

# A Receiver Structure that Performs Simultaneous Spectral Analysis and Time Series Channelization

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## 1 Abstract

Spectral analysis and channelization are two common tasks performed in many software defined and cognitive radios. These tasks are usually performed sequentially by controllers that reconfigure the algorithmic resources of a polyphase filter and a fast Fourier transform. We present here a receiver structure that performs both tasks simultaneously in the common software.

## 2. Introduction

Surveillance receivers perform two primary tasks. They form and scan the spectrum for signals of interest and then perform the appropriate signal conditioning that bandwidth limits, heterodynes, and down samples the input time series in preparation for secondary processing that extracts information from the time signals in the identified spectral regions. These two tasks are traditionally performed by two distinct instruments; a broadband FFT based spectrum analyzer and one or more steerable digital down converter receivers.

In modern collection systems the two tasks are merged by using the FFT as both the broadband spectrum analyzer and as a multiple channel receiver. The channelization is accomplished by overlap and add processing of the time series formed by a sequence of inverse transforms of the complex valued spectral samples from selected spectral spans in the broadband spectrum.

An alternate option combines the same two tasks in an efficient multi-channel receiver based on a modified form of a polyphase filter bank. The polyphase filter down samples the input time series M-to-1 by cyclically delivering successive input samples to the M-paths of the partitioned prototype low pass filter. The down sampling causes M-fold aliasing of the input spectra in each of the M-paths. Due to the offset and time base in each path the aliases from each Nyquist zone exhibit unique

phase profiles reminiscent of an M-blade paddle wheel. An FFT following the M-path filter phase aligns the M-aliases from select Nyquist zones which consequently survive the FFT summation. The non-phase aligned aliases, with phases distributed over the M-roots of unity, destructively cancel during the same summation. In this manner the time series associated with each of the aliased spectral components are extracted from the aliased maelstrom caused by the input down sampling.

By reconfiguring the conventional polyphase partition and processing chain, the same FFT can be used to first form the spectra of each channel of the channelizer as well as supplying the phase alignment required to channelize the composite signal by destructively canceling the undesired aliased spectral components. Post processing of the channelized spectra makes available the time series from these same channel bands.

Post processing can also interpolate the time series from each spectral band and can merge composite spectra from multiple adjacent channels to form wider band channels, and thus can deliver time series at different sample rates from multiple resolution channels in a single receiver structure.

## 3. Standard Polyphase Filter Structure

We first remind the reader that the standard polyphase filter, shown in figure 1, starts as a single channel M-path linear time invariant (LTI) filter structure that performs M-to-1 down sampling at the filter output after the M-to-1 bandwidth reduction.

This structure becomes the M-path periodically time varying filter structure of figure 2 that performs commutator based M-to-1 down sampling at the filter input prior to the bandwidth reduction.

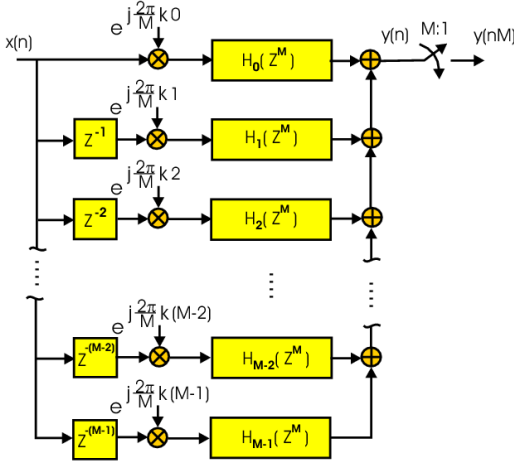


Figure 1. Output M-to-1 Down-Sampled, Polyphase Partitioned Single Channel Filter

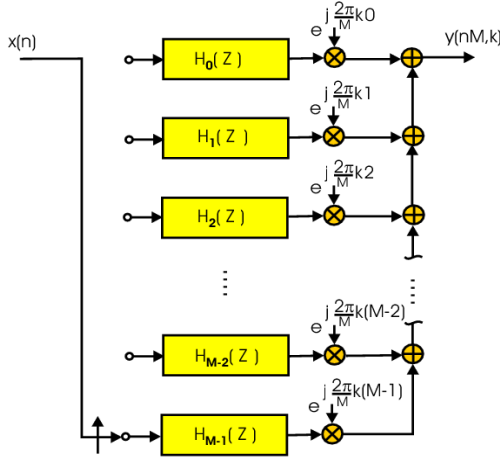


Figure 2. Input M-to-1 Down Sampled Polyphase Partitioned Single Channel Filter

In figure 3, we replace the phase rotators of figure 2 with an IDFT or its more efficient implementation an IFFT. Here the input resampling is responsible for M-fold aliasing of the input signal's spectrum to baseband. The multiple aliased signals in each path have unique phase profiles aligned with the M-roots of unity. Since the M-roots of unity sum to zero, all the spectral aliases aligned with these roots destructively cancel when summed in the IFFT. The IFFT phase shifters align the phase of any selected aliased frequency band to a common phase angle hence that selected band does not cancel but rather experiences the coherent gain of the sum. The IFFT extracts the time series of all M aliased bands to form a TDM output series.

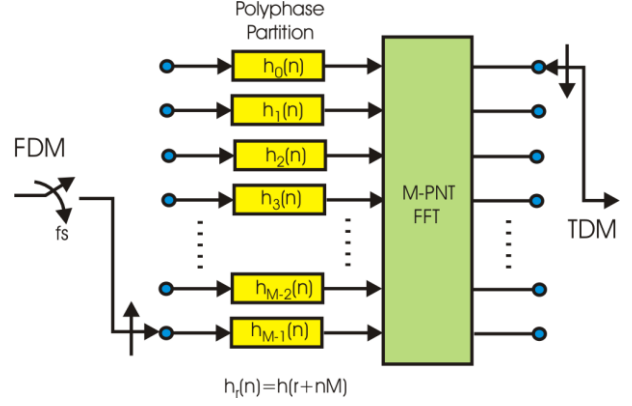


Figure 3. Input M-to-1 Down Sampled Polyphase Partitioned M-Path Filter Bank:

#### 4. Modified Polyphase Filter Structure

Each path of the M-path polyphase partition is a filter performing convolution with the sequence of input samples directed to its input port by the input commutator. There is an advantage to replacing these filters and their succession of inner products that form the convolution between the down sampled input streams and their path filters with an equivalent fast convolution process. Replacing direct convolution with the equivalent fast convolution using zero extended forward FFTs, spectral products, and inverse FFTs is shown in figure 4. When the direct convolution is replaced by the fast convolution segments of the form shown in figure 4, we have the structure shown in figure 5. We note that in this form, the input samples are no longer delivered as a succession of column vectors but rather as a matrix formed by numerous column vectors.

Observe that at the input port of the fast convolution blocks, the input streams are immediately converted to the frequency domain. The spectra seen at each port contain multiple aliased spectra due to the M-to-1 down sampling. The spectra of each path are modified by the gain and phase responses,  $H_{M-2}(k)$  of each M-path filter to obtain the spectra of the output filters. The filters are essentially all-pass frequency dependent phase shift filters. The row based IFFTs following the spectral products return us to the time domain where the scalar phase spinners required to align the selected alias terms are applied by the column based IFFT.

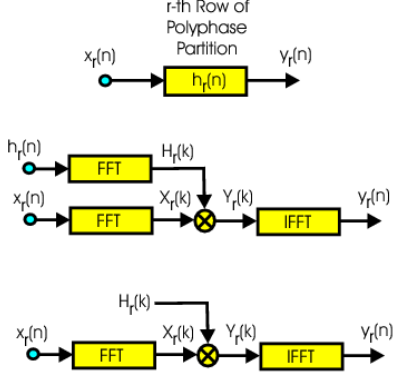


Figure 4. Direct Convolution Between the  $r$ -th Input Time Series and the Filter  $h_r(n)$ , and Fast Convolution of the two Series as the IFFT of the Spectral Products of their Zero Extended FFT's

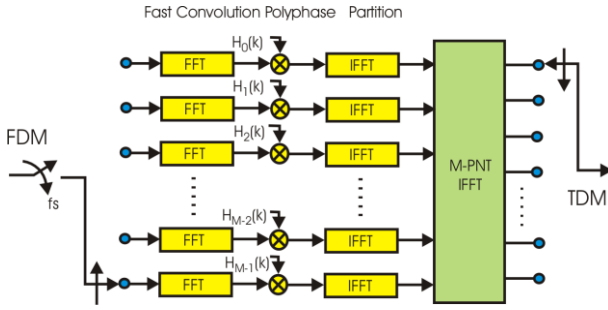


Figure 5. Polyphase Filter Partition with Fast Convolution of each path of M-Path Filter

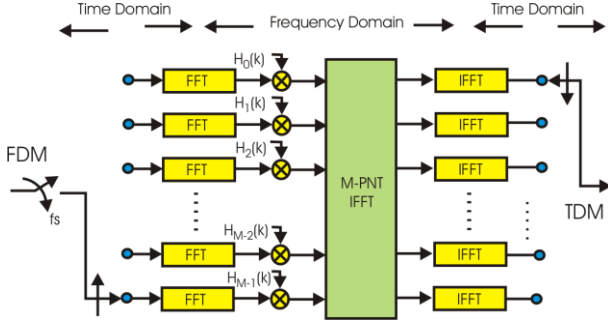


Figure 6. Reversed Order of IFFT's in Fast Convolution Based Polyphase Filter Bank

We note that the cascade of the row IFFTs and column FFT is in fact a two dimensional IFFT which can be applied to the input array in either order. When we reverse the order of the two IFFTs we obtain the structure shown in figure 6.

In this implementation the scalar phase rotators that align the  $M$ -aliases from a selected Nyquist zone are applied to the spectra of each path rather than to the time series of each path. Thus what we see at the output of the column transform is the de-aliased spectra from each Nyquist zone. We can tile a display by the juxtaposition of spectra from successive Nyquist zones to visualize the composite spectra spanning the input sample rate. Keep in mind that the spectra from each path of the column IFFT is not windowed. If we wish to see the windowed spectra we have to run a short circular convolver through the spectra to synthesize a multiplicative window in the time domain. Figure 7 shows the spectra obtained from a circular convolution of the Hann window weights for 16 specific rows from the column transform of figure 6. Some of the channels were intentionally left empty to illustrate adjacent channel isolation. Figure 8 shows the times series obtained from the corresponding row transforms. Each occupied channel contains a spread spectrum signal with bandwidth 4-times the chip rate.

## 5. Enhanced and Modified Filter Structure

With minor variation the  $M$ -path filter can operate in a mode that performing an  $M/2$ -to-1 input down sampling. This variant doubles the output sample rate of the down-sampled time series from each channel. The benefit of the increased output sample rate is that we avoid aliasing the transition bands at the edges of the equivalent band pass filters due to the down sampling. The filters operating in this mode are said to be non-critically down sampled. This option enables a particularly simple synthesis of perfect reconstruction wider bandwidth channels from adjacent fixed bandwidth output channels. Of course the reconstruction can also be performed with aliased critically sampled filters by destructive cancelling of the folded aliases. We chose to use the double rate option because it effortlessly permits the assembly of an un-aliased tiled spectrum formed from the outputs of the column IFFT.

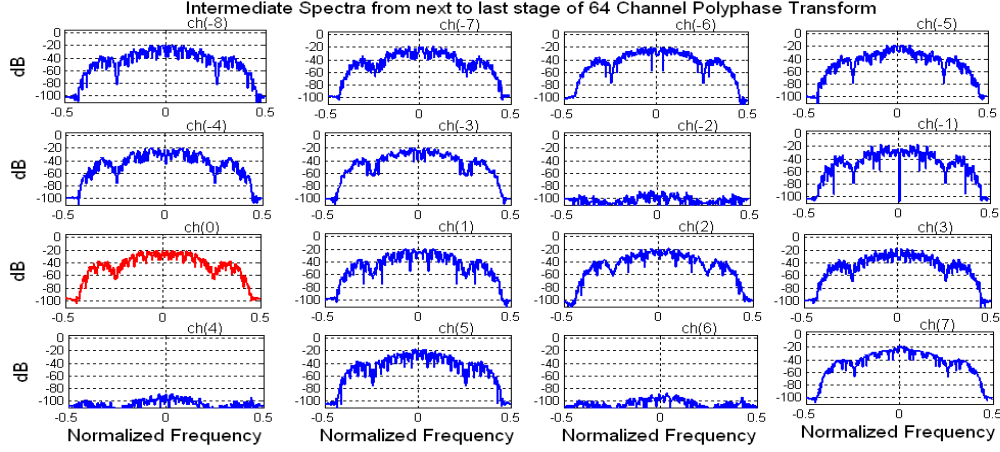


Figure 7. Spectra from 16 Channels formed by 64-Path Polyphase Filter Bank

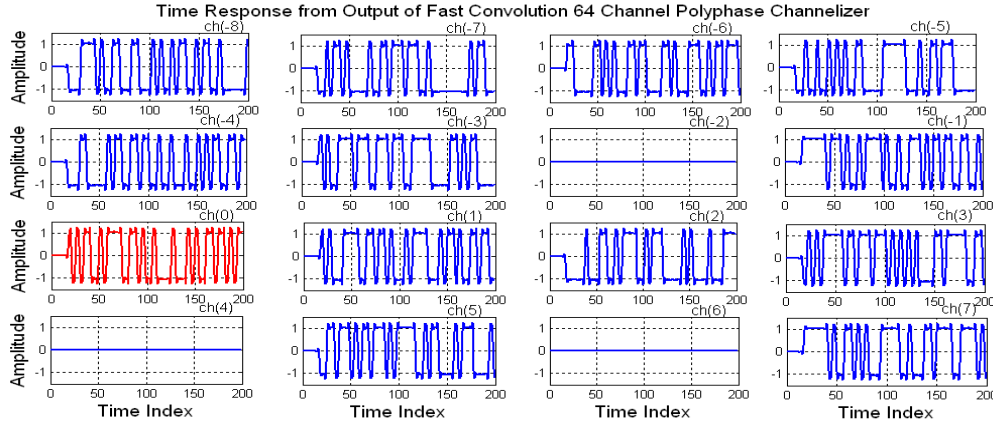


Figure 8. Time Series from 16 Channels formed by 64-Path Polyphase Filter Bank

It is a simple matter to accommodate the M/2-to-1 resampling of the polyphase filter by adding a cyclic buffer between the polyphase arms and the IFFT. There is no change in the cyclic buffer processing when the polyphase path filtering is performed via the fast filtering option operation. There is however significant changes when we interchange the order of the row and column transforms and have to account for the M/2-to-1 resampling in the frequency domain. Space limitations prevent us from presenting a full description of the altered process. A comment or two will give insight into the change which will be thoroughly described in a later paper. Let us assume that we had constructed a version of our process resampled M/2- to-1 rather than M-to-1. If we now take this process and perform a 2-to-1 down sampling we

would have two options, down sample 2-to-1 starting on index 0 and down sample 2-to-1 starting on index 1. Both of these options present M-to-1 resampling and are seen to be zero-packed and interleaved versions of M-to-1 down sampling with a time offset of 1 sample. The 1-to-2 zero packed time series of the two interleaved series have repeated copies of their spectra with one of the signals incorporating a frequency dependent phase shift to delay its time series the 1-sample offset. This relationship is seen to be the butterfly structure of a radix-2 FFT. Consequently a radix-2 butterfly structure is embedded in the middle processing task between the row and column transforms.

The final observation related to the change to M/2-to-1 down sample follows. When we shift

$M/2$  new input time samples in the  $M$ -path polyphase filter bank the spectral aliasing becomes frequency dependent. The even indexed frequency bins of the IFFT alias to DC while the odd indexed frequency bins of the IFFT alias to the half sample rate. Aware of this, the middle processing task shifts the spectrum of the odd indexed frequency bins so that the band centered at frequency  $fs/2$  is

translated to frequency 0. This is trivially done with calls to the MATLAB function `fftshift`. As they say in graduate school: “the rest of the derivation is left as an exercise for the reader!”

Figure 9 shows the spectra obtained from a circular convolution of the Hann window weights for the 16 specific rows from the column transform of the  $M/2-1$  down sampled version of figure 6.

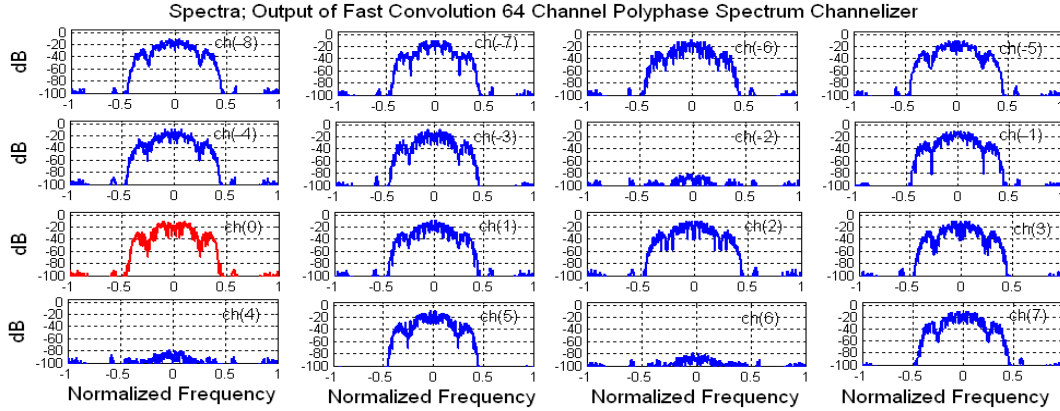


Figure 9. Spectra from 16 of the Channels Formed by 64-Path Polyphase Spectrum Channelizer, Non-Critically Down Sampled 32-to-1. Window Applied as a Circular Convolution in Frequency Domain, Polyphase Filter Applied as Spectral Products in Frequency Domain

## 6. Perfect Reconstruction Filters

When the polyphase filter bank is implemented as a fast channelizer or as a fast spectrum analyzer the channel center frequencies of the filter bank are separated by  $1/M$ th of the input sample rate and the filter bandwidth nominally matched the channel spacing.  $M$  is the number of paths in the polyphase partition as well as the length of the column transform embedded in the polyphase filter. If the signals of interest span more than a single channel width we can synthesize a wider effective channel by coherently combining multiple adjacent channels. If the sample rate of each channel is twice the Nyquist rate for the channel this is a particularly simple operation. The synthesis process involves raising the sample rate of the separate channels to that of the composite channel and then translating the spectra from adjacent channels back to their relative positions and merging their separate contribution by simple addition. When the processing is performed in the frequency domain on spectra formed by the fast spectrum channelizer the sample rate increase is performed by zero extending

the spectra and the spectral translation is performed as a circular shift of the spectral support. Both operations are trivial. Figure 10 shows the time and frequency response of the prototype filter used in the  $M$ -path partition. Figure 11 shows the spectra of three adjacent channel filters, up sampled 1-to-3 and offset to their nominal center frequencies. Figure 12 shows the reconstruction of a synthesized filter formed as the sum of three adjacent offset filters. Also shown is the pass band reconstruction error which is seen to be on the order of 0.001 dB.

## 7. Closing Comments

We have presented a novel architecture for a polyphase spectrum analyzer-channelizer. This paper started with a review of the polyphase channelizer in its traditional embodiment as a cascade of three fundamental transformations; these being an input commutator controlling the input down sampling operation, an  $M$ -path filter partition of a prototype low pass filter designed to satisfy a set of spectral requirements, and an  $M$ -point IFFT.

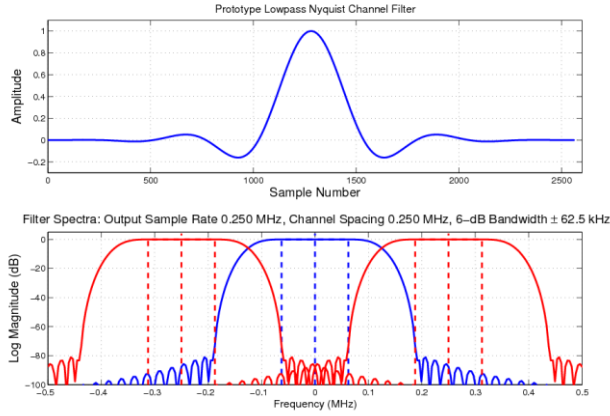


Figure 10. Time and Frequency Response of Prototype Low Pass Filter in Channelizer

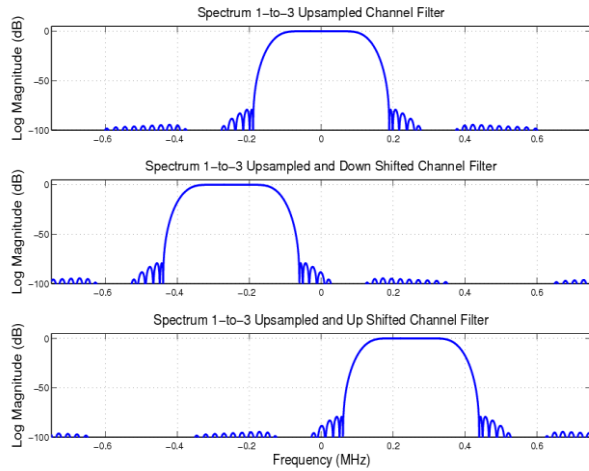


Figure 11. Spectra of three Adjacent Channel Filters: 1-to-3 Upsampled and Frequency Offset

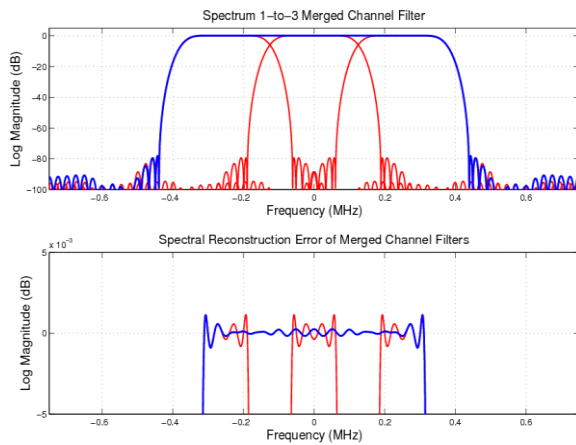


Figure 12. Sum of three Adjacent Offset Spectra with Zoom to Pass Band Reconstruction Error

We then replaced the inner product based filters in each arm of the M-path polyphase partition with a fast convolution structure. This structure performs the filtering operation in the frequency domain as a spectral product rather than in the time domain as a set of inner products. The traditional justification for using fast convolution to perform traditional filtering tasks is the reduced computational load and the reduced processing time. Simulations performed for this paper showed that the fast convolution techniques required approximately one-fifth of the time required for the direct convolution polyphase partition. As a reference point, all filtering done in Matlab is performed by fast convolution techniques.

We then interchanged the order of row and column IFFTs and noted the availability of the channelized spectrum prior to the row IFFTs. A circular convolution with a 3-or-5 tap spectral window gives of easy access to the spectra of each channel. We finally modified the input commutator to obtain an M/2-to-1 down sample M-channel system. This modification required the insertion of mid process alias correction. This proved to be a minor cost to obtain a significant gain in ease of spectral tiling and of adjacent channel merging to synthesize non uniform filter banks.

## 8. References

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