmb}n} e^{j\theta_{dpl}n} \cdot e^{-j\theta_{prmb}n} e^{j\theta_{dpl}n}$$

$$= (e^{+j\theta_{prmb}n} \cdot e^{-j\theta_{prmb}n}) (e^{j\theta_{dpl}n} \cdot e^{j\theta_{dpl}n})$$

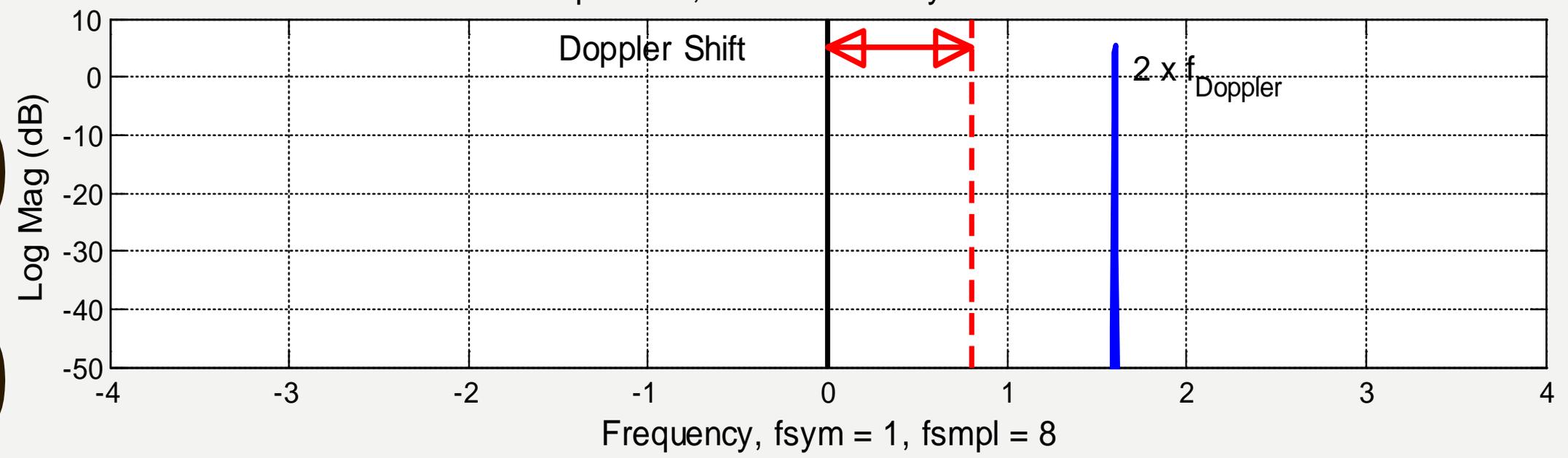
$$= e^{j2\theta_{dpl}n}$$



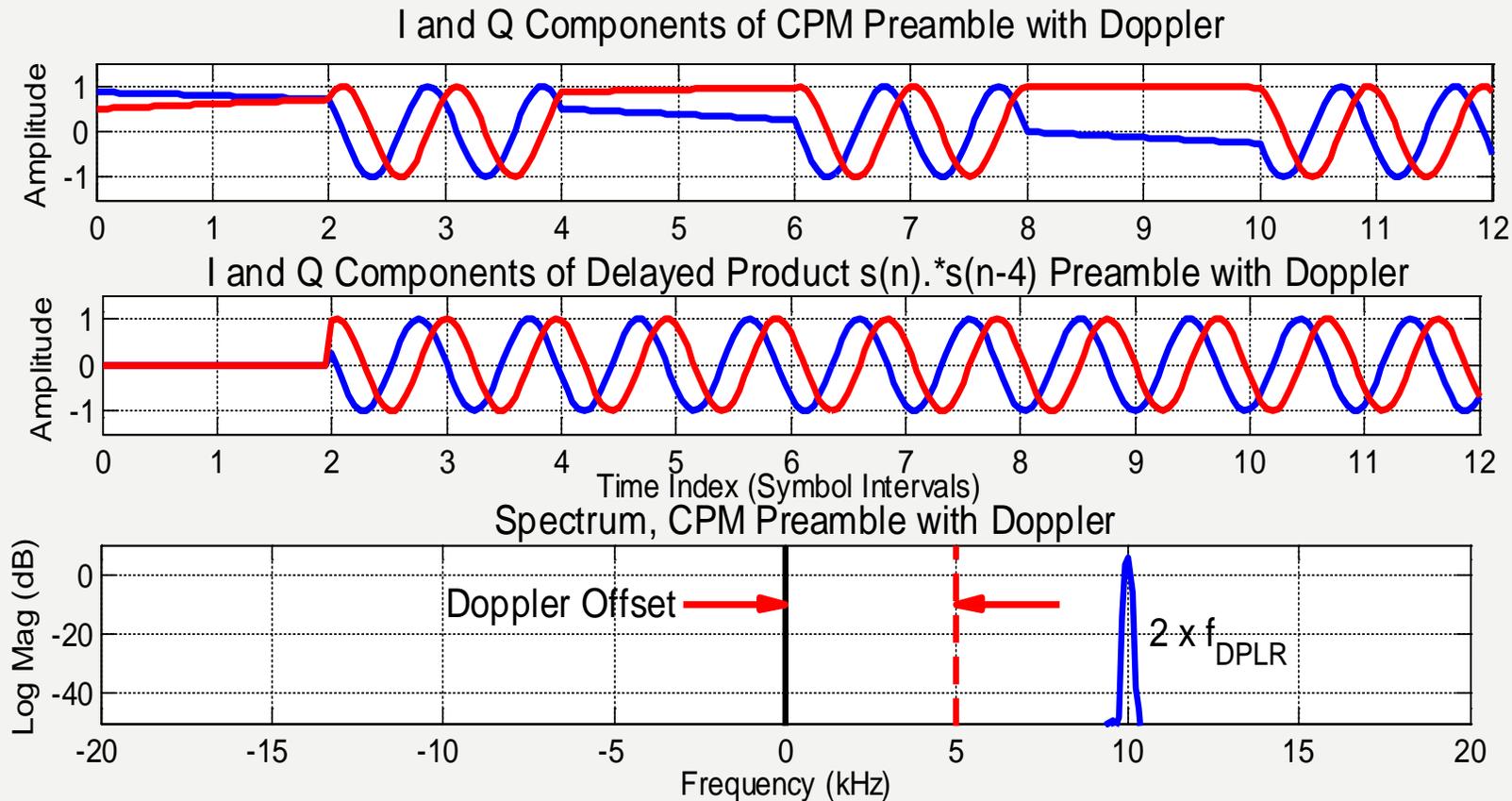
$s(n) \cdot s(n-16)$ , Preamble Delayed Product



Spectrum, Preamble Delayed Product



Top Subplot. Doppler Distorted Tones in Successive Preamble Intervals Center Subplot, Product of Successive Conjugate Symbol Intervals Forms Complex Tone at Twice Doppler Frequency. Third Subplot Spectrum of Doppler Tone Formed.



four Modulation Complex Envelopes, N-Samples per Symbol

$h1a = \exp(+j\frac{\pi}{2N}(0:N-1))$ : Positive Rotation, Quarter Cycle

$h1b = \exp(-j\frac{\pi}{2N}(0:N-1))$ : Negative Rotation, Quarter cycle

$h2a = \exp(+j\frac{\pi}{N}(0:N-1))$ : Positive Rotation, Half Cycle

$h2b = \exp(-j\frac{\pi}{N}(0:N-1))$ : Negative Rotation, Half cycle

four Matched Filter Complex Envelopes, N-Samples per Symbol

*Time Reversed and Complex Conjugate*

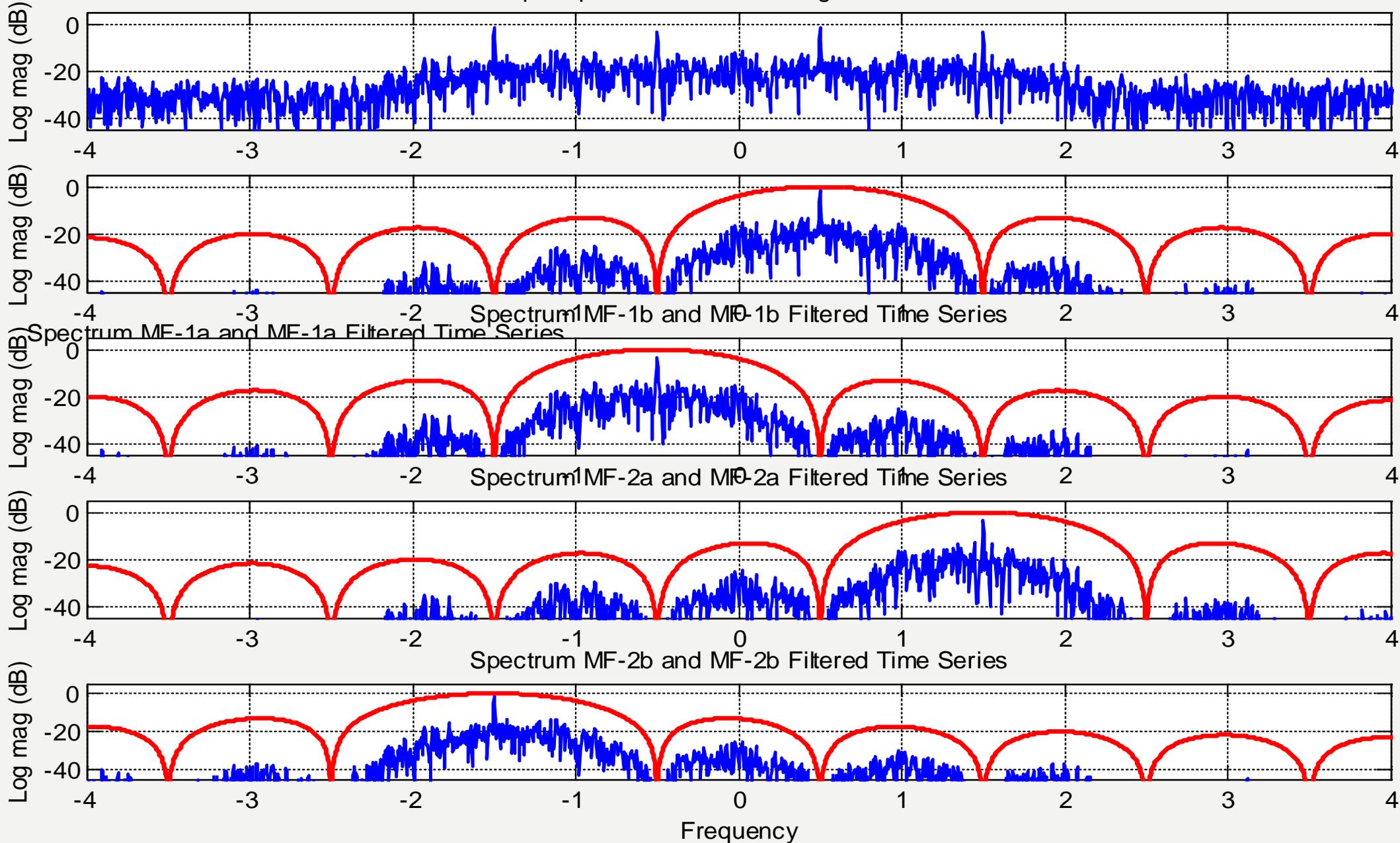
$MF1a = \exp(-j\frac{\pi}{2N}(N-1:-1:0))$ : Positive Rotation, Quarter Cycle

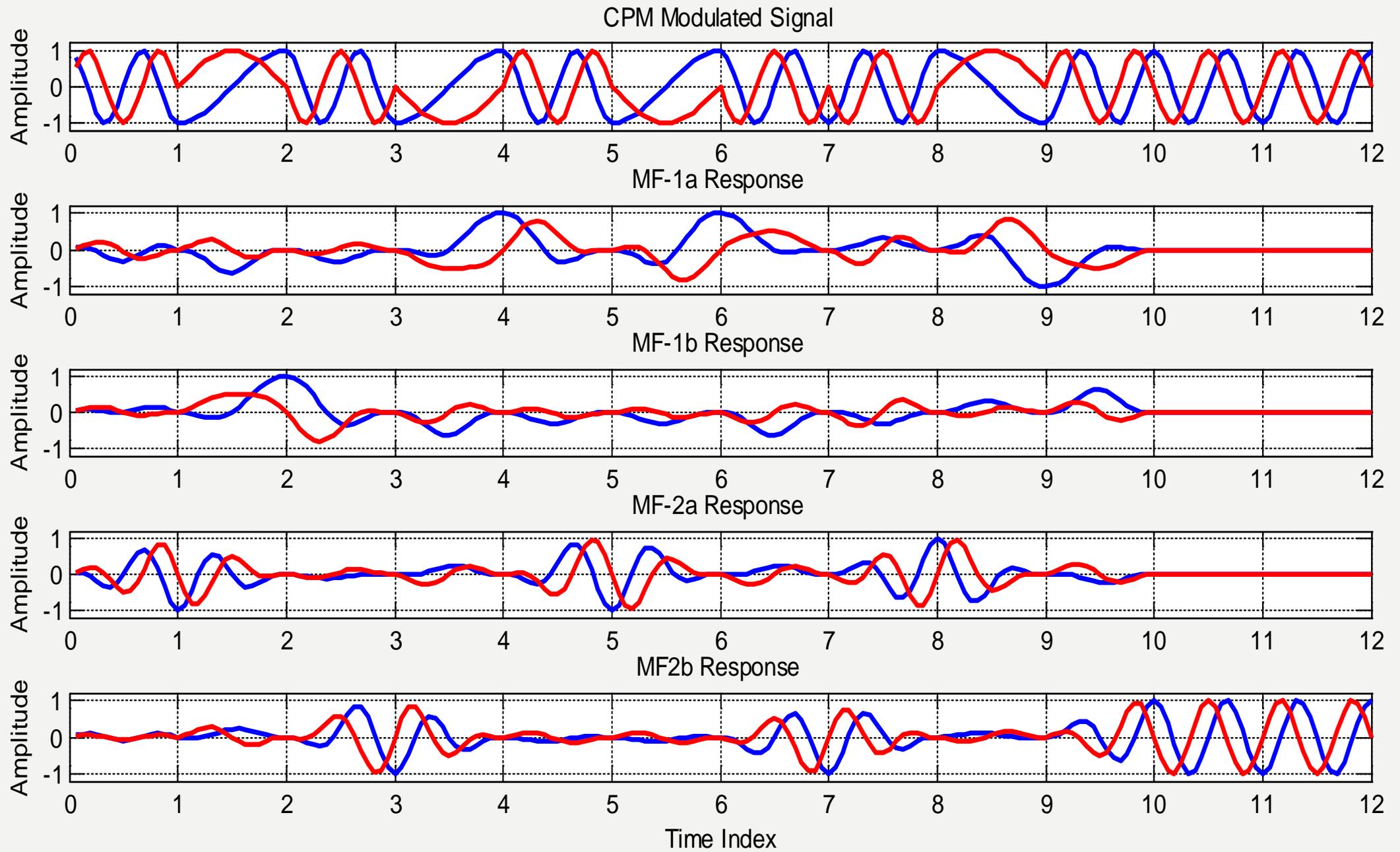
$MF1b = \exp(+j\frac{\pi}{2N}(N-1:-1:0))$ : Negative Rotation, Quarter cycle

$MF2a = \exp(-j\frac{\pi}{N}(N-1:-1:0))$ : Positive Rotation, Half Cycle

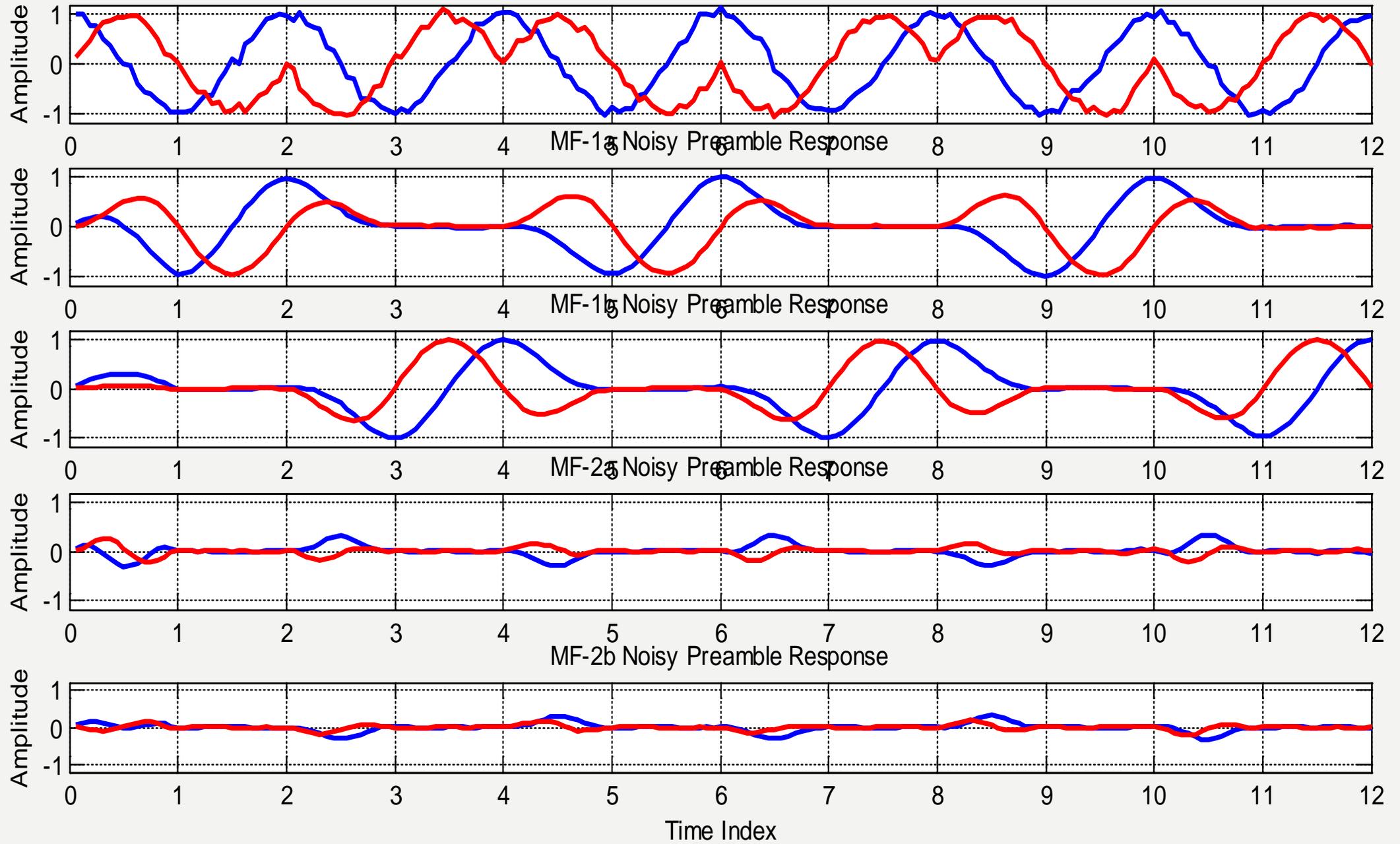
$MF2b = \exp(+j\frac{\pi}{N}(N-1:-1:0))$ : Negative Rotation, Half cycle

# Input Spectrum Modulated Signal with Noise



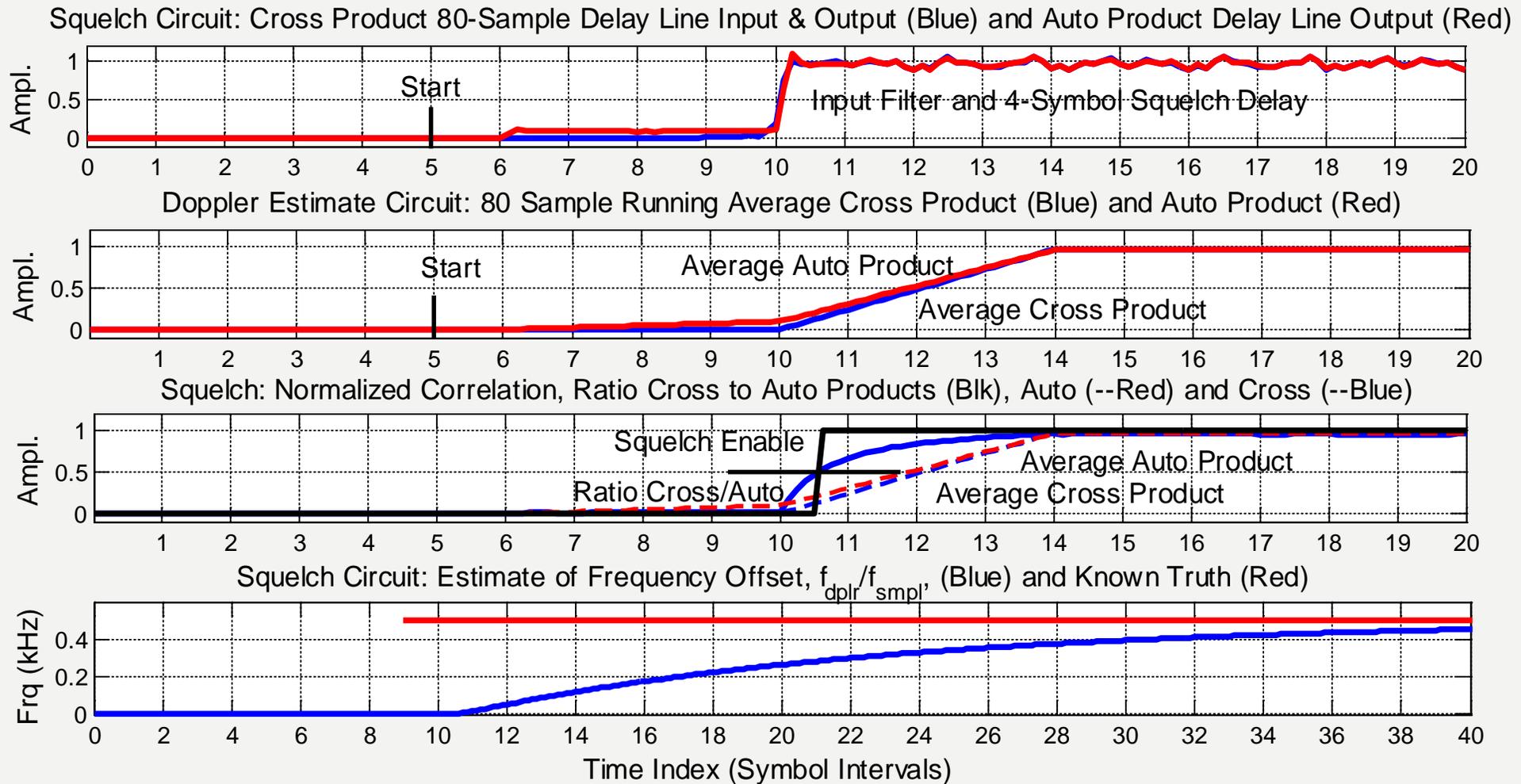


Noisy Preamble Time Series

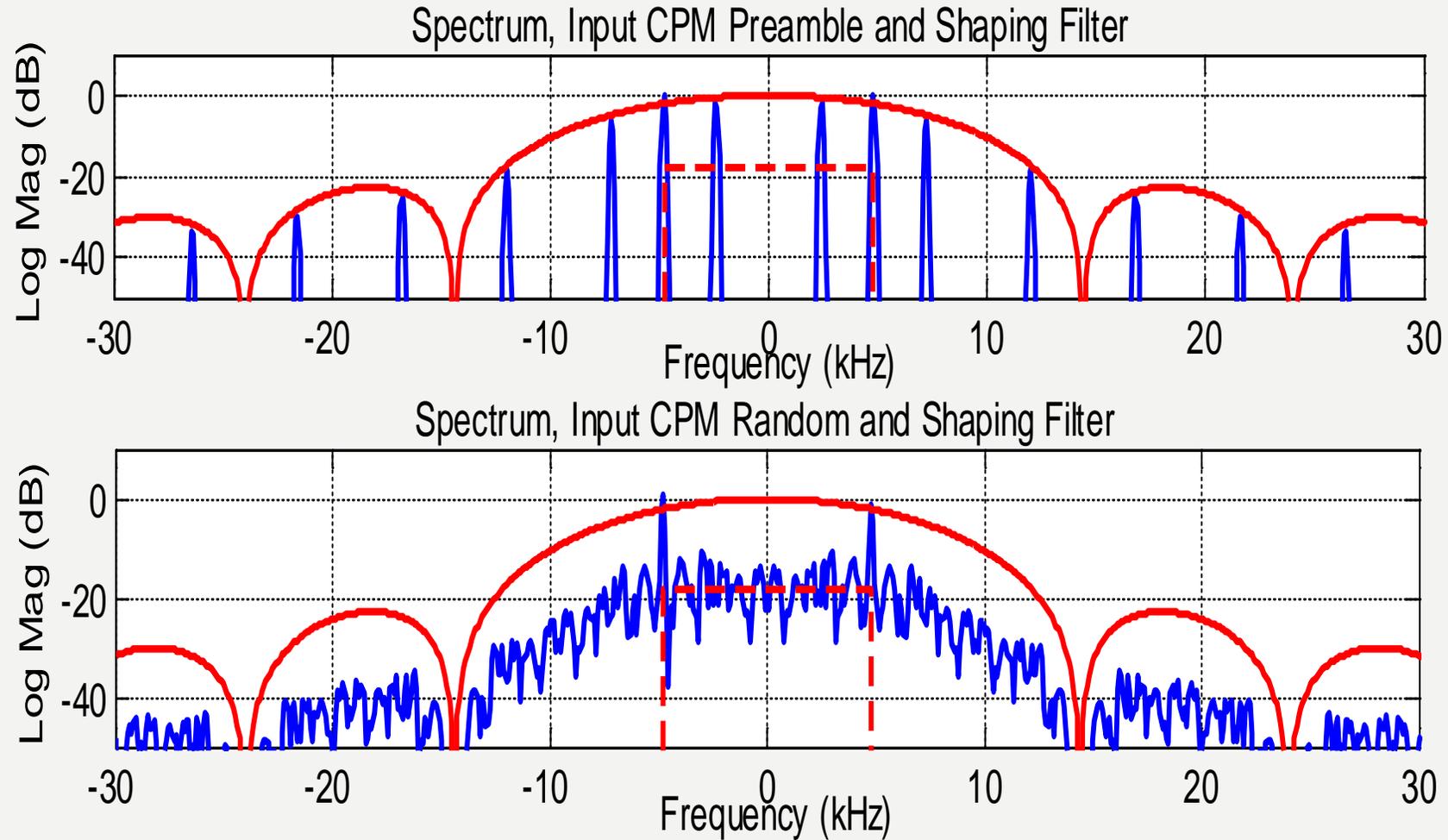


# Squelch Network, Auto and 4-Symbol Delayed Conjugate Cross Product and Auto and Cross Correlation

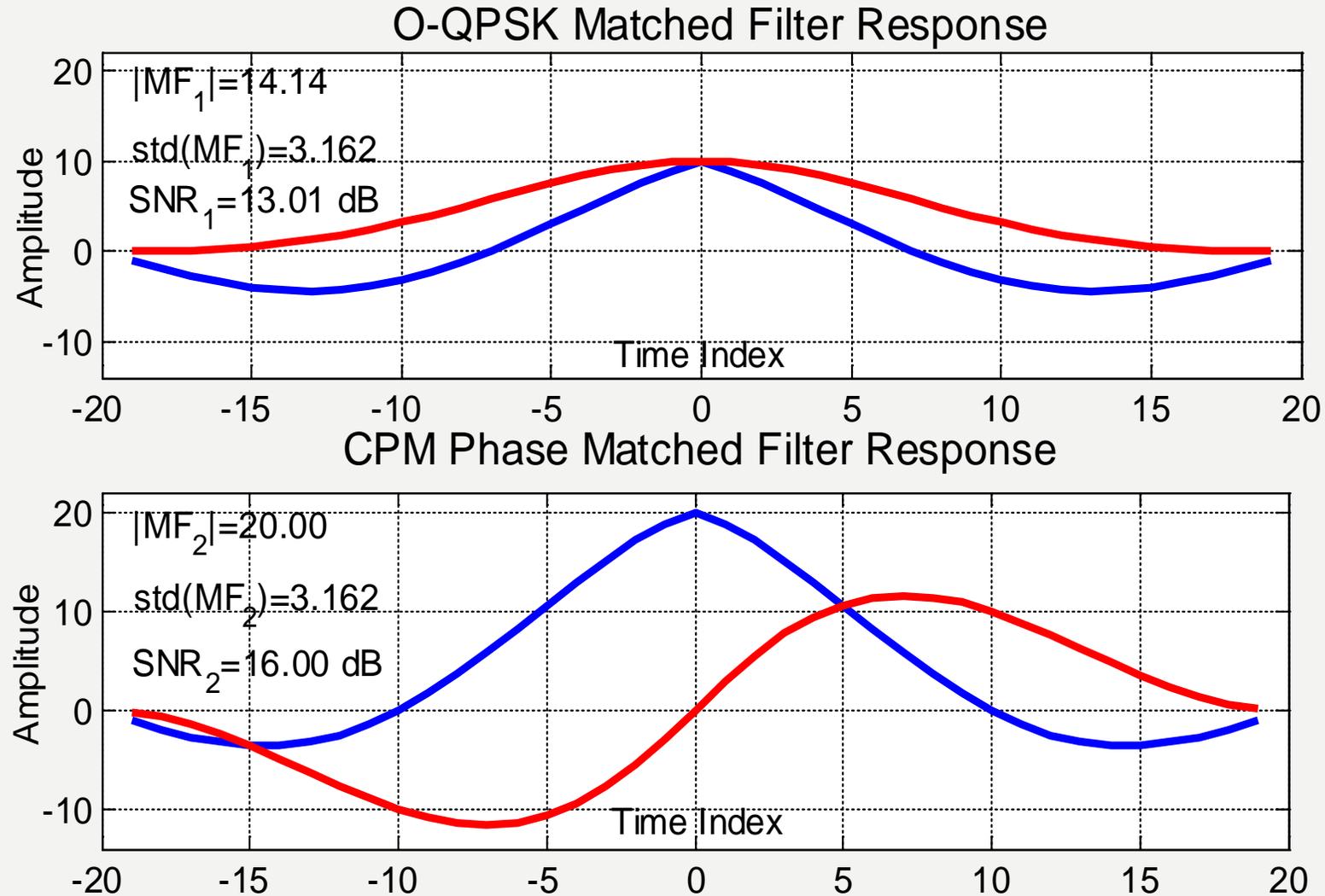
## Normalized Correlation and Threshold Crossing, 2-Symbol Delayed Product, and 0.3 kHz Doppler Frequency Estimate.



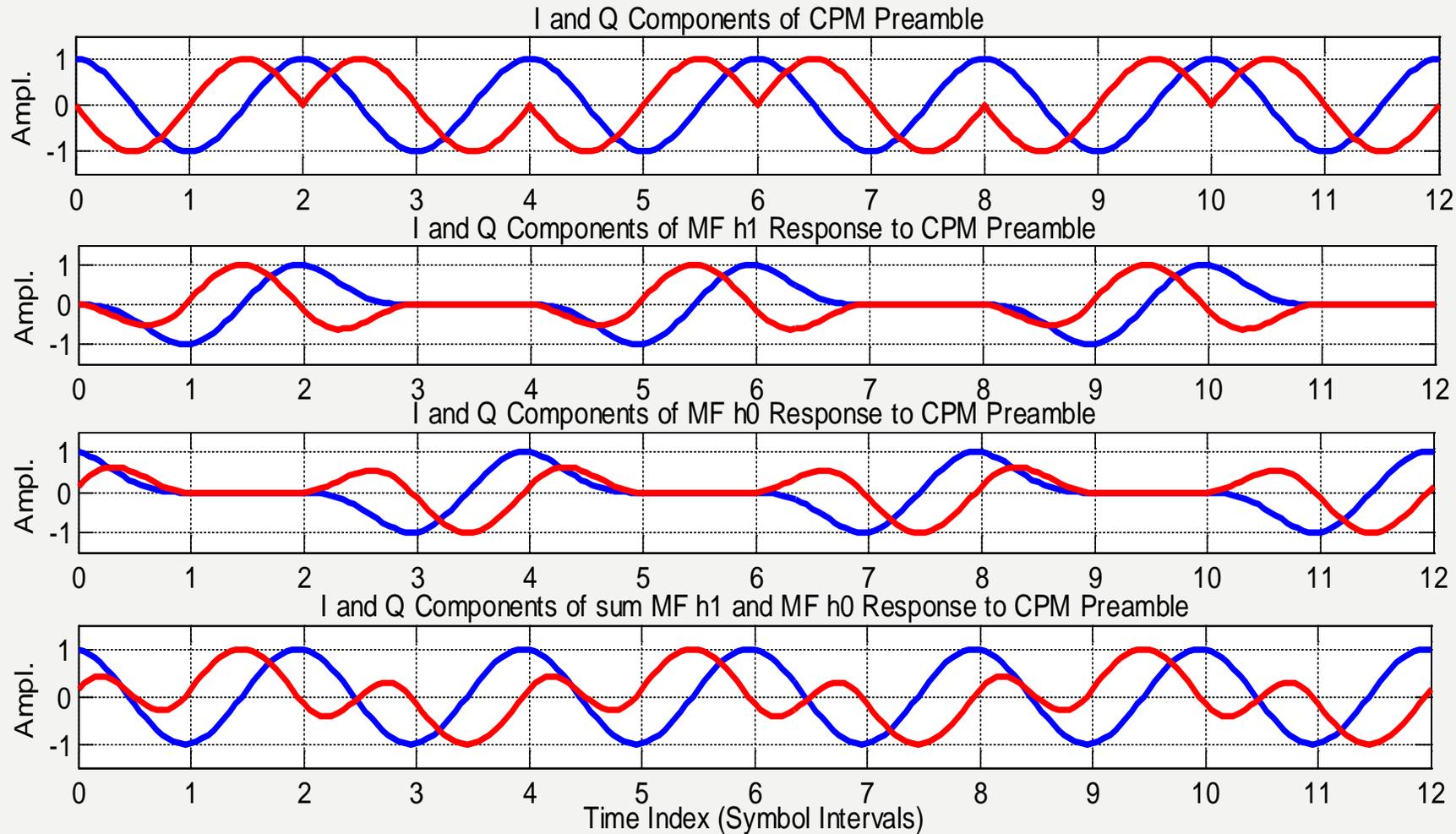
Upper Subplot: Spectra of Periodic Preamble and of Half Sinewave Shaping Filter  
Lower Subplot: Spectra of Random CPM Data and of Half Sinewave Shaping Filter.



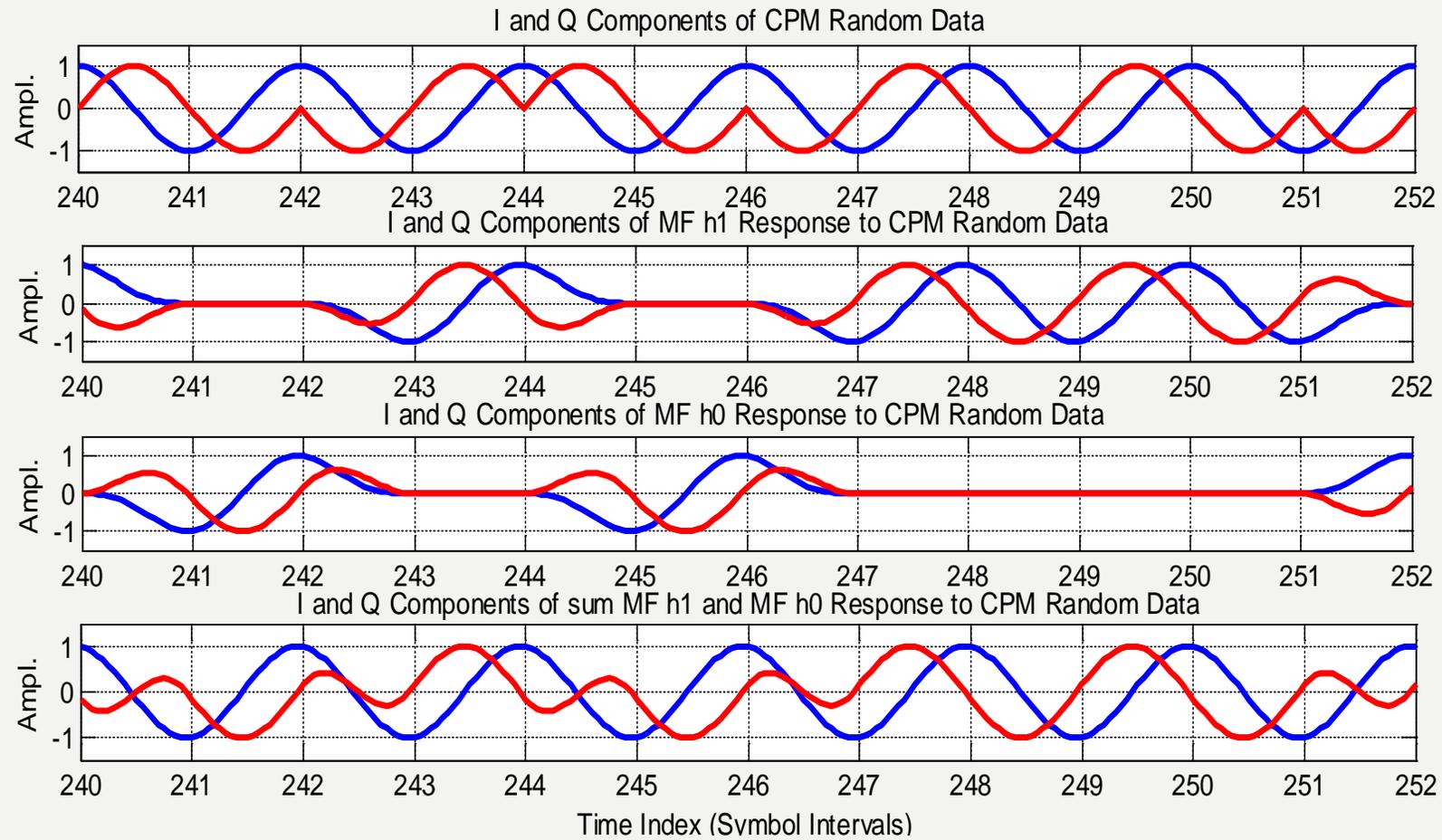
# Matched Filter Response, OQPSK and CPM Process.

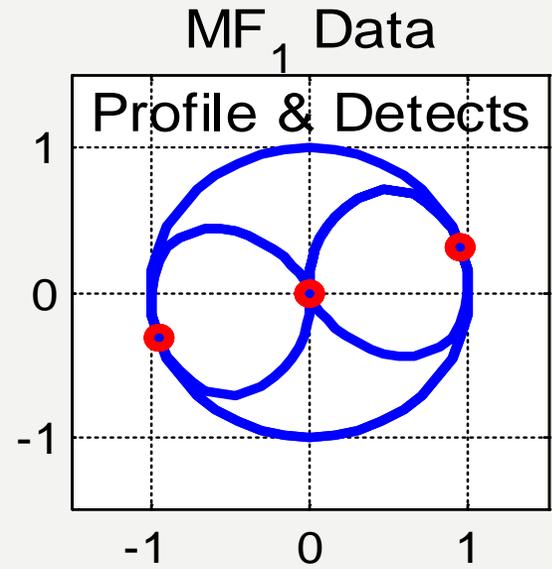
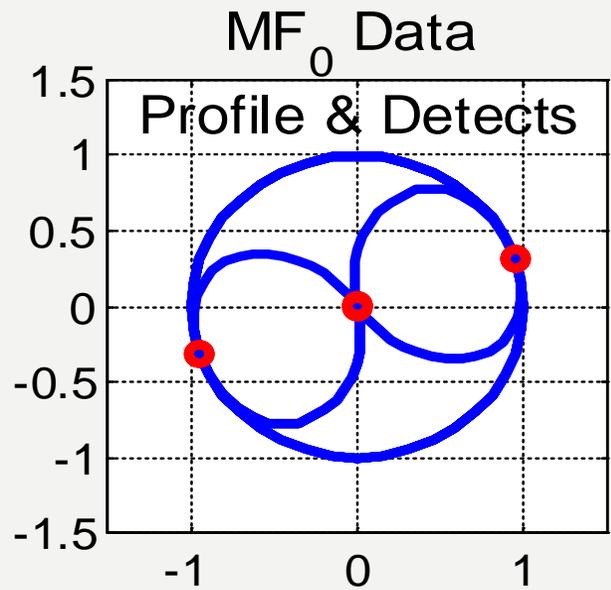
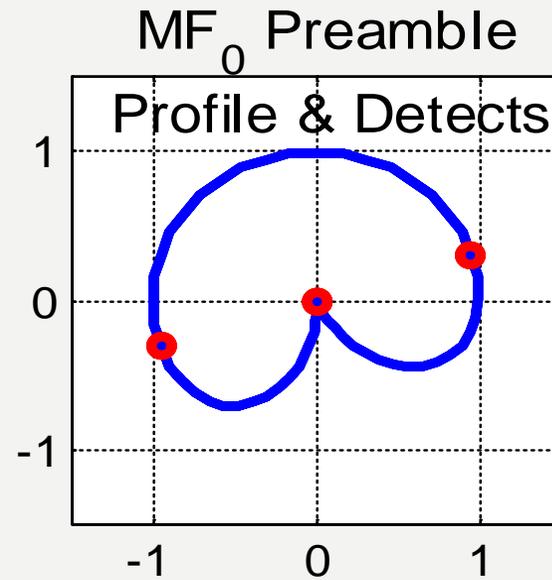
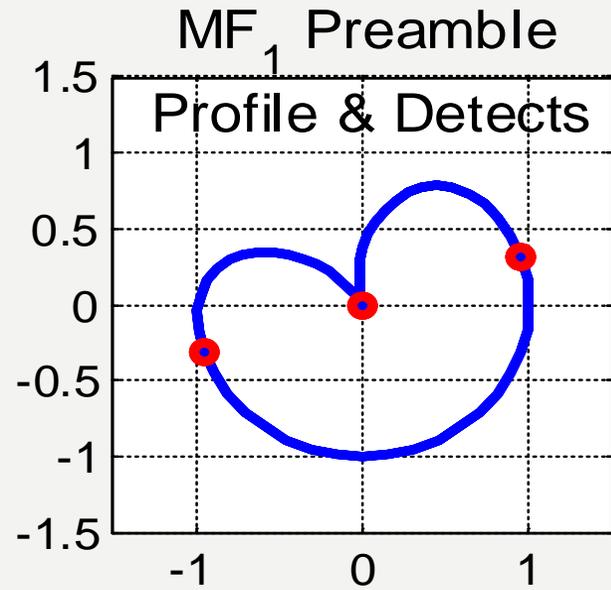


# I & Q Input Signal and Two Matched Filter and the Sum of I & Q Responses of CPM Preamble Process.

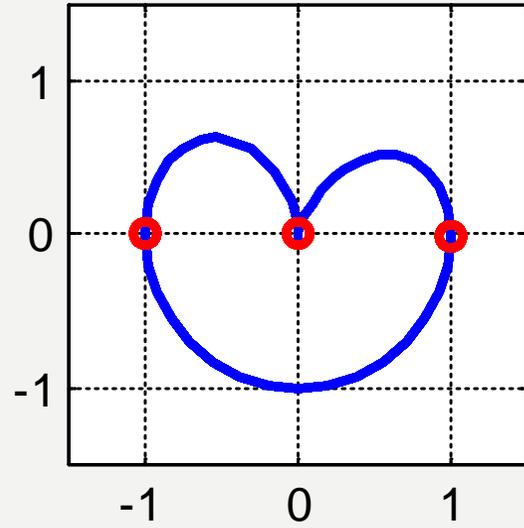


# I & Q Input Signal, Two Matched Filters, and the Sum of I & Q Responses of CPM Random Data Process.

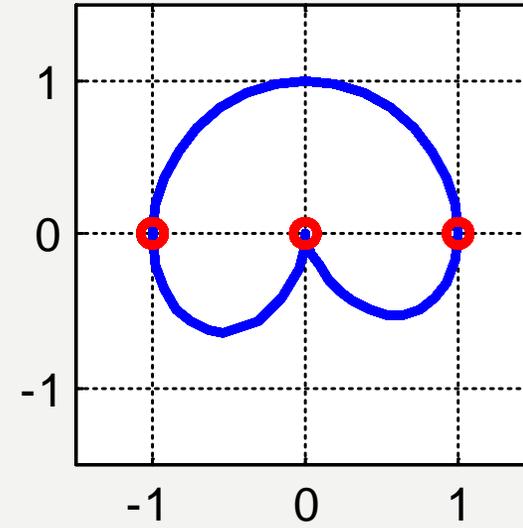




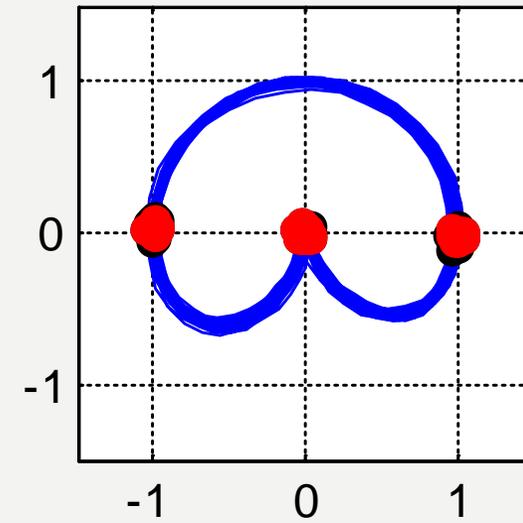
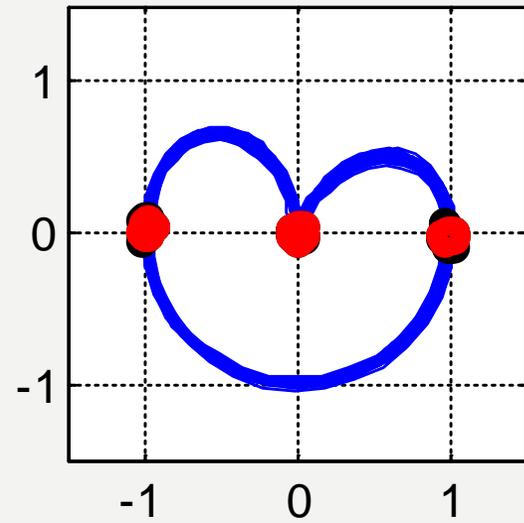
MF<sub>1</sub>a Preamble Response



MF<sub>1</sub>b Preamble Response

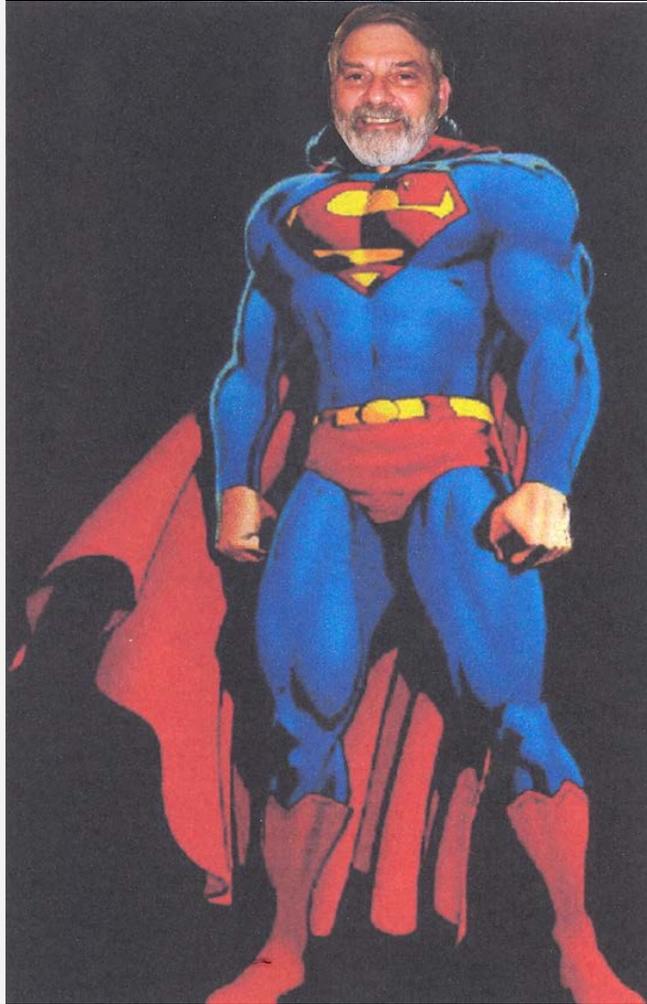


MF<sub>1</sub>a Doppler Preamble Response MF<sub>1</sub>b Doppler Preamble Response



# Interesting Thoughts about CPM

- Matched filter for pairs of symbols
- Preamble  $+1+1-1-1$ , can implement  $+1+1$  filter and  $-1-1$  filter
- Preamble  $+1-1-1+1$ , can also implement  $+1-1$  filter and  $-1+1$  filter  
can use them for random data as well
- Binary Random data,  $+1, -1$ : 2 matched filters
- Can form matched filters for all three-tuples:  $[A +1 B]$ , and  $[A -1 B]$ .  
4 possible 3-tuple filters for each input symbol
- Quaternary Random data,  $+3, +1, -1, +3$ ; 4 matched filters  
16 possible 3-tuple filters for each input symbol
- May be an alternate to trellis decoding of multi-h CPM



# SOFTWARE DEFINED RADIO MAN

Is Open For Questions

