

## A TRANSMIT BEAMFORMING TECHNIQUE FOR MIMO COGNITIVE RADIOS

L. Bixio, L. Ciardelli, M. Ottonello, M. Raffetto, C. S. Regazzoni, Sk. S. Alam  
(Department of Biophysical and Electronic Engineering , University of Genoa, Genoa,  
Italy; {bixio, ciardelli, ottonello, raffetto, carlo, alam}@dibe.unige.it);  
and C. Armani  
(Selex Communications S.p.A., Genoa, Italy; claudio.armani@selex-comms.com)

### ABSTRACT

In the present paper one of the main problems of cognitive radios, that is how to allow the secondary (cognitive) users to exploit the unused resources by primary users, is faced by introducing multiple antennas at the cognitive terminals.

Under the assumption of perfect channel state information (CSI), a transmit beamforming scheme based on a linear algorithm is proposed for the exploitation of the degrees of freedom offered by the spatial diversity. A closed form expression for the achievable rate obtainable by employing two antennas is derived and numerical results regarding the effects of different fading channels are provided.

### 1. INTRODUCTION

In the last years, the most popular spectrum allocation strategy for wireless services and applications has been the fixed spectrum allocation [1].

In fact, to guarantee the coexistence, different wireless services are assigned to different portions of the radio spectrum [1]. With the increasing number of wireless technology along with the fixed spectrum allocation methodology the unlicensed frequency bands are going to disappear [1]. However, many studies have shown that, although the radio spectrum is allocated, it is highly underutilized [1].

To improve radio spectrum utilization, the Cognitive Radio (CR) networks have been proposed. In particular, unlicensed (secondary) users are allowed to dynamically access to licensed resources if primary (legacy) users are not using them at a given time and in a given location [1]. Although this novel paradigm leads to a dynamic and effective management of the spectrum, many problems arise since primary users have to be protected from detrimental interference while assuring an acceptable Quality-of-Service (QoS) to the secondary systems [2].

In fact, CR networks operate in a heavy interference corrupted environment and effective interference

management has to be addressed to permit the coexistence among primary and CR networks [3]. Most of prior research about opportunistic spectrum access and interference mitigation for CR networks focuses on the detection of primary users' activity in frequency or time domain considering single antenna at both primary and secondary transceivers [1].

However, it is well known that the introduction of multiple antennas at the transmitter and/or receiver can provide many desirable functionalities, such as capacity enhancement, effective co-channel interference reduction, spatial division multiple access, etc [1].

In spite of the introduction of multiple antennas in CR networks has gained attention from theoretical and practical perspectives, only few researches have been carried out and some open issues persist [1]. In the open literature, in order to face the complex problem of the coexistence among primary and CR networks, some simplifying hypotheses are considered. As an example, in [4]-[7] it is assumed that a secondary system, equipped with multiple antenna, is provided with the message sent by the primary transmitter.

Under this assumption the capacity of the secondary system is evaluated and it is shown that significant improvements can be obtained with respect to traditional system. However, such an approach suffers in practical opportunistic scenarios where primary users are unaware of the presence of the secondary system and cooperation (i.e. sharing of the transmitted message) cannot be assumed. In [8], although the secondary system does not know the primary message and an interference free CR networks is obtained by implementing properly designed filters at both primary and secondary transceivers, it is required some modifications to the legacy terminals which is unpractical in real environment.

In some other works, presented in [2], [3], no information regarding the primary networks is required and the capacity of the secondary system is evaluated while ensuring an acceptable level of interference to the primary system. In particular, an interference cancelation technique, known as transmit precoding or transmit beamforming [3], is

exploited by a CR network "for mitigating the interference to primary system by adaptively choosing weights on the transmit antenna elements to form an emission pattern with nulls toward the direction of primary receivers" [3].

Such a strategy allows to minimize the interference caused at the primary users, and to maximize the Signal-to-Interference-Noise Ratio (SINR) for the secondary users [3].

In this work, a multiple antenna cognitive terminal which exploits transmit beamforming to completely avoid interference from secondary transmitter toward primary receiver is considered.

In particular, by considering a linear precoding at the secondary transmitter, in the case of terminal equipped with two antennas, the achievable rate at the secondary system is obtained by assuming perfect Channel State Information (CSI). It is shown that the degrees of freedom (DOFs) at the CR networks necessary to cancel the interference at the primary receiver are equal to 2, if simplex communications are considered.

Finally, numerical simulations are carried out to show the effectiveness of the proposed solution in multipath channel.

## 2. TRANSMIT BEAMFORMING

Generally speaking, the simplest CR problem can be represented by a communication scenario in which a couple of primary terminals and a couple of cognitive radios wish to communicate over the same resource [9], [10].

As remarked in Section I, in spite of the simplicity of the model, few algorithms have been proposed.

In this section, the benefits coming from the introduction of multiple antennas at the cognitive terminals is analyzed. In particular a transmit beamforming scheme is introduced to satisfy the constraint imposed by the CR communications.

For the sake of simplicity, the primary terminals are equipped with a single antenna system, while the cognitive terminals are equipped with two antennas, but the analysis can be easily extended to a high number of antennas, both at the primary and cognitive systems.

### 2.1. Channel Model

Transmit beamforming can be used by a CR system to steer the power towards the direction of interest (i.e. secondary receivers) while minimizing the interference to primary receivers [3].

In particular, this technique, employed by different approaches [2], [11], allows minimizing the interference caused to primary users while maximizing the SINR for the cognitive users.

In the proposed approach transmit beamforming is implemented, by introducing a linear pre-processing scheme

which guarantee, under specific conditions, to perform complete interference cancelation at the primary receiver.

The equations which describe the channel of interest, known in the open literature as the MIMO Z channel [12] and usually assumed for treating the problem of interest [1], [11] shown in Fig. 1, are the following

$$y_p = \mathbf{g}_r^T \mathbf{x}_c + hx_p + n_p \quad (1)$$

$$\mathbf{y}_c = H\mathbf{x}_c + \mathbf{n}_c \quad (2)$$

in which  $y_p \in \mathbb{C}$  and  $x_p \in \mathbb{C}$  are respectively the received and transmitted complex baseband signals of the primary terminals,  $\mathbf{y}_c \in \mathbb{C}^2$  and  $\mathbf{x}_c \in \mathbb{C}^2$  are respectively the received and transmitted complex baseband signal vectors (represented in bold, in the entire paper) of the cognitive terminals,  $H \in \mathbb{C}^{2 \times 2}$  is the complex channel matrix between the cognitive terminals,  $h \in \mathbb{C}$  is the complex channel coefficient between the primary terminals,  $\mathbf{g}_r \in \mathbb{C}^2$  is the complex channel vector between the cognitive transmitter and the primary receiver ( $\cdot^T$  stands for transpose), and  $n_p \in \mathbb{C}$  and  $\mathbf{n}_c \in \mathbb{C}^2$  are the zero-mean complex Gaussian noise quantities [13] respectively for the primary and the cognitive receivers. In the following,

$$E\{n_p n_p^*\} = \eta_p^2$$

$$E\{\mathbf{n}_c \mathbf{n}_c^\perp\} = \eta_c^2 I_2$$

will be assumed [14] where  $\cdot^\perp$  stands for transpose and complex conjugate,  $I_2$  is an 2 x 2 identity matrix and  $E(\cdot)$  is the expectation operator.

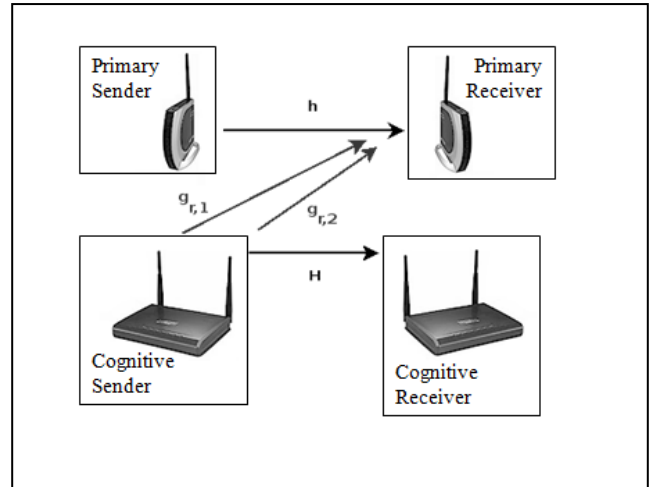


Fig. 1 Considered channel model.

It is important to note that, in this case, the interference caused by primary users is included in the additive noise term. Moreover, although the channel model in equations (1) and (2) refers to the narrowband case (all channel coefficients are frequency independent), it can be easily

extended to multi-carrier systems by applying it on a sub-carrier basis [15]. To perform the transmit beamforming, let us introduce a transmit precoding matrix  $A \in \mathbb{C}^{2 \times 2}$  such that  $\mathbf{x}_c = A\mathbf{x}_a$ . By substituting it in the channel model expressed by (1) and (2), one can obtain

$$y_p = \mathbf{g}_r^T \mathbf{x}_c + hx_p + n_p = \mathbf{g}_r^T A\mathbf{x}_a + hx_p + n_p \quad (3)$$

$$\mathbf{y}_c = HA\mathbf{x}_a + \mathbf{n}_c \quad (4)$$

To guarantee that the cognitive transmitter causes no interference to the primary receiver

$$\mathbf{g}_r^T A = 0 \quad (5)$$

has to be enforced, together with  $\|A\|_2 = 1$ , where the symbol  $\|\cdot\|_2$  stands for 2-norm, in order to avoid signal amplification or reduction, to obtain

$$y_p = hx_p + n_p \quad (6)$$

$$\mathbf{y}_c = \tilde{H}\mathbf{x}_a + \mathbf{n}_c \quad (7)$$

in which  $\tilde{H} = HA$ .

Such a process allows an effective decoupling of the scalar additive white Gaussian noise (AWGN) channel of the primary users (6) from that one of the cognitive users (7).

## 2.2. Derivation of the Achievable Rates

As suggested by the large amount of literature dedicated to MIMO transmissions [13], [15], [16] the  $2 \times 2$  channel expressed by (7) can be exploited through the singular value decomposition (SVD). Hence by writing  $\tilde{H} = U\Sigma V^\perp$ , with  $\Sigma$  diagonal matrix and  $U$  and  $V$  unitary matrices, and by introducing  $x = V^\perp \mathbf{x}_a$  and  $y = U^\perp \mathbf{y}_c$ , from (7)

$$\mathbf{y} = U^\perp \mathbf{y}_c = U^\perp \tilde{H} V \mathbf{x} + U^\perp \mathbf{n}_c = \Sigma \mathbf{x} + U^\perp \mathbf{n}_c \quad (8)$$

can be obtained. Equation (8) represents two parallel Gaussian channels

$$\mathbf{y} = \mathbf{z} + \mathbf{n} \quad (9)$$

with input  $\mathbf{z} = \Sigma \mathbf{x}$  and complex Gaussian noise  $\mathbf{n} = U^\perp \mathbf{n}_c$ . The noise has zero-mean and covariance matrix still  $\eta_c^2 I_2$ , since the multiplication by a unitary matrix does not change the distribution of the noise [15], while the input has covariance

$$E\{\mathbf{z}\mathbf{z}^\perp\} = \Sigma K_x \Sigma$$

where  $K_x = E\{\mathbf{x}\mathbf{x}^\perp\}$

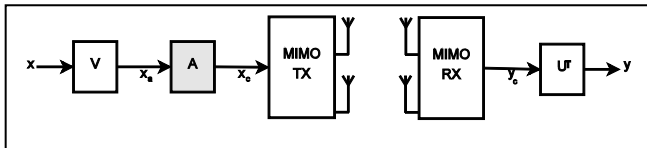


Fig. 2 Adopted processing scheme.

The obtained linear processing scheme, shown in Fig. 2, allows, under the hypotheses of a perfect knowledge of the channel between cognitive terminals and the channel from cognitive transmitter to primary receiver, to exploit the degrees of freedom of the  $2 \times 2$  MIMO channel for the transmission of the cognitive users, and, at the same time, to cancel the interference to the primary receiver. It is important to note that, in order perform such a cancelation, the available degrees of freedom of the MIMO Z channel which models the CR problem, expressed by (1) and (2), are reduced.

In particular, since the number of DOFs of the considered MIMO Z channel is 2 [12] and the number of DOFs of the primary link is 1, one can deduce that the number of DOFs available for the cognitive link is 1 and for this reason  $\Sigma$  will have at most one non trivial diagonal entry  $\varepsilon$ .

Such a property allows simplifying the computation of the achievable rates of the proposed processing scheme. As a matter of fact, one can deduce that the covariance of the input

$$E\{\mathbf{z}\mathbf{z}^\perp\} = \Sigma K_x \Sigma$$

will have at most one entry

$$\varepsilon^2 E\{|x_1|^2\}$$

where

$$E\{|x_1|^2\}$$

is the first entry of  $K_x$ , being  $x_1$  the first component of  $\mathbf{x}$ . Therefore, no signal power will be received on the last component of  $\mathbf{y}$  for any choice of  $K_x$  and the achievable rates of the channel have to be evaluated by taking into account only the non trivial component of  $\mathbf{x}$ , accordingly. To this end the channel of interest represented by (9) can be rewritten as a scalar equation

$$y = z + n \quad (10)$$

where the variance of the input  $z$  is

$$E\{zz^*\} = \varepsilon^2 E\{|x_1|^2\} = \Phi$$

and the variance of the noise  $n$  is

$$E\{nn^*\} = \eta_c^2$$

The capacity of such a Gaussian channel is well established in the literature [16] and can be easily deduced for (10), by modifying the power constraint. In fact, while in the classical theory [16] the power constraint  $P$  is given in terms of  $\mathbf{z}$ , in the proposed approach it has to be imposed on the power transmitted by the cognitive terminal and is given in terms of  $\mathbf{x}$  as

$$E\{|x_1|^2\} = \frac{\Phi^2}{\varepsilon} \leq P \quad (11)$$

By following the technique in [16] and by taking into account the statistical variations of the channel, one deduces that the achievable rates of the cognitive link obtained by employing the proposed linear scheme can be expressed by

$$C = E_{H, g_r} \left[ \max_{A, \Phi} \frac{1}{2} \log \left( 1 + \frac{\Phi}{\eta_c^2} \right) \right] \quad (12)$$

under the constraints (11).

For fixed  $A$ , which has to satisfy (5) and the condition  $\|A\|_2 = 1$ , the solution to (12) is found as the optimal input variance  $\Phi$  for the Gaussian channel subject to (11). Therefore, the achievable rates of the MIMO cognitive link with the proposed linear processing scheme and  $\varepsilon = 0$  can be expressed by

$$C = E_{H, g_r} \left[ \frac{1}{2} \log \left( 1 + \frac{\Phi}{\eta_c^2} \right) \right] \quad (13)$$

where all power has to be transmitted over  $x_1$  (the first component of  $\mathbf{x}$ ) since, from (11)

$$\Phi = \varepsilon^2 E\{|x_1|^2\} = \varepsilon^2 P$$

### 2.3. Computation of Matrix A

In order to complete the analysis, an explicit expression for  $C$  has to be found.

To this end, the expression for matrix  $A$  (and consequently  $\varepsilon$ ) which guarantees the maximum achievable rate has to be computed. By assuming that  $g_{r,i} \neq 0$  ( $i = 1, 2$ ) (otherwise, a partial spatial orthogonalization is already performed by the channel) and by enforcing (5), one can obtain

$$A = \begin{bmatrix} a_{11} & \frac{-a_{22}g_{r,2}}{g_{r,1}} \\ \frac{-a_{11}g_{r,1}}{g_{r,2}} & a_{22} \end{bmatrix} \quad (14)$$

Hence  $\varepsilon$ , which is the (possibly) non-trivial singular value of  $\tilde{H} = HA$  can be calculated in symbolic terms as

$$\varepsilon = \left( \left( |a_{11}|^2 |g_{r,1}|^2 + |a_{22}|^2 |g_{r,2}|^2 \right)^{\frac{1}{2}} \cdot \frac{\left( |h_{12}g_{r,1} - h_{11}g_{r,2}|^2 + |h_{22}g_{r,1} - h_{21}g_{r,2}|^2 \right)^{\frac{1}{2}}}{|g_{r,1}| |g_{r,2}|} \right)^{\frac{1}{2}} \quad (15)$$

As a consequence, by substituting (15) in (13), an explicit expression for the achievable rates can be obtained as follows

$$C = \frac{1}{2} E_{H, g_r} \left[ \log \left( 1 + P \frac{|a_{11}|^2 |g_{r,1}|^2 + |a_{22}|^2 |g_{r,2}|^2}{\eta_c^2 F(H, g_r)} \right) \right] \quad (16)$$

where

$$F(H, g_r) = \frac{|g_{r,1}|^2 |g_{r,2}|^2}{|h_{12}g_{r,1} - h_{11}g_{r,2}|^2 + |h_{22}g_{r,1} - h_{21}g_{r,2}|^2} \quad (17)$$

In order to find the values of  $a_{11}$  and  $a_{22}$  which guarantees the maximum achievable rate  $C$ , the expression  $|a_{11}|^2 |g_{r,1}|^2 + |a_{22}|^2 |g_{r,2}|^2$  has to be maximized since the other factors depend only on the channels  $H$  and  $g_r$ . For the maximization process, it is important to recall that  $\|A\|_2$  has to be imposed equal to 1, in order to avoid signal amplification or reduction. Hence

$$\|A\|_2 = \max\{s_i\} = 1 \quad (18)$$

for the property of the singular values  $s_i$ ,  $i = 1, 2$ .

The maximum singular value can be computed as

$$\max\{s_i\} = \sqrt{\frac{\left( |g_{r,1}|^2 + |g_{r,2}|^2 \right) \left( |a_{11}|^2 |g_{r,1}|^2 + |a_{22}|^2 |g_{r,2}|^2 \right)}{|g_{r,1}|^2 |g_{r,2}|^2}} \quad (19)$$

and by imposing it equal to 1 from (18) one can obtain

$$\left( |a_{11}|^2 |g_{r,1}|^2 + |a_{22}|^2 |g_{r,2}|^2 \right)^2 = \frac{|g_{r,1}|^2 |g_{r,2}|^2}{|g_{r,1}|^2 + |g_{r,2}|^2} \quad (20)$$

Hence the expression  $|a_{11}|^2 |g_{r,1}|^2 + |a_{22}|^2 |g_{r,2}|^2$ , which had to be maximized in order to guarantees the maximum achievable rate  $C$ , is bound from (20) by the channel coefficients  $g_{r,1}$  and  $g_{r,2}$ .

For this reason, at each channel variation, the choice of the optimal  $A$  does not require an optimization, but just a selection of  $A$  according to (14) and (20), since the maximum achievable rate  $C$  of the cognitive link depends only on  $\|A\|_2$ .

### 3. NUMERICAL RESULTS

In this section we provide a set of results which show the effectiveness of the proposed linear processing scheme.

The achievable rates guaranteed by employing the transmit beamforming precoding has been evaluated by varying the multipath conditions and the signal to noise ratio (SNR) of the involved channels. In particular, a set of numerical simulations have been carried out for different values of the Ricean factor  $K$  [17]-[19] of MIMO Rice channels. To this end, each channel entry has been modeled as an independent complex Gaussian variable with mean  $\sqrt{K}/(K+1)$  and variance  $1/(K+1)$  [18].

The simulations have been performed with a SNR of 0 and 20dB both at the cognitive and at the primary receiver. In order to provide a fair comparison, the capacity of the single-input single-output (SISO) channel used for the primary transmission, has been reported. Moreover, it is added with that one obtained by employing the proposed

processing scheme, to provide an overall evaluation of the performances of the ensemble of the primary and cognitive systems.

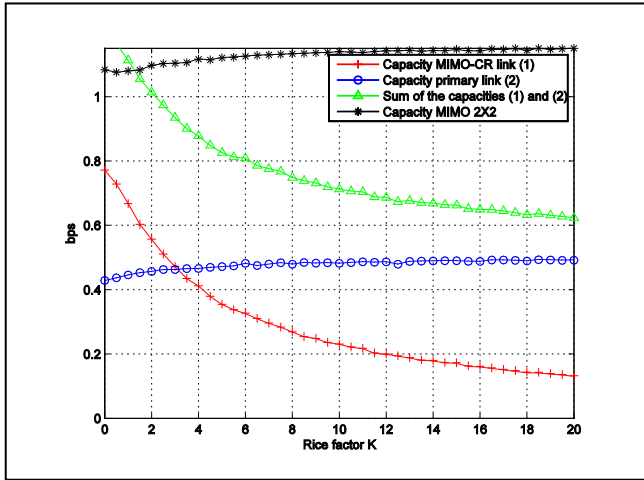


Fig. 3 Achievable rates by the four considered transmission schemes at  $SNR=0dB$

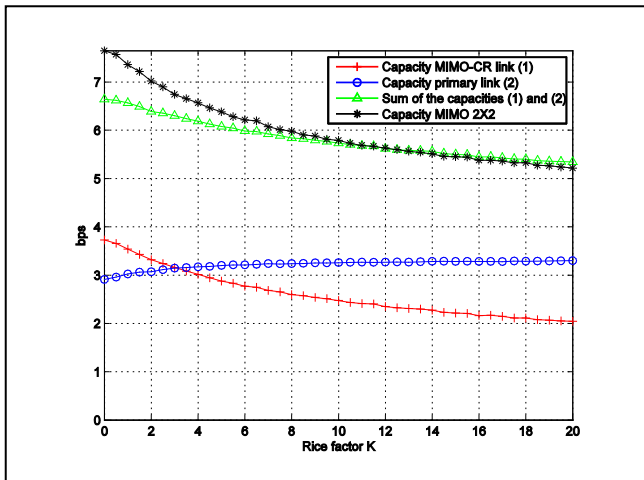


Fig. 4 Achievable rates by the four considered transmission schemes at  $SNR=20dB$

Finally, an additional evaluation is provided by comparing also the capacity of a  $2 \times 2$  MIMO channel [15], [20], [21], i.e. the channel used by the cognitive terminals for the transmission if the primary terminals are absent.

The obtained results for  $SNR = 0dB$  are shown in Fig. 3. As it can be seen by comparing the proposed scheme with the more traditional ones, satisfactory performance can be obtained also in a low SNR environment, although it can be noted a steep decrease along with the increase of the Ricean factor  $K$ . As expected, better performances can be obtained by increasing the SNR. As shown in Fig. 4, at  $20dB$  the achievable rates increase, although a slighter decrease can

be noted again in the performance of the proposed precoding technique if the Ricean factor increases.

As can be noted by observing both Fig. 3 and Fig. 4, all the capacities are maximum for the Rayleigh case ( $K=0$ ): this is due to the distribution of the singular values in the deterministic part of the channel matrices.

#### 4. CONCLUSIONS

In this paper, a transmit beamforming technique is applied in the cognitive radio context in order to evaluate the advantages of the introduction of multiple antennas at the secondary terminals.

The channel model and the cognitive radio problem have been stated for the MIMO Z channel, and the explicit solution for the problem of maximizing the transmission of information has been derived while avoiding interference among primary and secondary users. Finally, numerical results regarding the effects of different Rice fading channels have been reported.

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