Map-Reduce Based Hybrid Beamforming:Trade-Off between Complexity and Cost

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***Abstract*—High data rates up to 10 Gbps can be achieved for next generation wireless communication by using the millimeter wave (mmWs) bands. Hybrid beamforming, which combines analog beamformers in the RF domain and digital beamformers in the baseband domain, allows the reduction of RF chains while achieving high performance gains in mmWs. Therefore, wireless systems operating at mmWs are expected to use hybrid beamforming. Analog and digital beamformers, which achieve the maximum mutual information over the channel, should be designed in hybrid beamforming. The computational complexity of finding optimal beamformers is significantly high since it grows exponentially with the number of subarrays at the transmitter and the receiver. MapReduce is a framework which can be used to process large data sets in a parallel fashion. Optimization of precoders in hybrid beamforming is actually a distributed sorting problem. The MapReduce framework can be used to increase the speed of the optimum precoder design in hybrid beamforming by executing the algorithm distributively among the multiple cores. We want to show the optimum number of cores to run the MapReduce based hybrid beamforming algorithm based on the trade-off between the cost and complexity. The optimum number of cores to use in the MapReduce based hybrid beamforming has not been studied to the best of our knowledge. In this paper, we analyze the optimum number of cores in terms of the computational complexity and the cost due to the communication load.**

I. Introduction

The millimeter-waves (mmWs) is a promising technology which is expected to play a critical role towards the 5G systems. The available mmW spectrum is 200 times larger than the spectrum below 3 GHz, in which today‘s cellular systems operate [1]. The main drawback of using higher frequencies is the increase in path loss due to Friis' Law [2]. However, the performance degradation due to the path loss can be compensated by using appropriate beamforming in mmWs [3]. Beamforming can boost the signal-to-noise ratio (SNR) at the receiver and decrease the co-channel interference when there are multiple users [4].

Conventionally, beamforming is implemented in analog or digital domain. In analog beamforming, time delaying or phase shifting can be used to apply antenna weights [5]. The data stream is split among array elements and the signal in each substream is processed by a time delay element or a phase shifter, is amplified and fed into the array element. Even though it is the most cost-effective way of building beamforming, one data stream can be handled with a single analog beamformer. In order to form multiple beams, multiple analog beamformers must be used. Digital beamforming can handle multiple data streams. By feeding each array element with a separate transceiver and data converter, multiple beams can be generated simultaneously. Since each array element requires a complete dedicated RF chain, it is less cost-effective than analog beamforming [6]. Hybrid beamforming, introduced in [7], [8], has been proposed to strike a balance between the system performance and cost objectives. Hybrid beamforming is the combination of analog and digital beamformers. The advantage of hybrid beamforming over conventional methods is due to the fact that number of RF chains can be lower-bounded by the number of data streams while the beamforming gain can be still set as high as the number of array elements. Hybrid beamforming can be implemented by combining multiple array elements into subarray modules.

The hybrid beamforming architecture consist of RF and baseband precoders at the transmitter and the receiver [9], [10]. In order to obtain the highest data rates in these systems, optimum precoders need to be designed based on the channel conditions and available beams. In particular, the optimum RF and baseband precoders achieve the maximum mutual information over the channel. Computational complexity of finding the optimum precoders by searching all different combinations of the precoders, which are constructed by using available beams, grows exponentially with the number of subarrays at the transmitter and the receiver [11]. With even a modest number of subarrays, a great number of computations is required to calculate the optimum precoders. For instance, the number of computations for 6 transmitter and receiver subarrays with 3 available beams at the each transmitter and receiver subarray would be $3^6 \times 3^6 = 531441$.

MapReduce is a framework that allows to process the large

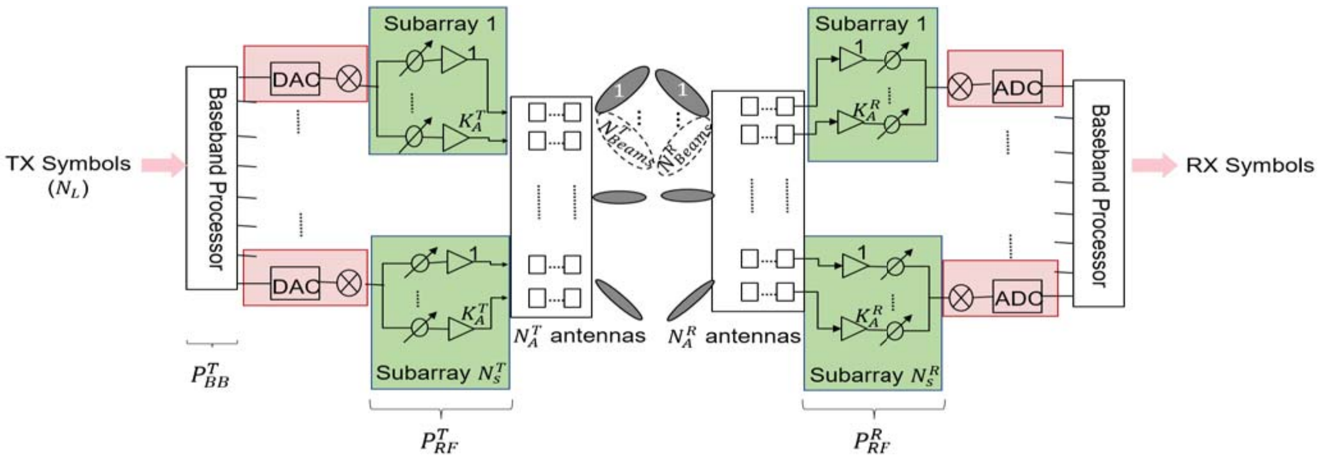

Fig. 1: Above the hybrid beamforming architecture is shown with RF and baseband blocks at the transmitter and the receiver. N_L transmitter (TX) symbols are processed by the baseband precoder P_{BB}^T. Then, each baseband signal is processed by the RF precoder P_{RF}^T and fed to the one of the N_S^T subarrays with K_A^T antennas each. K_A^T antennas in a particular subarray at the transmitter can form a RF signal with a direction towards one of the N_{Beams}^T beams. The reverse of this operation is performed at the receiver.

datasets with a distributed manner on a cluster of servers [12]. It is commonly used to execute data-intensive tasks such as data sorting, which is a key step in most of the machine learning algorithms [13]. Therefore, this framework would be very suitable to find the optimum RF and baseband precoders at the transmitter and the receiver, which is actually a distributed data sorting problem. In our recent paper, we propose a MapReduce based hybrid beamforming to reduce the computational complexity of finding the optimum RF and baseband precoders by solving this problem in a parallel fashion [14]. At the beginning of this algorithm, all possible combinations of RF and baseband precoders are divided into multiple cores. Map or reduce tasks are assigned to each core. The core which executes map and reduce task is called as mapper and reducer, respectively. Each mapper computes mutual information obtained by the assigned precoders and generates intermediate key/value pairs which denote precoder indexes/mutual information. The intermediate key/value pairs are passed to the reducer which sorts the mutual information and finds precoders that achieve the maximum mutual information. The reducer returns the indexes of the optimum precoders as the output. [14] shows that a linear relationship is obtained between speed-up in hybrid beamforming algorithm and the cores. In this paper, they also propose an optimized MapReduce based hybrid beamforming algorithm which partitions the precoder matrices into submatrices and runs the conventional algorithm on each submatrix separately. They show that the nearly same performance is achieved with the optimized algorithm in terms of bit error rate (BER).

In this paper, we focus on deeply analyzing the optimized and the conventional MapReduce based hybrid beamforming algorithms. This paper's main contributions are:

1) We give an analysis for the computational complexity and the communication load of the optimized and the conventional MapReduce based hybrid beamforming algorithms.
2) We show the optimum number of cores to run the optimized and the conventional algorithms in terms of the computational complexity and the cost due to the communication load.

The paper is organized as follows. In Section II, the system and the channel models that we use in this paper are summarized. In Section III, we explain MapReduce based hybrid beamforming algorithm. In Section IV, we analyze the computational complexity and the communication load of the optimized MapReduce based hybrid beamforming algorithm. We also give the optimum number of cores in terms of computational complexity and the communication load to run the conventional and the optimized algorithms. In Section V, we present our simulation results. We conclude our work in Section VI.

II. System and Channel Models

In this paper, we consider a hybrid beamforming architecture in which an array of antennas is divided into multiple subarrays at the transmitter and the receiver. The hybrid beamforming architecture is shown in Figure 1. Each RF chain at the transmitter and the receiver feeds one of the subarrays in this architecture. The transmitter and the receiver antenna arrays consist of N_A^T and N_A^R antennas, respectively. Antenna array at the transmitter (receiver) is divided into N_S^T (N_S^R) subarrays with K_A^T (K_A^R) antennas, respectively. We consider N_{Beams}^T and N_{Beams}^R number of beams at the transmitter and the receiver subarray, respectively. We denote MIMO channel in this architecture with a complex matrix $H \in \mathbb{C}^{N_A^T \times N_A^R}$. Transmitter baseband and RF precoders are denoted by $P_{BB}^T \in \mathbb{C}^{N_S^T \times N_L}$ and $P_{RF}^T \in \mathbb{C}^{N_A^T \times N_S^T}$, respectively. The receiver RF precoder is shown with $P_{RF}^R \in \mathbb{C}^{N_A^R \times N_S^R}$. N_L denotes the number of layers.

The received symbols vector of dimension $N_S^R \times 1$ is given as:

$$y = {P_{RF}^R}^* H P_{RF}^T P_{BB}^T x + n, \quad (1)$$

where $(.)^*$ denotes the conjugate transpose operation, x is the transmitted symbols vector $N_L \times 1$ and n is the noise vector of dimension $N_S^R \times 1$ with i.i.d. $CN(0, \sigma^2)$ entries.

In order to model mmW channel, various measurements have been carried on [3], [15], [16]. Ray-cluster based channel models are well-suited for mmW channel which is characterized by a finite number of scatterers. In ray-cluster based channel model, each scatterer produces a cluster of channel rays. The authors of [11] give a ray-cluster based spatial channel model for mmWs. We use the model which is given in [11] in this paper. The channel representation based on this model is shown as:

$$H = \sqrt{N_A^T N_A^R} \sum_{i=0}^{C-1} \sum_{j=0}^{r-1} G_{i,j} \mathbf{a}^R(\phi_{i,j}^{AoA}, \theta_{i,j}^{AoA}) \mathbf{a}^T(\phi_{i,j}^{AoD}, \theta_{i,j}^{AoD}), \quad (2)$$

where $G_{i,j}$, $\phi_{i,j}^{AoA}$, $\phi_{i,j}^{AoD}$, $\theta_{i,j}^{AoA}$, and $\theta_{i,j}^{AoD}$ denote the complex gain, azimuthal AoA, azimuthal AoD, elevation AoA, and elevation AoD of ray j in cluster i, respectively. We consider there are C clusters and each of the cluster consists of r rays. The array response vectors of the receiver and the transmitter arrays are defined as $\mathbf{a}^R(.)$ and $\mathbf{a}^T(.)$.

III. MapReduce based Hybrid Beamforming

The general problem of hybrid beamformer design is to jointly optimize the transmitter/receiver RF precoders and the transmitter baseband precoder based on channel measurements. Channel measurements can be done by using training sequences, which are transmitted from a particular beam of each transmitter subarray to a particular beam of each receiver subarray. By using the measured channel, the achieved mutual information with all possible RF precoders at the transmitter and the receiver and the baseband preocder at the transmitter is calculated. The solution to the optimization problem given in (3) are the optimum precoders which maximize the mutual information.

$$\underset{P_{BB}^T, P_{RF}^T, P_{RF}^R}{\text{argmax}} \log_2 det\left(I + \frac{1}{\sigma^2} \widetilde{H}^* \widetilde{H}\right), \quad (3)$$

where $\widetilde{H} = {P_{RF}^R}^* \hat{H} P_{RF}^T P_{BB}^T$ and $\hat{H}$ is the estimated channel based on the measurements obtained with training sequences. Least-squares (LS) channel estimation method is used in this paper. P_{BB}^T, P_{RF}^T, and P_{RF}^R are selected from codebook C_{BB}^T, C_{RF}^T, and C_{RF}^R, respectively.

The number of all different precoder combinations is given below:

$$N = (N_{Beams}^R)^{N_S^R} \times (N_{Beams}^T)^{N_S^T} \times N_B^T, \quad (4)$$

where $(N_{Beams}^R)^{N_S^R}$, $(N_{Beams}^T)^{N_S^T}$, and N_B^T are the number of different precoder matrices to possibly build for the RF receiver, the RF transmitter, and the baseband transmitter, respectively. The mutual information, which involves four matrix multiplications and one determinant operation, needs to be calculated for each precoder combinations. Therefore, we should execute $4 \times N$ number of matrix multiplications and N number of determinant operations to solve the hybrid optimization problem, given in (3). As it is explained in Section I, the number of computations for finding the solution to (3) grows rapidly.

One can use MapReduce framework to easily implement the parallel version of an algorithm which involves data-intensive tasks. In order to design the optimum precoders in hybrid beamforming, the mutual information obtained with all different combination of precoder matrices need to be sorted. The precoder matrices, which achieve the maximum mutual information, are the optimum among all. Since this is simply a data sorting problem, hybrid beamforming algorithm can be implemented by using MapReduce framework. We propose a MapReduce based hybrid beamforming algorithm, which parallelizes the computations required to find the optimum precoders, in our recent paper [14]. The MapReduce based hybrid beamforming algorithm is summarized in Figure 2.

There are N possible combinations for the precoder matrices P_{RF}^T, P_{RF}^R, and P_{BB}^T to design the hybrid beamformer. Let $p_i = 1, ..., (N_{Beams}^R)^{N_S^R}$, $p_j = 1, ..., (N_{Beams}^T)^{N_S^T}$, and $p_k = 1, ..., N_B^T$ denote the indexes of P_{RF}^R, P_{RF}^T, and P_{BB}^T, respectively. First, a master controller divides the input data into multiple subsets, then it assigns map and reduce tasks to the idle cores. Each subset consists of N/L different combinations of matrices. Each mapper calculates the mutual information for N/L different matrix combinations as given below:

$$I = \log_2 det(I + \frac{1}{\sigma^2} \widetilde{H}^* \widetilde{H}). \quad (5)$$

The channel frequency response $\hat{H} \in \mathbb{C}^{N_A^T \times N_A^R}$ can be estimated with a pilot-based channel estimation method such as LS. We assume that each mapper knows the channel frequency response. Mutual information of the assigned precoder matrices, and the corresponding indexes of the precoders p_i, p_j, and p_k are generated as intermediate key/value pairs by each mapper. These intermediate key/value pairs are stored in local disks whose locations are known by the master. Then, the master sends these locations to the reducer. Reducer sorts the mutual information obtained by different precoder matrix selections and finds the maximum mutual information. The indexes of the precoder matrices that achieve the highest mutual information are returned as output values by the reducer.

IV. Analysis for Computational Complexity and Communication Load in MapReduce based Hybrid Beamforming

The computational complexity of MapReduce based hybrid beamforming has been studied in [14]. For N number of precoder combinations, the equation given in (5) needs to be computed. $M_1 = {P_{RF}^R}^* \hat{H}$ has complexity $O(N_S^R \times N_A^R \times$

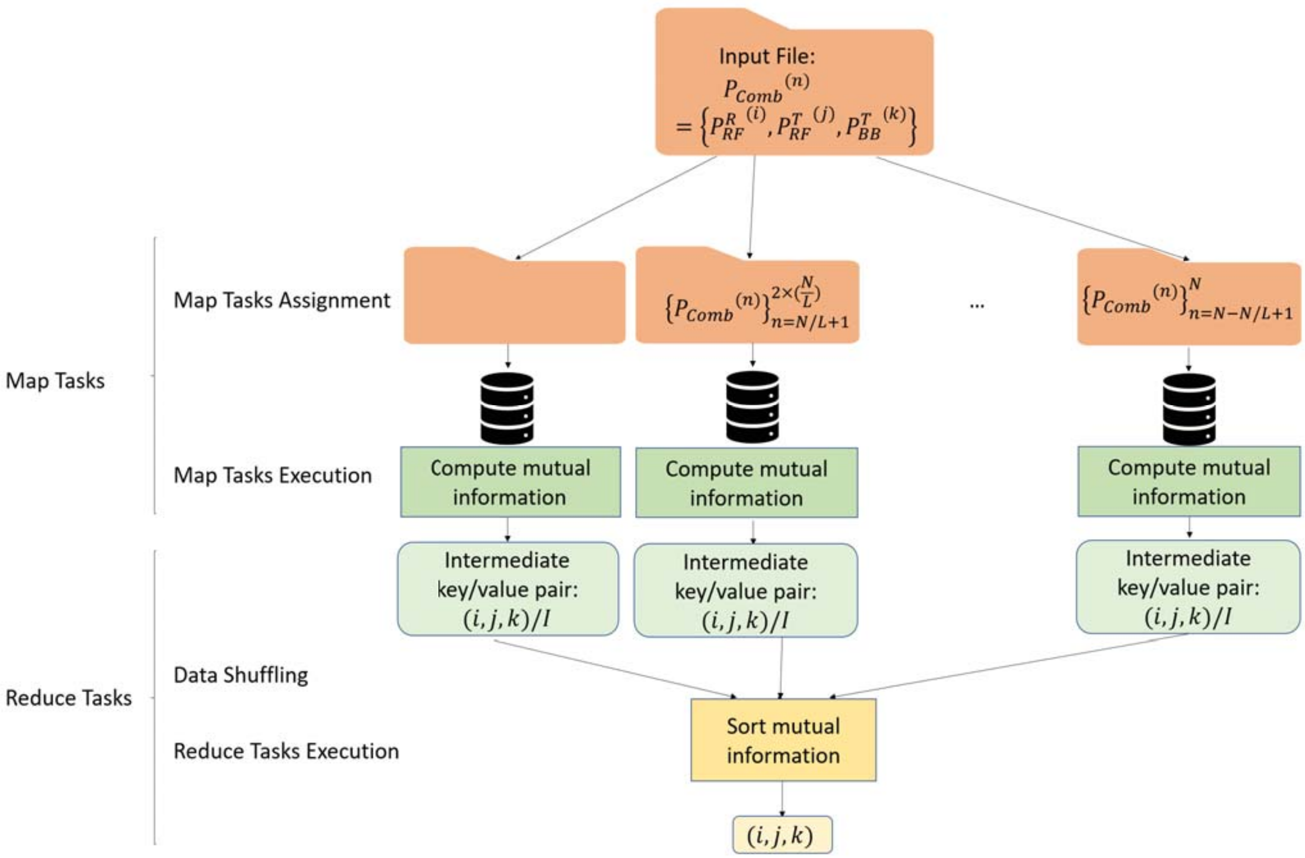

Fig. 2: MapReduce based hybrid beamforming algorithm structure is shown. $i = 1, ..., (N^R_{Beams})^{N^R_S}$, $j = 1, ..., (N^T_{Beams})^{N^T_S}$, and $k = 1, ..., N^T_B$ denote indexes of P^R_{RF}, P^T_{RF}, and P^T_{BB}, respectively. There are N possible combinations of the precoder matrices and n^{th} combination is shown by $P^{(n)}_{Comb}$. N/L number of precoder combinations are assigned to one of L cores. I denotes the mutual information obtained by the selected precoder matrices.

N^T_A). The complexity of $M_2 = M_1 P^T_{RF}$ is $O(N^R_S \times N^T_A \times N^T_S)$. $\widetilde{H} = M_2 P^T_{BB}$ has complexity $O(N^R_S \times N^T_S \times N_L)$. The complexity of $\widetilde{H}^* \widetilde{H}$ is $O(N_L \times N^R_S \times N_L)$. The determinant operation has complexity $O(N^3_L)$. The total computational complexity for calculating (5) is $O(N^R_S \times N^R_A \times N^T_A + N^R_S \times N^T_A \times N^T_S + N^R_S \times N^T_S \times N_L + N^2_L \times N^R_S + N^3_L)$. The computational complexity of hybrid beamforming, which calculates (5) for N different combinations of precoders, is $O(N \times (N^R_S \times N^R_A \times N^T_A + N^R_S \times N^T_A \times N^T_S + N^R_S \times N^T_S \times N_L + N^2_L \times N^R_S + N^3_L))$. The computational complexity of MapReduce based hybrid beamforming is $O(N \times (\frac{N^R_S \times N^R_A \times N^T_A}{L'} + \frac{N^R_S \times N^T_A \times N^T_S}{L'} + \frac{N^R_S \times N^T_S \times N_L}{L'} + \frac{N^2_L \times N^R_S}{L'} + \frac{N^3_L}{L'}))$. Here $L' \approx L$ and L is the number of cores. We achieve L', which approximately equals to the number of cores L, speed-up gain in hybrid beamforming algorithm.

We also show in [14] that the communication load of MapReduce based hybrid beamforming due to the data shuffling between different cores is given as:

$$CommLoad = QN\left(1 - \frac{1}{L}\right). \tag{6}$$

We assume that each core maps N/L subfiles and reduces Q/L keys. Q is the number of total intermediate values. Since the communication load increases linearly with N, the speed-up gain decreases with the large values of N. In this case, there is a trade-off between the computational complexity and the communication load. The communication load is the bottleneck of MapReduce based hybrid beamforming algorithm.

In [14], we also show that we can further reduce the computational complexity of MapReduce based hybrid beamforming algorithm by dividing the precoder matrices into submatrices. Let us analyze the computational complexity of the optimized MapReduce based hybrid beamforming algorithm which partitions precoder matrices into multiple submatrices. We assume that RF precoder matrix P^T_{RF} at the transmitter is divided into dimension of $K^T_A \times (N^T_S)'$ submatrices and RF precoder matrix P^R_{RF} at the receiver is divided into dimension of $K^R_A \times (N^R_S)'$ submatrices. $(N^T_S)'((N^R_S)')$ can be 1 and $N^T_A(N^R_A)$ as minimum and maximum, respectively. We set $(N^T_S)' = (N^R_S)' = 1$ and solve the optimization problem given in (3) separately for each submatrix. Optimization problem given in (3) is solved for each submatrix in the optimized MapReduce based hybrid beamforming algorithm. In this case, the total number of precoder combinations is given as:

$$N_{Opt} = N_S^R \times N_S^T \times N_{Beams}^R \times N_{Beams}^T \times N_B^T. \quad (7)$$

In each computation, the mutual information is calculated for the selected precoder matrices. This calculation consists of four matrix multiplications and one determinant operation. Let us calculate the computational complexity of calculating mutual information for the selected precoder matrices. ${P_{RF}^T}^{'} \in \mathbb{C}^{K_A^T \times 1}$ and ${P_{RF}^R}^{'} \in \mathbb{C}^{K_A^R \times 1}$ denote one submatrix of RF precoders at the transmitter and the receiver, respectively. ${P_{BB}^T}^{'} \in \mathbb{C}^{1 \times N_L}$ and $\hat{H}^{'} \in \mathbb{C}^{K_A^T \times K_A^R}$ denote one submatrix of the baseband precoder at the transmitter and the estimated channel coefficients matrix, respectively. $M_1 = {P_{RF}^R}^{'*} \hat{H}^{'}$ has complexity $O(K_A^R \times K_A^T)$. The complexity of $M_2 = M_1 {P_{RF}^T}^{'}$ is $O(K_A^T)$. $\widetilde{H}^{'} = M_2 {P_{BB}^T}^{'}$ has complexity $O(N_L)$. The complexity of $(\widetilde{H}^{'})^* \widetilde{H}^{'}$ is $O(N_L^2)$. The determinant operation in (3) has complexity $O(N_L^3)$. In this case, the total number of computations that occurs in calculation of mutual information is $O(K_A^R \times K_A^T + K_A^T + N_L + N_L^2 + N_L^3)$. These computations occur for N_{Opt} number of precoder combinations, so the computational complexity of the optimized hybrid beamforming is $O(N_{Opt} \times (K_A^R \times K_A^T + K_A^T + N_L + N_L^2 + N_L^3))$. These computations can be sped up by a factor of $L^{'} \approx L$ on L cores by using MapReduce framework. Therefore, the computational complexity of optimized MapReduce based hybrid beamforming algorithm becomes $O(N_{Opt} \times (\frac{K_A^R \times K_A^T}{L^{'}} + \frac{K_A^T}{L^{'}} + \frac{N_L}{L^{'}} + \frac{N_L^2}{L^{'}} + \frac{N_L^3}{L^{'}}))$.

The communication load can be decreased with the optimized MapReduce based hybrid beamforming algorithm since N_{Opt} is less than N. The communication load of the optimized MapReduce based hybrid beamforming can be calculated as:

$$OptCommLoad = QN_{Opt}\left(1 - \frac{1}{L}\right). \quad (8)$$

V. Simulation Results

We obtained the results of our analysis in Section IV by using MATLAB. We assume a MIMO system with 16 and 8 antennas at the transmitter and the receiver, respectively. There are 3 available beams in the codebooks of the RF precoders at the transmitter and the receiver. The baseband precoder is chosen from 2×2 codebook which is defined in [17].

In Figure 3, we show number of operations required to run the MapReduce based hybrid beamforming and the optimized MapReduce based hybrid beamforming algorithms while number of cores (L) increases from 1 to 16. We obtain these results when N_S^T and N_S^R are chosen as 2 and 4. We observe that the number of operations decreases for both of the algorithms while L increases. The computational complexity of the optimized algorithm is less than the computational complexity of the conventional algorithm. For example, the number of operations is decreased by a factor of 14 when $L = 16$ and $N_S^T = N_S^R = 2$. According to the results in Figure 3, the computational complexity of the conventional algorithm increases more than the computational complexity of the optimized algorithm when the number of subarrays is doubled at the transmitter and the receiver. Therefore, the improvement in the reduction of the computational complexity increases as the number of subarrays increases.

In Figure 4, we observe the communication load, which denotes the number of data shuffling occurs between any two cores, while L increases from 1 to 16. N_S^T and N_S^R are set as 2 and 4. It can be seen in Figure 4 that the communication load of both of the algorithms increases with the number of cores. The communication load is decreased with the optimized MapReduce based hybrid beamforming algorithm. For instance, the communication load of the optimized algorithm is 2 times of the communication load of the conventional algorithm when $L = 16$ and $N_S^T = N_S^R = 2$. The communication load of both algorithms also increases significantly for the larger number of subarrays. The gain in decrease in the communication load also further improves with the increasing number of subarrays at the transmitter and the receiver. For example, the communication load is decreased by almost a factor of 46 with the optimized algorithm when number of cores is 16 and $N_S^T = N_S^R = 4$.

There is a trade-off between the computational complexity and the communication load for both of the algorithms when the number of cores increases. According to the results in Figure 3 and Figure 4, the computational complexity significantly decreases while the communication load increases with the number of cores for both algorithms. Our aim is to show the optimum number of cores based on this trade-off. The optimum number of cores depends on the upper bound on the number of operations and the communication load that is allowed in the system. For example, when the maximum number of operations and the communication load are restricted to 2000 and the number of subarrays is 2 at the transmitter and the receiver, the optimum number of cores is 4 and 8 for the optimized and the conventional algorithms, respectively. In general, the optimum number of cores required for the optimized algorithm is less than the conventional algorithm.

VI. Conclusion

In this paper, we studied MapReduce based hybrid beamforming algorithms, and investigated thetrade-off between computational complexity and communication load. We analyze these tradeoffs for two algorithmic variations, namely, conventional and optimized versions of hybrid beamforming. Our results show that both computational complexity and communication load can be reduced at the expense of increasing the number of cores. Furthermore, we also analyze the optimum number of cores for both algorithmic variations as a function of the operating point on the trade-off between the computational complexity and the communication load.

VII. Acknowledgements

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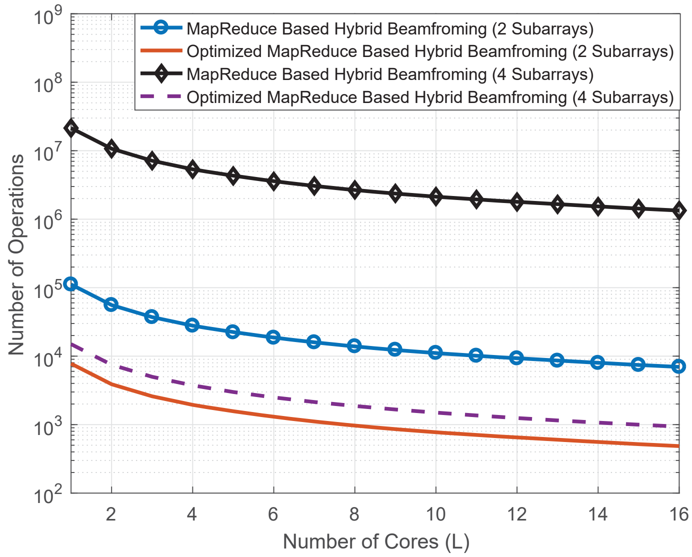

Fig. 3: Computational complexity comparison of the optimized and the conventional MapReduce based hybrid beamforming algorithms when the number of subarrays at the transmitter and the receiver is 2 and 4

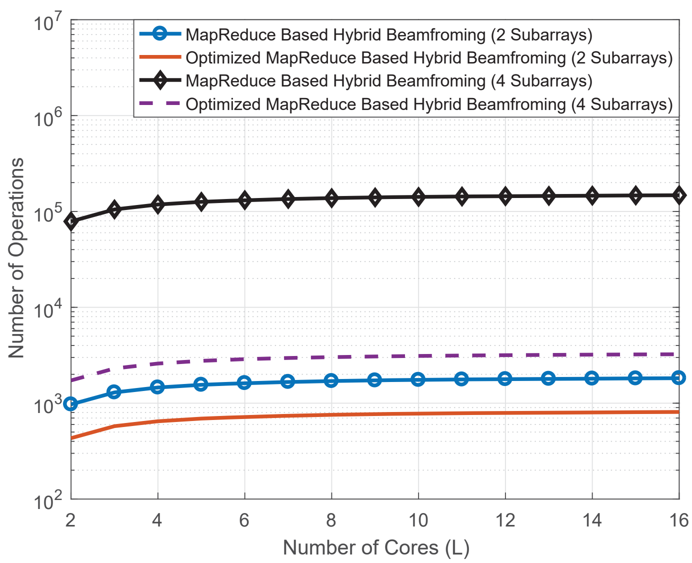

Fig. 4: Communication load comparison of the optimized and the conventional MapReduce based hybrid beamforming algorithms when the number of subarrays at the transmitter and the receiver is 2 and 4

Award No. 1265960.

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