Statistical Framework for Parametric Optimization of Cognitive Radio Systems

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Abstract—**Enhancing access to radio spectrum is a growing need across the country that cognitive radio (CR) can address. CR systems incorporate learning and decision making into wireless and networking systems with the goal of improving performance and interoperability. Optimizing radio and cognitive engine parameters based on operating performance is a crucial requirement. This step is also a needed element within functional testing and overall evaluation for validating overall systems and specific algorithms. However, the realities of limited time, manpower, and resources require efficient methodologies for performing these tasks. While CR techniques continue to advance, functional testing methodologies remain largely stagnant and rely on ad hoc approaches relevant to a specific platform and application space. Recent CR testing methods are briefly surveyed in this paper. A generalized statistical framework that is independent of radio platform and application theater is introduced that enables a methodological process for configuration optimization and functional testing of CR systems and components. The process identifies a minimal number of experiments required to achieve an understanding trends and interaction of configuration elements. The need to calibrate transmit-and-receive gain settings on a software-defined radio illustrates the use of response surface methodology (RSM). The results indicate that as little as 15 data points can develop a representative model of system performance. The RSM analysis shown focuses on response model estimates, statistical fit of the models, and graphical representation of output responses.**

I. INTRODUCTION

Cognitively inspired radio (CR) mates simple decision making, learning algorithms and situational awareness with reconfigurable wireless communications platforms. Goals include enabling improved performance and long-term learning based on past experiences. CR techniques are viewed as enablers to enhanced access to radio spectrum as well as improving interoperability and military radio applications such as electronic warfare. However, this wide diversity in CR applications has led to a lack of consistency in testing methodologies.

Testing and evaluation are vital elements for the development and validation of cognitive engine (CE) architectures. Before a system can even be run through overall system tests, operating ranges of individual configuration parameters of the CE must be identified. However, testing is bounded by realistic constraints on time and human resources. Analysts cannot realistically test every possible configuration of system parameters. They must also distill the data to a concise form that decision makers can act on without losing the

understanding of the statistical variations inherent to wireless systems. Current gaps in CR validation include: 1) lack of an abstract framework that can be applied across varied architectures and applications; 2) focus on physical (PHY)-centric metrics with little emphasis on cognition metrics; 3) lack of systematic methodologies for making the best use of limited time and resources; and 4) precise metrics that frame application specific measures within a context of human decision making. Such a framework can provide a foundation for the comparison of architecture and algorithms developed within varying application spaces.

The identification of operating ranges of internal CE configuration parameters is akin to multi-objective optimization. This area has been well researched within CR [1]. Some initial efforts to attack this issue from an empirical perspective include design of experiments (DOE) approaches [2] that utilize statistical methods. This paper builds upon statistical empirical methods of parameter evaluation and system testing to answer two key questions: 1) How does one identify initial operating configurations for a CE; and 2) How does one define a test plan that validates CR architectures?

Contributions of this paper include an abstract CR testing framework that is independent of the specific hardware platform and application space as well as an application of RSM to CR testing. This systematic process provides a missing formalism to assist in evaluating overall CR system performance, calibration, and tuning of configuration parameters. This paper moves beyond factorial DOE methods to implement Response Surface Methodology (RSM) on a USRP software defined radio where most existing statistical works have focused simulation and 802.11 platforms. The method is utilized to identify hardware configuration parameters that are often overlooked and selected by ad-hoc approaches.

This paper is organized as follows. Section II surveys existing CR testing methodologies. Section III proposes an abstract framework for testing and system parameter tuning capable of extension to specific applications. Section IV focuses on applying RSM to a calibration of three SDR hardware configuration factors: 1) transmitter gain; 2) receiver gain; and 3)transmit signal amplitude from the perspective of optimizing three responses: 1)output power; 2)bit error rate (BER); and 3)packet error rate (PER). The results of the RMS analysis are discussed included model development of each response,

statistical fit of each model, and identification of a favorable setting for the three input factors. Section V summarizes the paper and identifies next steps.

II. REVIEW OF CR/SDR TESTING

This section reviews current literature about testing methodologies for CR, dynamic spectrum access (DSA) and white space communications. The majority are typically founded upon military radio frequency (RF) testing standards developed in the 1960s [3]. Table I summarizes the reviewed methods. In general, any of these testing methods can be abstractly viewed in terms of application of a load placed upon a system with the corresponding response measured. The entire system can be viewed as black box or individual system elements upon which a load and corresponding response can be measured.

While DSA systems do not always constitute CR, they are often viewed synonymously. Given their popularity, the majority of existing testing methodologies focus here. The Defense Advanced Research Projects Agency (DARPA) next generation (XG) communications program led to the earliest mature deployment of DSA technology. The XG radios utilized a policy-based CE to enable nodes to operate in an environment populated by primary users. Performance metrics included temporal measures such as abandonment time and network join time. Posherstnik et al. have proposed an extensible framework for testing DSA policy-based reasoners [4]. The plan focuses on RF aspects as well as network performance and describes key procedural steps, needed configuration selection and the system model.

Arlsan proposed SDR and CR testing that focuses on waveform analysis, coding and modulation with suggested metrics to include cyclostationarity, interference temperature, and interference statistics [5]. A system model is presented that places a signal analyzer in parallel with SDR components and excited through a signal generator. Test equipment manufacturers also propose similar strategies for testing CR and SDR [6].

Government and independent consortia provide important driving forces for testing and evaluation. Both the Federal Communications Commission (FCC) and National Telecommunications and Information Administration (NTIA) have active programs for testing and measurement for whitespace devices (WSD). The FCC has published test plans and results of preliminary WSDs [7]. The NTIA has proposed a test-bed pilot program to evaluate DSA devices for sharing spectrum in the land mobile radio 410-420 MHz federal band and 470-512 MHz non-federal band. The test bed includes equipment characterization, evaluation of capabilities, and a field operations evaluation [8]. The NTIA presents detailed procedures for conducting a variety of RF tests, including presenting probability of detection results. The Wireless Innovation Forum, also known as the SDR Forum, has a standing Test and Measurement Task group which recommends additional procedures for the evaluation of cognitive radio for DSA [9]. Metrics and test procedures are still in development phases.

Zhao et al. introduce metrics, utility functions and methodology for the performance evaluation of CR [10]. In addition to the traditional physical layer metrics, the need for cognition-centric metrics are discussed. The authors generally consider an evaluation of CR at either the node or network level from the perspective of one of four domains: cognitive functionality, overall node performance, complexity, and technical maturity. Cognitive metrics proposed include "Radio IQ", while node-level metrics include the traditional physical metrics such as bit error rate (BER).

TABLE I
SUMMARY OF CR-SDR TESTING

Name	Focus	Metrics	Ref.
DARPA XG	DSA	PHY	[11]
		Temporal	
Posherstnik	DSA	Temporal	[4]
Arslan	RF testing	PHY	[5]
FCC	TV-WSD	% Detection	[12]
		PHY	
NTIA	Spectrum Sharing	PHY	[8]
		Detection	
		Vacating	
SDR Forum	DSA	not available	[9]
Zhao	Performance Evaluation	Detection Rate	[10]
		PHY	
		Cognition	

III. FRAMEWORK FOR TESTING AND EVALUATION OF CR

This section introduces a concept framework for evaluating CR. A basic abstraction of testing consists of an application of a defined load upon a system and measuring the response output. A load is viewed as stress placed on the system, such as sending a data file across a network or wireless link. A supplemental view of load may also include an external influence such as a noise spike at a specific frequency, or the presence of a primary user.

A general flowchart for a testing strategy for CR is shown in Figure 1. This framework provides a systematic approach and enables customization to the specific platform, cognitive architecture and application space. Testing and evaluation of a CR system must start with a frame of reference for desired functionality. A concept of operations (CONOPS) defines the overall mission objectives and purpose of the system. These definitions drive the specific functional requirements needed to achieve the objective. These requirements then drive applications-specific performance metrics. Defined test cases create a controlled setting in which implementation of the CONOPS is verified against each functional requirement utilizing the performance metrics. The framework builds upon two philosophies from computer and software performance testing known as *black box* and *white box* testing [13]. The former views the internal components of the system, in this case the CE or radio platform, as one element, and the overall response to a test load is measured. With the latter, internal structural elements of the system are analyzed individually along with interactions. The goal of white box testing is to
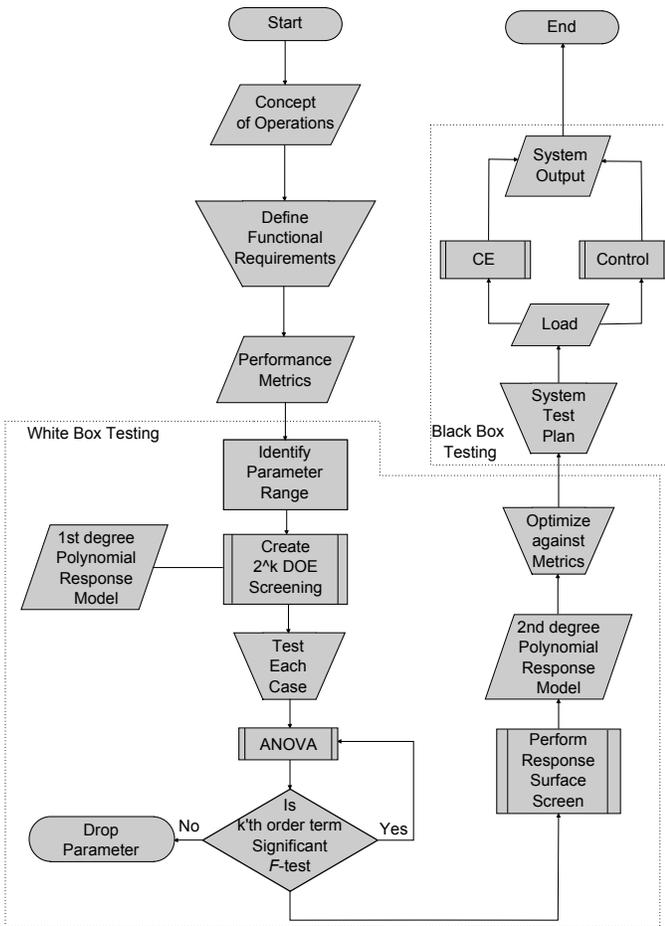
Fig. 1.   Testing Flowchart

tune the internal parameters for desired performance prior to overall black box system testing.

Performance metrics, as seen from Table I, vary depending on the functional requirements. In general, metrics are a system response to a load and can be viewed in two categories: Higher is Better (HIB) and Lower is Better (LIB). This distinction has bearing towards visualization methods for presenting overall results. The Kiviat diagrams technique alternates HIB and LIB metrics in a circular plot, also known as a spider plot [14]. This enables visual comparisons between system configurations and identification of balanced operation.

The first step in the white box testing is to identify the internal configuration parameters that have the potential to affect the system response. One can manipulate one configuration parameter at a time to elicit the single variable effects; however, this does not take into account interaction effects incurred by changing multiple variables at the same time.

With performance metrics defined for the specified application, the next step is tuning the internal elements of the system to ensure that overall system testing occurs from a sound foundation. This white box-level testing may be required for calibrating radio knobs and setting specific CE parameters such as case-based reasoning settings or genetic algorithm

settings. The concept of load application and corresponding response is implemented across defined test settings of these internal parameters. The first goal is to identify which parameters induce the most effect on the response and to identify interaction effects as multiple settings are changed simultaneously. Depending on the complexity of the system, the number of parameters could be high. An initial screening can help minimize the number of tests needed in later steps by identifying effect parameters that are statistically insignificant to the response outcome. DOE is a technique used across the sciences that is suggested at this stage to identify statistically significant effect parameters. DOE has been utilized as a method to dynamically configure a CR [2] and identify routing and protocol factors that impact the performance of ad hoc networks [15],[16]. In this analysis all effect parameters had statistical significance to the output response either as a main factor or as a part of a two-way factor, therefore this paper focuses more on the RSM analysis.

A. 2^k Factorial DOE

One can consider hardware and CE configuration parameters to consist of a variable value between a minimum and maximum range, designated as -1 and $+1$ and 0 for a nominal midpoint value. Given k factors it is unrealistic to be able to test every combination of k factors across each factor's range. The 2^k factorial DOE methodology tests the boundary settings of the factors as an initial screening to identify which parameters impact system performance, gauge impact magnitude and identify interactions between parameters. Figure 2 illustrates three parameters and boundary points consisting of the corners of a cube. 8 points are required in a three factor system. Face and center points, indicated in gray, are used in central composite design (CCD) RSM as discussed later.

The theory behind DOE is founded in Analysis of Variance (ANOVA) and is detailed in [17]. Fundamentally, each main factor setting, paired combinations, and the joint combination of all factors are analyzed to quantitatively determine the effect that each factor or combination of factors has on the output. This enables identification of the most important parameter with regards to affecting the output as well as identifies interaction effects between parameters that would be impossible to uncover when varying only one factor at a
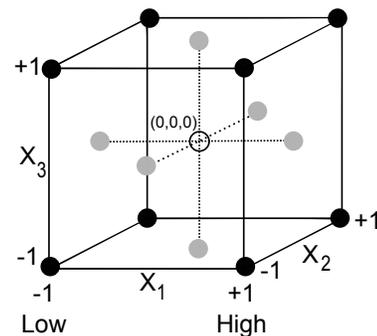

Fig. 2.   Data points for 2^3 Factorial and Response Surface Method

time. An ANOVA procedure seeks to partition the variation in the output response into components that tie to the sources of the variation. The procedure breaks the total variation in the output response into the part due to random error and the part due to changes in the each of the individual factors as well as the combinations of factors. A linear model of the system response as a function of the input factors is developed through the method of least squares and linear regression. This model will be expanded to incorporate curvature in the RSM procedure.

B. RSM

The RSM builds upon the initial factorial DOE screening by adding test configurations beyond the factorial design which contribute to the development of a second order quadratic model capable of identifying curvature in the response. The quadratic approximation of the response for a three-effect parameter system is shown in Equation (1). Y is the output response under study, X_i is input factor i, β_0 is the intercept, β_i is the coefficient for X_i, and ϵ is the statistical error. While this model is still only an approximation, it provides a foundation for the pursuit of minimization of LIB metrics and maximization of HIB metrics. A basic summary of the theory behind solving for the coefficients is presented here. For more details refer to [18].

$$
\begin{aligned}
Y =\ & \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{11} X_1{}^2 + \beta_{12} X_1 X_2 \\
& + \beta_{22} X_2{}^2 + \beta_{13} X_1 X_3 + \beta_{23} X_2 X_3 + \beta_{33} X_3^2 + \epsilon
\end{aligned} \tag{1}
$$

Equation (1) is a multiple-regression model and can be rewritten into the matrix form shown in Equation (2), where $\mathbf{y}$ is an $n \times 1$ vector of the responses where n is the number of responses, $\mathbf{X}$ is $n \times k$ matrix of the factors where k is the number of factors, and $\boldsymbol{\beta}$ is a $k \times 1$ vector of the regression coefficients. A least squares solution can be identified by Equation (3). An estimation of the regression equation is Equation (4) which can be written in a final form of the estimation as shown in Equation (5).

$$
\mathbf{y} = \mathbf{X}\boldsymbol{\beta} \tag{2}
$$

$$
\hat{\boldsymbol{\beta}} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{y} \tag{3}
$$

$$
\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}} \tag{4}
$$

$$
\hat{y_i} = \hat{\beta}_0 + \sum_{j=1}^{k} \hat{\beta}_j x_{ij} \quad i = 1, 2...n \tag{5}
$$

Section IV-B identifies the coefficients for models of the three responses under study based upon analysis of the experimental results. Typically, a three dimensional graph of the response surface is combined with analytical results to draw conclusions.

The experimental design, or the choice of what input configurations to implement, has important bearing on the effectiveness of fitting the response surface. Several methods exist for how to approach the experimental design. This paper utilizes the Box-Wilson CCD approach. Other popular techniques include the Box-Behnken design that collects data points along the midpoints of the cube edges.

C. Drawbacks / Limitations

These techniques are based on statistical models conceived from empirical data. Given the stochastic nature of wireless communications, all collected data contain a level of uncertainty. Models generated from the RSM, while better approximations than first-order DOE representations, are still only estimates. Furthermore, the limited data points utilized in the 2^k factorial and the RSM, regardless of the particular method used (CCD versus Box-Behnken), may fail to capture key configurations that affect overall system response. These methods suffer from the potential of uncertainty in collected data combined with limitations in the RSM. A common criticism of the proposed methods is that they do not identify true optimality; rather, they point the user towards the potential ranges for improved performance. The black box methodology does not compare the CE decisions against a theoretical optimum. Rather, it serves to provide a performance snapshot essential to draw conclusions about system verification and evaluation.

IV. Calibration of Receiver/Transmit Gain

A hybrid case-based reasoner-genetic algorithm CE designed for a railway application is in implementation on a universal serial radio peripheral (USRP) SDR. Performance measures include BER and Packet Error Rate (PER).

Transmit power amplitude is a reconfigurable parameter that the CE may modify during operation. Initial testing showed that two other initialization elements on the USRP, transmitter (Tx) gain and receiver (Rx) gain, impacted the useful range of available output power measured at minimum and maximum settings of the amplitude. Some combinations of Rx/Tx gains showed limited change in output power regardless of whether the amplitude was set to the minimum or maximum setting. Determining both gain settings represents a white box testing calibration step. Proper operation of the CE requires this calibration prior to performing overall black box system testing. The goal is to identify a general range for Rx and Tx gain settings that provide adequate change in power output across the available amplitude settings without negatively affecting performance metrics such as the BER and PER.

A. Experimental Procedure

The empirical procedure and associated results illustrate the response surface screen, development of polynomial response model, and optimization against metrics portion of the Flowchart in Figure 1. A Tx/Rx USRP1 link utilizing the RFX900 daughter card transmitting at 920 MHz was set up at a fixed distance of 35 feet. The *benchmark.ty* script enabled transmission of a photograph in bitmap format modulated with Gaussian minimum shift keying (GMSK). One hundred packets, consisting of a 1,500 bytes each, were sent across the link during each test creating a total $1.2e^6$ bit transmission. A spectrum analyzer recorded average output power from the Tx

during transmission, while a comparison between the received bits and sent bits provided an estimation of the BER and PER. In some cases, no link could be established, and the BER was listed as 0.5 while the PER was listed as 1.

A 2^3 CCD RSM design was followed where the three parameters under consideration were Tx gain, Rx gain, and the USRP transmit amplitude. Table II lists the response factor, the associated coded variable, and the min, max and nominal values. The upper and lower bounds of the effect parameters are: 1) Rx gain(dB): [0,50]; 2) Tx gain(dB): [0,25]; and 3) Amplitude(V): [0.05, 0.99]. These ranges are represented in the DOE methodology as -1,+1. Nominal values, represented as 0, were 25 for Rx gain, 12.5 for Tx gain, and 0.5 for Amplitude. Table III shows the configuration settings for each test run. For example, point (-1,-1,-1) represents the configuration (Rx gain=0, Tx gain=0, Amplitude=0.05) while the point (0,0,0) represents (RxGain=25, TxGain=12.5, Amplitude=0.5). The listed settings are configurations that are entered into GNU radio, the software controller for the USRP, at the beginning of transmission.

BER, PER and Output Power were recorded for each test. Two replicates of the CCD design shown in Table III were implemented. Statistical analysis was performed using the model fitting capability of *SAS JMP* version 9.0. An effect leverage standard least squares fit was followed. Models for each response output matching the format of Equation (1) are reported as well as a measure of how well the model fits the collected data known as R^2. R^2 estimates how much of the variation around the mean is caused by the model rather than random error. A perfect fit would have an R^2 value of 1.

TABLE II
MAPPING FACTORS TO CODED VARIABLES

Factor	Code	-1	0	1
Tx Gain(dB)	X_1	0	12.5	25
Rx Gain(dB)	X_2	0	25	50
Tx Amplitude(V)	X_3	0.05	0.5	0.99

TABLE III
CCD RESPONSE SURFACE DESIGN

Test	TxGain	RxGain	Transmit Power
1	1	-1	-1
2	0	0	0
3	-1	-1	-1
4	-1	1	1
5	-1	1	-1
6	0	0	1
7	1	1	1
8	1	0	0
9	0	-1	0
10	0	1	0
11	0	0	-1
12	1	-1	1
13	-1	0	0
14	1	1	-1
15	-1	-1	1

B. Results

The results presented here focus on the RSM analysis of this system, specifically response model estimates, statistical fit of the models, and graphical representation of output responses.

Table IV lists each output response and the identified coefficients that define a prediction model for the response as a function of the input factors. These coefficients map to Equation (1). Also listed is the R^2 value of each response model that corresponds to a measure of statistical fit of the RSM model. In general, R^2 measures the percentage of the variation of the response around the mean identified from the regression analysis. Graphically this is represented by Figure 3 which illustrates the actual values of PER on the y-axis and the estimated values of PER on the x-axis. The dotted horizontal line is the mean, and the solid diagonal line represents a perfect fit where actual values equal predicted values. The vertical distance from the data point to the perfect fit line is called the residual and is used in the calculation of R^2. Refer to [18] for more details on the mathematical theory behind the model fitting and statistical analysis. The *JMP* analysis calculated that the models for BER and PER achieved an R^2 value of 0.82522 and 0.844152 respectively. The model prediction for output power obtained an R^2 value of 0.930937.

A benefit of RSM is the capability to graph the response output in three dimensions enabling the analyst to visualize how the output response changes across varied settings of effect factors as shown in Figure 4. Only two effect factors at a time can be graphed for a single response while the other effect factors are kept constant. In this case, the effect parameter of transmit amplitude is kept constant, while TxGain and RxGain are varied. A surface contour, similar to a topographic map, can be generated to provide a two dimensional view of the response profile. This is equivalent to viewing the response surface from above.

The contour profile of this response surface has been placed underneath the surface to assist in identifying where the minimum ranges are. If one desires a target for a response
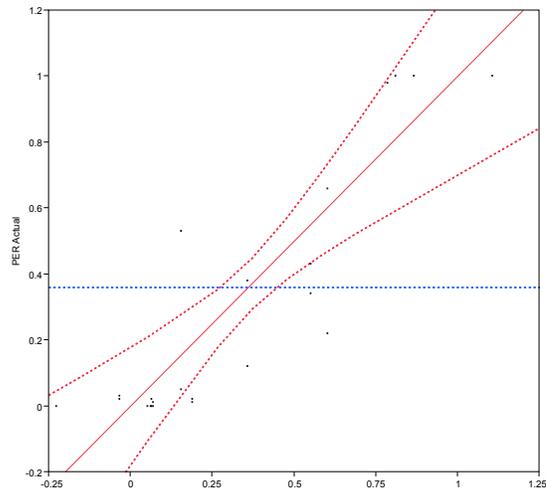

Fig. 3. PER Actual by Predicted Plot

TABLE IV
RSM MODEL COEFFICIENTS AND R^2 MEASURE OF FIT

Response	β_0	β_1	β_2	β_3	β_{11}	β_{12}	β_{22}	β_{13}	β_{23}	β_{33}	R^2
BER	-0.08301	0.645543	-0.03084	0.007735	0.106261	0.132056	0.123761	0.081327	0.210156	0.108761	0.825222
PER	-0.22778	-0.008	-0.1105	-0.0015	0.287222	0.173125	0.304722	-0.33125	0.191875	0.289722	0.844152
Output Power	-20.8689	2.6211.02	11.02	8.335	-2.38889	3.24375	-6.53889	3.4125	-3.68125	-7.11389	0.930937

output rather than the absolute minimum, then contour profiles illustrate how there exist multiple sets of solutions to meet the desired target.

Contour profiling is a useful tool when attempting to overlay several response outputs at at time. By overlaying multiple profile's contour responses on top of another, one can identify desired regions based on combinations of output responses. Intersections of desired contours can show regions where several responses meet desired values.
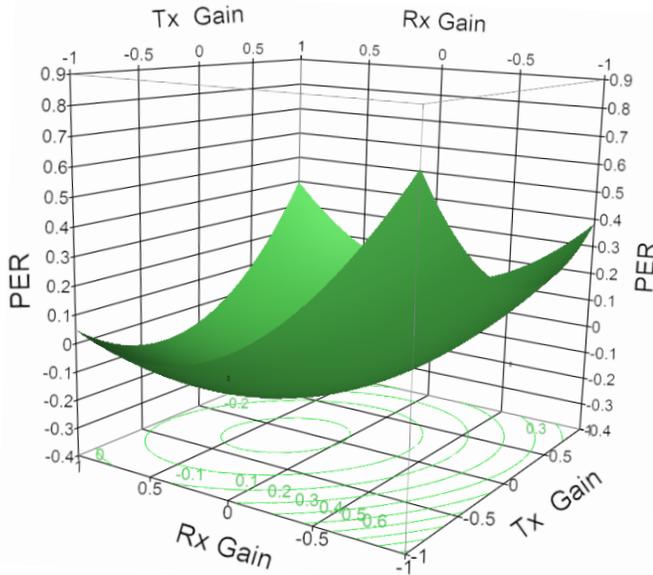

Fig. 4. PER Response Surface

V. SUMMARY AND NEXT STEPS

This paper presented an abstract framework for approaching the testing of CR systems. This framework included a systematic approach to setting internal configuration parameters based on statistical methods as well as an overall system testing method. These methods support the realistic constraints placed upon testing and validation that often include limited time, manpower, and the need to minimize costs. An illustrative example of parameter tuning for an SDR utilized in a CE was presented that utilized 15 data points. Next steps include expanding the application of this framework to more internal CE configuration parameters and performing full system level tests.

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REFERENCES

[1] T. W. Rondeau, "Application of artificial intelligence to wireless communications," Ph.D. dissertation, Virginia Tech, Blacksburg, 2007.

[2] T. Weingart, D. Sicker, and D. Grunwald, "A predictive model for cognitive radio," in *Military Communications Conference, 2006. MILCOM 2006. IEEE*, 2006, pp. 1–7.

[3] "Measurements of radio frequency spectrum characteristics mil-std-449d," Department of Defense, Tech. Rep., 1973.

[4] Y. Posherstnik, M. Totaro, T. Mai, and J. Molnar, "Testing of policy-based dynamic spectrum access radios," in *Military Communications Conference*, 2010.

[5] H. Arslan, "Testing and measurement of cognitive radio and software defined radio systems," in *SDR Forum Technical Conference*, 2007.

[6] G. Jue and B. Cutler, "A design-to-test methodology for sdr and cognitive radio," in *Aerospace and Defense Symposium*, 2010.

[7] S. K. Jones, T. W. Phillips, H. L. V. Tuyl, and R. D. Weller, "Evaluation of the performance of prototype tv- band white space devices (phase ii)," FCC, Tech. Rep. FCC/OET 08-TR-1005, 2008.

[8] "Spectrum sharing innovation test-bed pilot program," NTIA, Tech. Rep., 2009. [Online]. Available: http://www.ntia.doc.gov/frnotices/2006/spectrumshare/comments.htm

[9] J. Ziegler, "Sdr forum: Test and measurement task group: Ieee 802.11-09/1123r0," Wireless Innovation Forum, Tech. Rep., 2009.

[10] Y. Zhao, S. Mao, J. Neel, and J. H. Reed, "Performance evaluation of cognitive radios: Metrics, utility functions, and methodology," in *Proceedings of the IEEE*, vol. 97, no. 4, April 2009.

[11] M. McHenry, E. Livsics, T. Nguyen, and N. Majumdar, "Xg dynamic spectrum access field test results," *IEEE Communications Magazine*, vol. 45, no. 6, 2007.

[12] S. Jones and T. Phillips, "Plan for tests of prototype personal/portable tv white space devices (phase ii)," FCC, Tech. Rep., 2008.

[13] B. Beizer and J. Wiley, "Black box testing: Techniques for functional testing of software and systems," *Software, IEEE*, vol. 13, no. 5, p. 98, Sep. 1996.

[14] R. Jain, *The Art of Computer Systems Performance Analysis: Techniques for Experimental Design, Measurement, Simulation, and Modeling*. Wiley New York, 1991.

[15] C. Barrett, A. Marathe, M. V. Marathe, and M. Drozda, "Characterizing the interaction between routing and mac protocols in ad-hoc networks," in *Proceedings of the 3rd ACM international symposium on Mobile ad hoc networking & computing*, ser. MobiHoc '02. New York, NY, USA: ACM, 2002, pp. 92–103. [Online]. Available: http://doi.acm.org.ezproxy.lib.vt.edu:8080/10.1145/513800.513812

[16] K. Vadde and V. Syrotiuk, "Factor interaction on service delivery in mobile ad hoc networks," *Selected Areas in Communications, IEEE Journal on*, vol. 22, no. 7, pp. 1335 – 1346, 2004.

[17] D. C. Montgomery, *Design and Analysis of Experiments*. John Wiley & Sons, 2001.

[18] N. Bradley, "The response surface methodology," Master's thesis, Department of Mathematical Sciences Indiana University South Bend, 2007.