# POWER CONSUMPTION MINIMIZATION FOR MIMO SYSTEMS USING COGNITIVE RADIO

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## ABSTRACT

This paper shows how cognitive radio (CR) can help to optimize system power consumption of multiple input multiple output (MIMO) communication systems. This paper mathematically formulates the system power consumption minimization problem under a rate constraint for MIMO systems. Optimal and suboptimal algorithms are developed to solve the optimization problem numerically. The simulation results show that significant power savings (up to 75% for a $4\times4$ MIMO system with Class A power amplifiers) can be achieved compared to conventional allocation schemes. The results also show that the presented suboptimal algorithms can achieve power savings comparable to the optimal algorithm with lower complexity.

## 1. INTRODUCTION

This paper shows how cognitive radio (CR) can be used to optimize system power consumption of multiple input multiple output (MIMO) wireless communication systems by dynamically reconfiguring the radio for the required Quality of Service (QoS) based on channel conditions and radio (component) capabilities and characteristics.

Recently, significant power reduction through radio reconfiguration based on channel conditions and QoS requirements has been reported for wireless communications, mostly for short range and sensor network applications [1]-[6]. For MIMO systems, various power and bit loading algorithms have been proposed to solve the two classic problems [7], [8], rate maximization (maximizing rate under a power constraint), and power minimization (minimizing power under a rate constraint). However, most investigations focused on the received or radiated power [7]-[10]. The system power consumption of MIMO systems, on the other hand, has only received limited attention. In [2], Alamouti based MIMO scheme and modulation are jointly adapted to minimize system energy consumption per bit under throughput and delay constraints in sensor networks. In [5], the number of transmit antennas, receive antennas, spatial streams, and MIMO detection scheme are adapted with other radio parameters, to minimize energy per successfully received bit under packet error rate (PER) and transmission bit rate constraints. However, the fundamental relationship between rate and system power consumption for MIMO systems has not been fully investigated.

With the advance of cognitive radio (CR) technologies, some capabilities of a CR have been adopted to optimize system power consumption further. A CR [11], [12] is an intelligent wireless communication system which is able to determine the most favorable operating parameters for application QoS requirements (cognition) based on the knowledge of the radio environment and its capability (awareness) and reconfigure the radio accordingly (reconfigurability). By doing this, radio resources can be used more efficiently. A CR can not only learn the channel conditions as in conventional radios, but is also aware of the radio (component) capabilities and characteristics. The knowledge of radio capabilities and characteristics as well as channel conditions helps system power consumption optimization. In [4], in addition to modulation adaptation, the best available channel is dynamically detected and chosen for transmission to minimize power consumption under a bit error rate (BER) constraint in a CR sensor network setting. A power optimization framework using CR for given QoS requirements based on channel conditions and radio capabilities and characteristics for single channel communication systems is demonstrated in [6]. In addition, this framework has been applied to multichannel communication systems [13]. A system power consumption minimization problem under a rate constraint is formulated and numerical algorithms are proposed in [13] for multichannel systems.

This paper extends our previous work on multichannel systems in [13] to a specific multichannel system, the MIMO system. This work leverages information theory and cognitive radio technologies (e.g., learning capability in obtaining radio capabilities and characteristics) for the development of a theoretical framework and algorithms for system power consumption minimization of MIMO systems. To be specific, instead of focusing on specific MIMO techniques, such as those investigated in [2], [5], the system power consumption minimization problem with rate constraint for general MIMO systems is mathematically formulated and numerical solutions are developed. This paper focuses on medium and long range applications where the power amplifier (PA) usually dominates the system power consumption. Other power consuming components can be integrated into the analysis as future work.

This paper is organized as follows. Section 2 discusses the MIMO system model. Section 3 formulates the system power consumption minimization problem with a rate constraint for MIMO systems. Section 4 discusses the optimal and suboptimal algorithms to solve this problem. Section 5 evaluates the power reduction of the proposed schemes by simulation. Section 6 concludes the paper.

## 2. MIMO SYSTEM MODEL

We consider a MIMO system with additive white Gaussian noise (AWGN) and flat Rayleigh fading. A linear model for a MIMO system with $t$ transmit antennas and $r$ receive antennas can be expressed as [14]

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}, \tag{1}$$

where $\mathbf{y} \in C^{r\times 1}$ is the received signal vector, $\mathbf{x} \in C^{t\times 1}$ the transmitted signal vector, $\mathbf{H} \in C^{r\times t}$ the channel gain matrix, and $\mathbf{n} \in C^{r\times 1}$ the zero mean complex Gaussian noise vector with independent and equal variance real and imaginary parts, $E\{\mathbf{nn}^+\} = \sigma_0^2 \mathbf{I}_r$ . $\mathbf{H}$ is a complex Gaussian random matrix with independent and identically distributed (i.i.d.) entries, each entry having independent real and imaginary parts with zero mean and variance 1/2. In other words, this models an uncorrelated Rayleigh fading channel. $\mathbf{H}$ is assumed to be independent of $\mathbf{x}$ and $\mathbf{n}$.

If the channel state information (CSI) is known at the transmitter and the receiver, the MIMO channel can be decomposed into several parallel single input single output (SISO) channels using singular value decomposition (SVD) [14]: $\mathbf{H} = \mathbf{UDV}^+$ , where $\mathbf{U} \in C^{r\times r}$ and $\mathbf{V} \in C^{t\times t}$ are unitary, and $\mathbf{D} \in R^{r\times t}$ contains the non-negative singular values on its diagonal with $\sqrt{\lambda_i}$ the i-th singular value of $\mathbf{H}$ . At the transmitter, let $\mathbf{s} \in C^{t\times 1}$ be the modulation symbol vector, then the transmitted signal vector is $\mathbf{x} = \mathbf{Vs}$. At the receiver, the received signal vector $\mathbf{y}$ is pre-multiplied by $\mathbf{U}^+$ . We then have

$$\tilde{\mathbf{y}} = \mathbf{Ds} + \tilde{\mathbf{n}}, \tag{2}$$

where $\tilde{\mathbf{y}} = \mathbf{U}^+\mathbf{y}$ and $\tilde{\mathbf{n}} = \mathbf{U}^+\mathbf{n}$ . Since $\mathbf{U}$ and $\mathbf{V}$ are unitary, $E\{\mathbf{s}^+\mathbf{s}\} = E\{\mathbf{x}^+\mathbf{x}\}$ and $E\{\tilde{\mathbf{n}}\tilde{\mathbf{n}}^+\} = E\{\mathbf{nn}^+\}$.

Let $m = \text{rank}(\mathbf{H}) \le \min(t, r)$ , only $m$ of the singular values of $\mathbf{H}$ are non-zero. Without loss of generality, we assume $\sqrt{\lambda_1} \ge \sqrt{\lambda_2} \ge \ldots \ge \sqrt{\lambda_m} > 0$ and $\mathbf{U}$ and $\mathbf{V}$ are determined accordingly. Then, (2) becomes

$$\tilde{y}_i = \begin{cases} \sqrt{\lambda_i}\, s_i + \tilde{n}_i, & 1 \le i \le m \\ \tilde{n}_i, & \text{otherwise} \end{cases}, \tag{3}$$

where the subscript $i$ refers to the i-th element of the vector. It is clear that the MIMO channel $\mathbf{H}$ has been decomposed into $m$ parallel virtual Gaussian channels with the i-th virtual channel having channel gain $\sqrt{\lambda_i}$ .

Although only $m$ elements of the modulation symbol vector $\mathbf{s}$ can deliver information from the transmitter to the receiver, all the transmit antennas are used in the transmission according to the following relationship:

$$\mathbf{x} = \mathbf{Vs} = \left[ \sum_{i=1}^{m} V_{1,i} s_i, \sum_{i=1}^{m} V_{2,i} s_i, \ldots, \sum_{i=1}^{m} V_{t,i} s_i \right]. \tag{4}$$

The average radiated power from branch $n$ can then be expressed as

$$\tilde{P}_n = E\{x_n x_n^*\} = \sum_{i=1}^{m} \left| V_{n,i} \right|^2 P_i , \tag{5}$$

where $P_i = E\{|s_i|^2\}$ is the allocated power on virtual channel $i$.

This paper focuses on medium and long range applications where the PA usually dominates the system power consumption. In this case, the power consumption of each transmit branch can be approximated as

$$\hat{P}_n = \hat{P}_{PA,n} = \frac{\tilde{P}_n}{\bar{\eta}_n}, \tag{6}$$

where $\hat{P}_{PA,n}$ is the power consumption of PA in branch $n$ and $\bar{\eta}_n$ is the average PA efficiency in branch $n$ and an engineering approximation model is adopted,

$\overline{\eta}_n = \eta\left(\tilde{P}_n\right)$, and $\eta\left(\tilde{P}_n\right)$ is the instantaneous PA efficiency at average output power $\tilde{P}_n$. However, in general, the average efficiency depends on the distribution of the output power of the underlying signal [15].

If we limit our analysis to linear PAs, such as, Class A and Class B PAs, similar to [13], the average PA power consumption for Class A and Class B PAs can be expressed as

$$\hat{P}_{PA} = \frac{P_{\max}^{\alpha}}{\eta_{\max}} \cdot \tilde{P}^{1-\alpha} = \begin{cases} \dfrac{P_{\max}}{\eta_{\max,A}}, & \text{Class A} \\ \dfrac{P_{\max}^{1/2}}{\eta_{\max,A}} \tilde{P}^{1/2}, & \text{Class B} \end{cases}, \quad (7)$$

where

$$\begin{cases} \eta_{\max,A} = 0.5 & \text{Class A} \\ \eta_{\max,B} = 0.785 & \text{Class B} \end{cases}. \quad (8)$$

From (7), the power consumption of a Class A PA is constant over its entire range of output power level.

Hence, for systems using Class A or Class B PAs, the power consumption of branch $n$ becomes

$$\hat{P}_n = \frac{P_{n,\max}^{\alpha_n}}{\eta_{n,\max}} \cdot \left( \sum_{i=1}^{m} \left| V_{n,i} \right|^2 P_i \right)^{1-\alpha_n}. \quad (9)$$

If we further assume the branches are identical, we have

$$\hat{P}_n = \frac{P_{\max}^{\alpha}}{\eta_{\max}} \cdot \left( \sum_{i=1}^{m} \left| V_{n,i} \right|^2 P_i \right)^{1-\alpha}. \quad (10)$$

We use (10) in the following system power consumption analysis.

## 3. SYSTEM POWER CONSUMPTION MINIMIZATION

Similar to [13], the system power consumption minimization problem for MIMO systems can be formulated as

$$\min_{\{P_i\}} \sum_{n=1}^{t} \hat{P}_n . \quad (11)$$

subject to

$$\sum_{i=1}^{m} b_i = b . \quad (12)$$

where $b_i$ is the achieved rate on virtual channel $i$. For an AWGN channel, $b_i$ is given by [7]

$$b_i = \log_2\left(1 + P_i \cdot g_i\right) = \frac{1}{\ln 2} \ln\left(1 + P_i \cdot g_i\right), \quad (13)$$

where $g_i = \lambda_i / \sigma_0^2$ is the channel signal to noise ratio (SNR).

As in [13], the solution to the constrained optimization problem defined in (11) and (12) generally needs to be obtained numerically except for the following special case, MIMO systems with Class A PAs.

### 3.1. Special Case: MIMO Systems with Class A PAs

For MIMO systems with Class A PAs, the constrained optimization problem in (11) and (12) is reduced to

$$\min_{\{P_i\}} \sum_{n=1}^{t} \hat{P}_n = \min_{\{P_i\}} \sum_{n=1}^{t} \frac{P_{\max}}{\eta_{\max,A}} \cdot U\left(\tilde{P}_n\right), \quad (14)$$

subject to

$$\sum_{i=1}^{m} b_i = b , \quad (15)$$

where $U(P) = \begin{cases} 1, & P > 0 \\ 0, & P \le 0 \end{cases}$.

Observe that for MIMO systems with Class A PAs the system power consumption only depends on the number of active antennas and different power allocation only affects the total radiated power and the total achieved rate under the same number of active antennas. An obvious solution to this constrained optimization problem is to minimize the required number of active transmit antennas.

## 4. NUMERICAL ALGORITHMS

This section discusses the optimal and the suboptimal numerical solutions to the constrained system power minimization problem defined in Section 3. It is assumed that any branch of the MIMO system can be deactivated as needed. Deactivating a branch results in a change in $\mathbf{H}$, i.e., the column of $\mathbf{H}$ corresponding to the inactive transmit branch is removed. In addition, for the same number of active transmit branches, different branch combination results in different $\mathbf{H}$. Changes in $\mathbf{H}$ may result in different SVD decomposition.

Conventionally, the water filling algorithm is used to minimize the total radiated power of a MIMO system for a given target rate when the CSI is known at both transmitter and receiver [7, 8]. It allocates power to the subchannels proportional to the inverse of the channel SNR as if water is poured into a container whose bottom is in the shape of the inverse of the channel SNR [7]. However, due to the nonlinear relationship between the radiated power and the power consumption (see (7)), the water-filling algorithm does not guarantee to generate the power allocation result that minimizes system power consumption. Therefore, several algorithms are proposed in this section to solve the constrained minimization problem. The water-filling algorithm is used as the comparison baseline.

Due to the similarity between the problem defined for MIMO systems and that for general multichannel systems [13], the algorithms proposed in [13] are adapted to solve the problem in this paper by integrating (10) in the calculation of power consumption.

### 4.1. Optimal Algorithm – Exhaustive Search

As in [13], the exhaustive search algorithm can find the optimal solution if the search step size, $P_{step}$, is sufficiently small by testing all possible power allocation combinations for all branch combinations.

The total number of power allocation combinations is [13]

$$N_{es} = \left(\left\lceil \left(P_{\max} - P_{\min}\right) / P_{step} \right\rceil + 2\right)^t - 1, \qquad (16)$$

where $\tilde{P} \in \left[P_{\min}, P_{\max}\right]$, and $\lceil \cdot \rceil$ is the ceiling function. The execution time and memory footprint of this algorithm can be prohibitive for large power range, a small step size, or a large number of branches.

### 4.2. Suboptimal algorithm 1 – Branch Adaptation

In order to reduce the computational burden and the memory requirement, as in [13], a suboptimal algorithm, the branch adaptation algorithm, uses the water-filling algorithm in power allocation for each branch combination. This algorithm differs from the conventional water-filling algorithm in a way that the branches in the conventional water-filling algorithm are fixed while the branches in this algorithm are adapted to minimize power consumption.

The total number of power allocation combinations is [13]

$$N_{so1} = \sum_{n=1}^{t} \binom{t}{n} = 2^t - 1, \qquad (17)$$

The branch adaptation algorithm results in much lower computational complexity and smaller memory requirement as compared to the exhaustive search algorithm.

### 4.3. Suboptimal algorithm 2 – Branch Minimization

The branch adaptation algorithm can be further simplified as in [13] based on the observation in the Class A PA case, i.e., the more transmit branches the MIMO system uses, the more power the system consumes. Hence, the suboptimal algorithm 2 finds the branch combination with the minimum number of active branches that satisfies the rate requirement. We call this algorithm branch minimization. The branch minimization algorithm uses the water-filling algorithm to allocate power for each branch combination.

In the worse case, the total number of power allocation combinations for branch minimization algorithm is the same as that for the branch adaptation algorithm [13].

## 5. PERFORMANCE EVALUATION AND SIMULATION RESULTS

In this section, the performance of the optimal and suboptimal algorithms discussed above is evaluated using simulation to understand the tradeoff between the power saving performance and the computational complexity.

The simulation assumes an uncorrelated flat Rayleigh fading channel with AWGN. The instantaneous channel gain is assumed to be known to the transmitter and the receiver. The number of branches at the transmitter and the receiver is 4, $t = r = 4$. The maximal radiated power on each branch is 1 Watt, $P_{\max} = 1$ W. The average channel SNR is set to 10 dB in order to achieve the desired rate.

The system power consumption reduction is defined as

$$P_{saving} = \frac{P_{con} - P_{cog}}{P_{con}} \cdot 100\%, \qquad (18)$$

where $P_{con}$ is the system power consumption with a conventional power allocation scheme (i.e., the water-filling algorithm), and $P_{cog}$ the system power consumption with the proposed power allocation schemes.

### 5.1. MIMO Systems with Class A PAs

The power saving for MIMO systems with Class A PAs and corresponding branch configuration are shown in Figures 1 and 2, respectively.

From Figure 1, up to 75% of power saving can be achieved depending on the rate requirements. The power saving decreases as the target rate increases. The conventional approach always uses all transmit branches no matter what the target rate is. Hence, the system power consumption is the same over all target rates. On the other hand, the proposed optimal and suboptimal algorithms tend to use as few transmit branches as possible. As we mentioned in Section III.A, for Class A PA case, the more active transmit branches the MIMO system uses, the more power the system consumes. At lower target rate, the proposed algorithms use fewer transmit branches. As the target rate increases, more transmit branches are activated (see Figure 2). In other words, the power consumption of the proposed algorithms increases as the target rate. Therefore, the power saving decreases as the target rate increases.

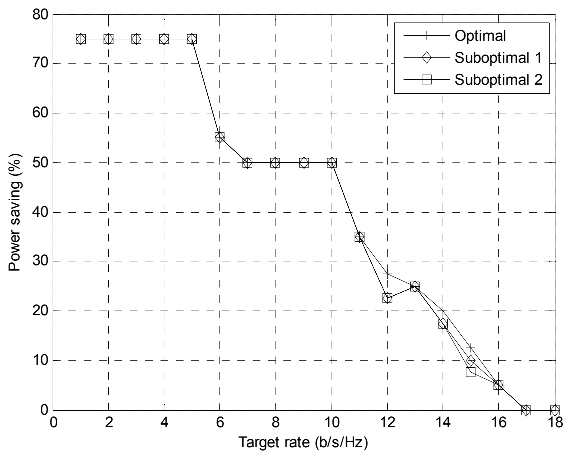


Figure 1. Power saving for systems with Class A PAs.

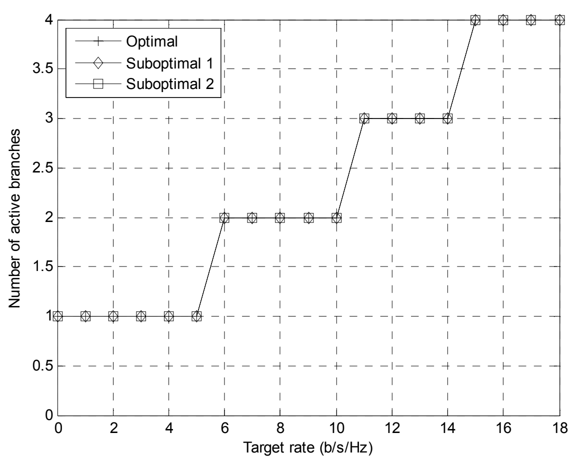


Figure 2. Number of branches for systems with Class A PAs.

From Figure 2 the optimal number of branches increases gradually as the target rate increases in all proposed algorithms. This result is consistent with earlier analysis. Note that the number of active antennas is averaged over many runs of simulation and rounded to the closest integer.

For MIMO systems with Class A PAs, the number of antennas is the same for the optimal and suboptimal algorithms and the power saving results are almost identical for the optimal and suboptimal algorithms. This suggests the branch minimization algorithm provides the best tradeoff between power saving and algorithm complexity.

### 5.2. MIMO Systems with Class B PAs

The power saving for systems with Class B PAs and corresponding branch configuration are shown in Figures 3 and 4, respectively.

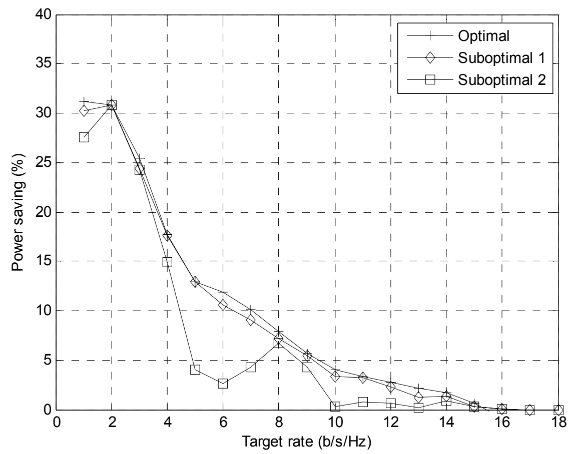


Figure 3. Power saving for systems with Class B PAs.

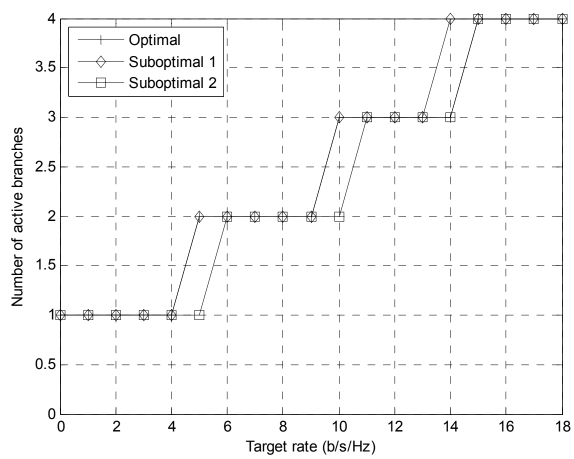


Figure 4. Number of branches for systems with Class B PAs.

As in Figure 3 the power saving can be up to 30% depending on the rate requirements. This power saving is lower than the Class A case. Because Class B PAs achieve higher efficiency than Class A PAs at all output power levels, the power consumption penalty for using more branches in water-filling algorithm is less severe in Class B case than the Class A case. Similar trend in power saving is observed in the Class B PA case as in the Class A PA case. The performance gap between the branch adaptation algorithm and the exhaustive search algorithm is larger than that in the Class A PA case. This is because in the Class B PA case, it is possible that the system power consumption using more transmit branches can be lower than that using fewer branches. For example, at target rate 5 b/s/Hz, in Figure 4, the exhaustive search algorithm and the branch adaptation algorithm use 2 branches while the branch minimization algorithm uses 1 branch. The exhaustive search algorithm achieves higher power saving.

As for branch configuration in Figure 4, similar to the Class A PA case, the optimal number of active branches increases as the target rate increases.

For MIMO systems with Class B PAs, the number of antennas and the power saving results are almost identical for the optimal algorithm and the branch adaptation algorithm. This suggests the branch adaptation algorithms provides the best tradeoff between power saving and algorithm complexity.

## 6. CONCLUSION

This paper formulates analytically the system power consumption minimization problem under a rate constraint for MIMO communication systems and derives optimal and suboptimal algorithms to solve this constrained minimization problem numerically. The power saving achieved by the proposed algorithms in comparison with the conventional power allocation scheme is evaluated by simulation for $4\times4$ MIMO systems with Class A PAs and with Class B PAs. The simulation results show that significant system power saving (up to 75% for systems with Class A PAs and 30% for systems with Class B PAs) can be achieved using the proposed schemes for MIMO systems.

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