# A Novel Interpolated Tree Orthogonal Multiplexing (ITOM) Scheme with Compact Time-Frequency Localization: an Introduction and Comparison to Wavelet Filter Banks and Polyphase Filter Banks


fred harris
San Diego State University
fred.harris@sdsu.edu

Erik Kjeldsen
Scientific Research Corporation
ekjeldsen.@scires.com



## ABSTRACT

A cascade of half-band filters form a branching tree whose branch responses constitute an orthogonal set of wave-shapes that enable multi-channel modulation. The filters and architecture in this process, also known as a wavelet transform, perform the dual tasks of shaping and interpolating. A variant of this process, called Interpolated Tree Orthogonal Multiplexing (ITOM), has a tree structure that interpolates sequences formed and shaped external to the tree. The branch responses of this tree also constitute an orthogonal set of wave-shapes that enable multi-channel modulation. The unique attribute of this signal set is their spectra have compact support spanning contiguous spectral regions. This permits us to vacate spectral intervals in the modulation process to bracket occupied spectral regions when scavenging spectrum. The signal set also shares a wavelet transform attribute in that data signals of different bandwidth can be mixed by entering the tree at different branch levels. We introduce ITOM and compare pertinent features with the Wavelet transform and the polyphase filter bank.


## 1. INTRODUCTION

Many of us are familiar with the discrete wavelet packet transform (DWPT). One form of the DWPT uses a pair of prototype half-band low-pass and high-pass filters in a repeated application of 1-to-2 up-sampling and filtering to pass one while suppressing the remaining spectral replicate. The structure of this familiar tree and its dual form is shown in figure 1.

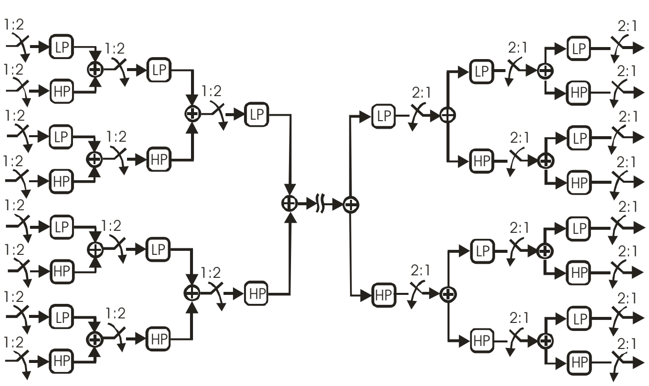


**Figure 1.** DWPT Binary Tree Modulator and Demodulator

The multiple impulse responses between each input port and the single tree root are mutually orthogonal and can separated by their corresponding matched filters which are implemented efficiently in the dual tree. Thus the DWPT supports a multi-channel communication system with flexible modulation options [1]. Usually, we have no interest in the spectra associated with each of the separate impulse responses but rather are concerned with the composite spectrum for its spectral structure in a communication channel. Lately there has been interest in the details of the separate spectra of the DWPT components. This interest is motivated by the desire to share spectral resources by an overlay of composite spectra containing intentional spectral gaps to bracket occupied spectral regions while accessing adjacent unoccupied spectra. This process is called spectral scavenging or opportunistic use [2].

We now take a quick look at the detailed spectra of individual components of the DWPT signal set. Figure 2 shows the spectra of the prototype half-band low-pass and high-pass filters. These filters are high quality square-root Nyquist filters (not cosine tapered) with small in-band ripple and deep out of band attenuation to support zero ISI at the matched filter output. The prototype filter pair (LP and HP) are seen to exhibit distinct, well defined pass-band and stop-band intervals.

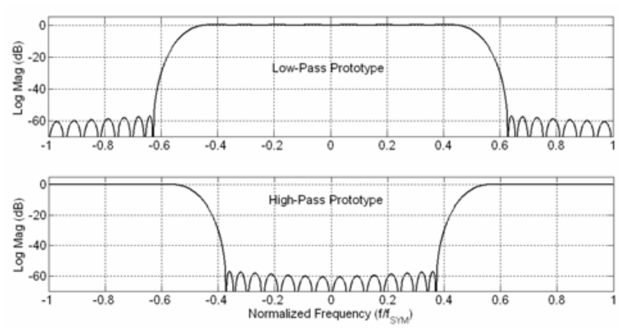


**Figure 2.** Spectra: DWPT Low-Pass and High-Pass Filters

Figure 3 shows the output spectra resulting from the first stage of up sampling and filtering. The subplots show the LP-LP, LP-HP, HP-LP and HP-HP spectra respectively. We note that the LP-LP and the LP-HP spectra are simple up sampled version of the LP and HP spectra obtained by rejecting the other spectral component. We also see that the HP-LP and HP-HP are different and that the up-sample HP spectra are band-pass spectra which are partially passed and partially rejected by the LP and HP filters. There is nothing wrong with this in the multi-channel application we just wanted to illustrate the spectral carving due to the interaction of the filters in the cascade. Where the spectral carving does affect us is in the process of accessing the channel like properties of the DWPT.

If we examine spectra from signal components deeper into the binary tree we see that many terms preserve their channel like structure but, due to the carving, others terms do not. In particular examine figure 4 where we show the spectra of selected tree paths at the third branch level of the tree. Notice the carved spectrum of subplot (4,2,6) and the high level spectral remnants of subplots (4,2,4) and (4,2,8). These terms may affect our ability to control the spectral occupancy of bands we have selected to vacate in order to bracket occupied regions when we scavenge spectra. We might say that while the DWPT offers an OK approximation to a spectral partition, it does not offer a great approximation.

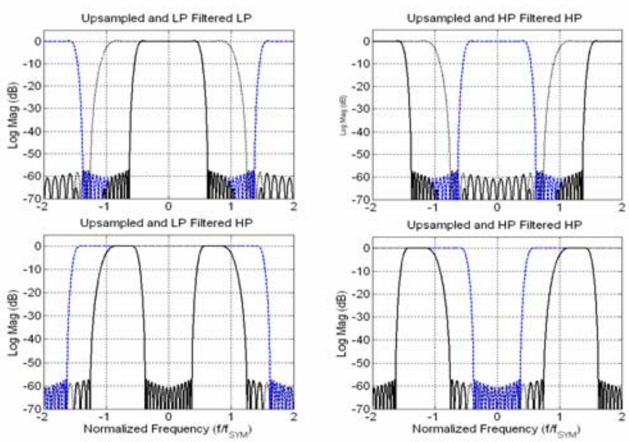


**Figure 3.** Spectra of Up-Sampled and Filtered LP-LP, LP-HP, BP-LP and BP-HP DWPT Filters

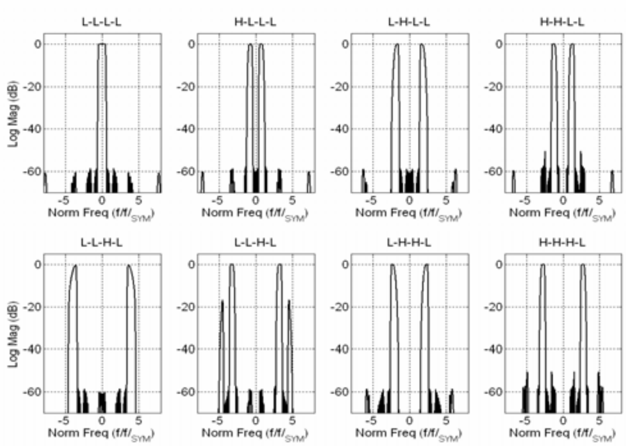


**Figure 4.** Spectra of Selected Equivalent Filters after Third Stage of Up-Sampling and Filtering in DWPT

Our final illustration of the unsuitability of the DWPT signal set to support spectral notching for spectral bracketing is seen in figure 5. Here the time series of a 64-channel DWPT has been assembled with select channels left vacant to avoid interfering with already occupied spectral regions. Note that disabling a specific signal component may not present a companion spectral notch in the enabled signal spectra.

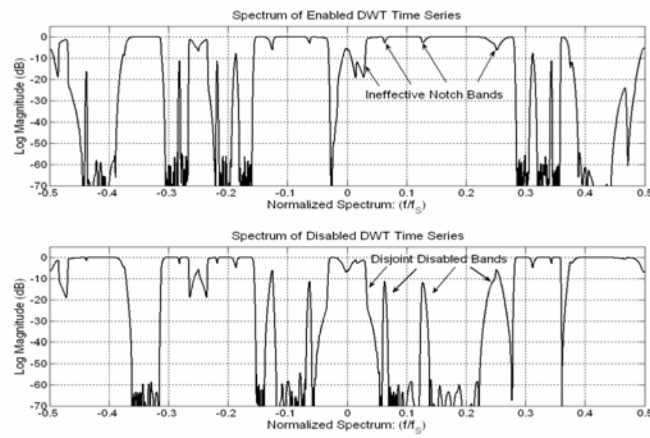


**Figure 5.** Spectra of Enabled and Disabled Spectral Bands of 64-point DWPT

## 2. INTERPOLATED TREE STRUCTURE

We now introduce an alternate to the binary tree DWPT we have chosen to call Interpolated Tree Orthogonal Multiplexing (ITOM). We note that in the DWPT, the filters perform the dual tasks of both shaping the spectra and of interpolating the time series. In this alternate structure we separate the two tasks. We perform the spectral shaping external to the tree structure in a bank of standard shaping filters and deliver their outputs to the tree for interpolation and structured spectral translation by the up-sampling induced aliasing [3]. In this structure the prototype baseband spectral shape is preserved during the interpolation and aliasing operations.

A block diagram of the alternate process is shown in figure 6. Here we see the up-sampling shaping filters at the input ports to the process. The signals from the input shaping filters are iteratively up sampled and filtered in a modified bank of half-band filters which by appropriate selection of half-band filters in each cascade path alias the input spectra to successive center frequencies of the composite spectrum. Note here that one of the shaping filters operates at twice the input rate as the others in the tree and its contribution to the composite signal is inserted in the tree at the second branch level. It is possible to insert signals with various data rates at appropriate branch levels to obtain a composite signal composed of a wide range of bandwidths. As we will see shortly, the leaving of one or more input branches vacant will have the effect of notching the associated compact spectral interval.

 

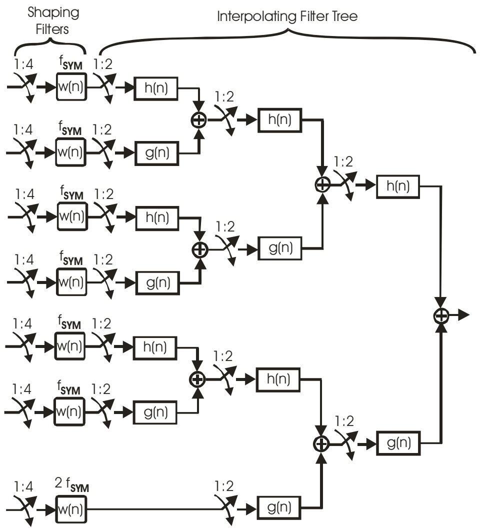


**Figure 6.** Modulator of ITOM Interpolated Tree Structure

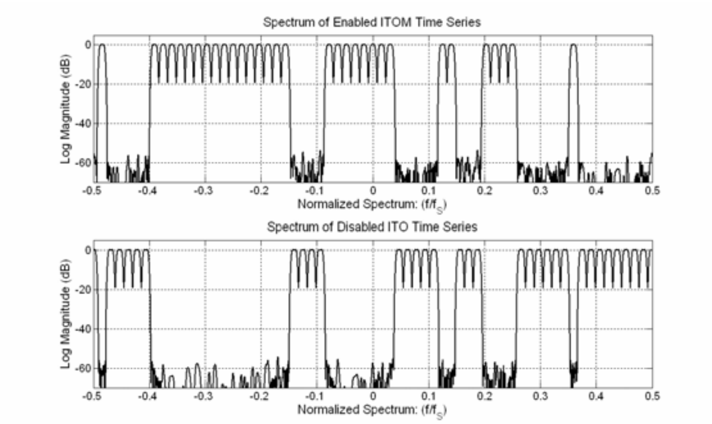


Figure 7. Spectra of Enabled and Disabled Spectral Bands of 64-point ITOM

We indicated earlier that the ITOM process delivers the spectra formed by the input shaping filters to specific center frequencies by sequentially forming aliased spectral replicas by up-sampling and selective half-band filtering the desired replica. Figure 7 illustrates how successful this process has proven to be. Here we see the spectrum of individual channels and of multiple adjacent channels spanning specific frequency bands. We also see the superior ability of the ITOM process to offer notched spectral intervals in the modulation band of the process by vacating specific channels. Be sure to compare figure 7 to figure 5.

Figure 8 illustrates how the non vacant spectral intervals can be assigned occupancy in the ITOM process. In the upper half of the figure signals of different bandwidth are modulated by the input shaping filters at rates appropriate to their bandwidths and are merged and further up sampled by the ITOM tree when inserted at the correct branch level of the tree. The lower half of the figure illustrates how multiple equal bandwidth signals formed by shaping filters operating at the lowest branch level of the interpolating tree branches can be mixed with a wider bandwidth inserted time series.

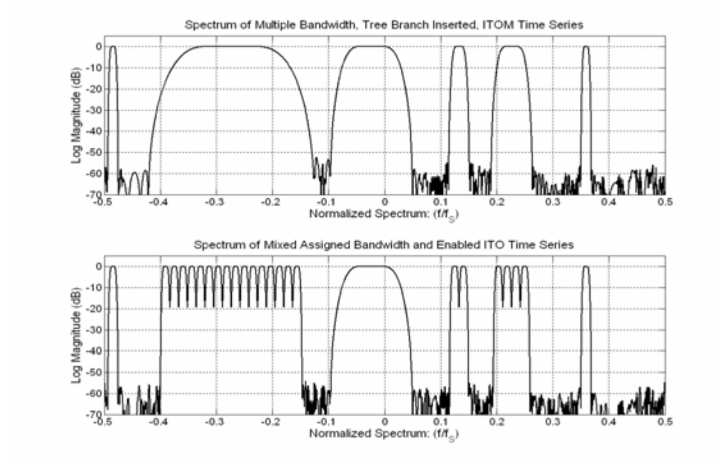


**Figure 8.** Spectrum of ITOM signal for Multiple Bandwidth, Tree Branch Inserted, Signals and for Mixed Inserted and Channelized Signals in Disjoint, Unequal Spectral Width Intervals

## 3. POLYPHASE CHANNELIZER

An obvious third alternate for spectral channelization is the standard multi-channel channelizer. The most efficient implementation of this DSP engine is the polyphase channelizer [3]. The polyphase channelizer is formed by a cascade of an inverse DFT and a polyphase partition of a prototype low-pass filter. The DFT is usually implemented by an FFT for its computation efficiency. At the highest level description, the filter defines the impulse response and the frequency response of the channelizer and the DFT defines the center frequencies of the channels.

The polyphase channelizer can operate in the two modes described in the previous two sections. In the first mode, the impulse response of the polyphase partitioned prototype filter performs the shaping of the individual time series formed by the up sampling FFT. To assure zero intersymbol Interference (ISI), the prototype shaping filter must be a square-root Nyquist pulse prior to its polyphase partition. This mode is known as *shaped OFDM*. In the second mode, the impulse response of the polyphase partitioned prototype filter performs the bandwidth limiting function of a channelized receiver. In this mode, the inputs to polyphase channelizer are shaped by external shaping filters prior to the channelizer. When the polyphase filter is omitted from the processing path the envelope defaults to a rectangle gating function. The spectrum associated with the rectangle envelope is the ubiquitous $\sin(\pi f/NT)/(\pi f/NT)$ and the system so formed is the standard OFDM modulator.

When operating in the shaped OFDM mode the DFT performs two simultaneous tasks, which are up-sampling and complex sinusoid signal generation. It converts a single input sample presented to input frequency bin "k" of an Npoint DFT into an output sequence containing N samples of a complex sinusoid exhibiting “k” cycles per interval of length N. Since the DFT is a linear device, it can simultaneously form a time series as the sum of multiple orthogonal sinusoids. The output of the DFT is processed by the polyphase partition of a prototype low-pass filter. The weights of the filter are imposed as an envelope to the DFT sinusoids and thus shape the spectral response of the equivalent filters.

The block diagram of a 16 channel shaped OFDM modulator and demodulator is shown in figure 9. Here the modulator outputs successive blocks of length N and the demodulator accepts successive blocks of length N. We gather insight to the shaped OFDM signal by examining the time and frequency response of the modulator-demodulator block. Figure 10 shows the impulse response and frequency response of the shaped OFDM equivalent filter bank. Here we see that the time envelope of the matched filter has zero response at the OFDM symbol rate. Thus the time samples of a single OFDM symbol are orthogonal to successive symbols. On the other hand, the spectra of adjacent frequency bins are seen to overlap their immediate adjacent neighbors but not their neighbors separated by more than one frequency bin. Thus the shaped OFDM exhibits cross talk or ACI (adjacent channel interference) to the immediate two spectral neighbors.

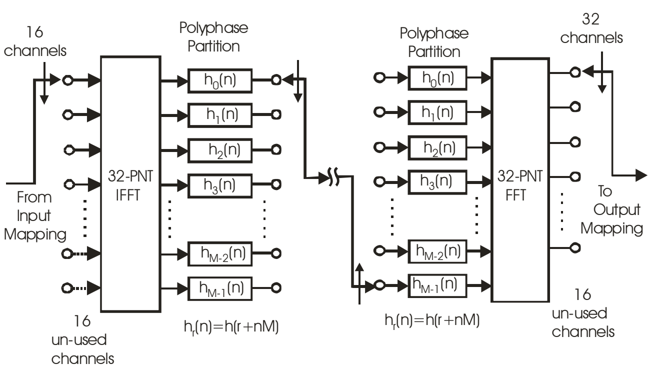


Figure 9. Shaped OFDM Modulator and Demodulator

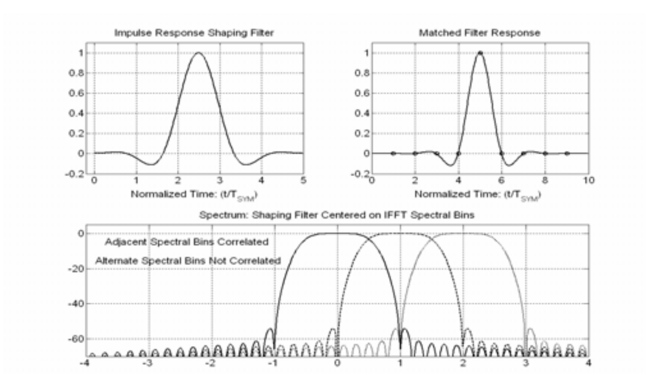


Figure 10. Impulse Response, Matched Filter response and Spectral Response of Shaped OFDM Demodulator

One possible response to the spectral overlap is to skip adjacent channels and only use alternate frequency bins. Operating in this mode makes the shaped OFDM a cousin of the standard channelizer with non overlapped bandwidths. The spectrum of the shaped OFDM alternate bin modulator is shown in figure 11 along with the spectral response of standard OFDM using alternate frequency bins. As expected the shaped OFDM spectrum exhibits bounded support due to the contained bandwidth of the shaping filters. Note that standard OFDM exhibits high out-of-band side-lobe levels due to the rectangle (gated) envelope of the modulation process. The shaped OFDM exhibits the side-lobe levels of the prototype shaping filter, the one presented in figure 10.

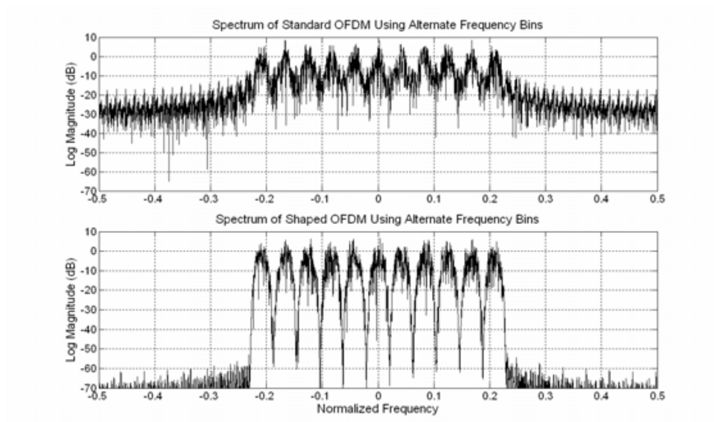


Figure 11. Spectra of Shaped OFDM and Standard OFDM Using Alternate Frequency Bins.

A second approach to the cross talk between adjacent center frequency channels of the shaped OFDM is a process known as offset OFDM. In this process successive shaped OFDM symbols are overlapped by 50%, hence the name offset OFDM [4]. We don’t have time or space here for a full exposition on offset OFDM but we do offer a short description. In shaped OFDM the cross talk is suppressed by partitioning the real and imaginary components into the separate overlapped symbols. The overlap between adjacent spectral bins causes cross talk between the real components and between the imaginary components of the signal in adjacent frequency bins. There is however, no cross talk between the real component of one bin and the imaginary component in the overlapped neighbor bin. The data partition uses this relationship. In the first shaped OFDM symbol the even indexed bins contain real components and the odd indexed bins contain imaginary components, these of course are mutually orthogonal! In the second of the overlapped shaped OFDM symbol the even indexed bin contain imaginary components and the odd indexed bins carry real components, these too are mutually orthogonal. We leave it to the reader to verify that, subject to the constrained bin assignments, the even and odd symmetric time signals in the real and imaginary components of the overlapped OFDM symbols are also orthogonal. The shaped OFDM offers a very efficient use of the available spectrum, permitting overlap of the adjacent channelized frequency bands, as does the non shaped OFDM, but by virtue of the shaping does not rely on the equally spaced spectral zeros of the overlapped spectral sinc functions to obtain mutual orthogonality.

As mentioned earlier, we can also use the polyphase channelizer as a simple filter bank carrying waveforms shaped by external shaping filters. This is so straight forward; we will not expand on the implementation [3].

## 4. COMPARING THE CHANNELIZERS

The three channelizers we examined in terms of their spectral occupancy and their degree of orthogonality are the Discrete Wavelet Packet Transform (DWPT), the Interpolated Tree Orthogonal Multiplexing (ITOM), and Polyphase Channelizer or Shaped OFDM. All we require of a multichannel communication system is that the time series of the separate channels are mutual orthogonality. The DWPT satisfies this requirement. In the spectral scavenging scenario, we require one additional property. We require that we can partition the spectra of the signal into a set of unequal width, non adjacent spectral intervals. We do this in order to occupy a frequency band in unoccupied intervals that bracket occupied intervals already in the spectral band.

We find that the individual time series of the DWPT do not exhibit spectra that would permit the spectral partition we require for spectral scavenging. Thus we are drawn to traditional channelizers with signal sets that naturally exhibit spectra on distinct compact supports or with localized spectral width and position. The ITOM modulator and the polyphase channelizer each exhibit this property. They both are capable of bracketing occupied frequency bands by omitting the channels spanning the not allowed frequency bands. Both are also able to tile the remaining, unoccupied, spectral intervals with equal bandwidth signal spectra.

**TABLE 1.** CHANNELIZER GRADE SHEET

| | ITOM | DWPT | OFDM | S-OFDM | O-OFDM |
|---|---|---|---|---|---|
| ISI | A | A | A | A | A |
| ACI | A | A | A | B | A |
| BW-Efficiency | A | A | A | B | A |
| Notched Bands | A | F | F | A | A |
| Mixed BW | A | A | F | C | C |
| Complexity | A | A | A | B | B |
| TOTAL GRADE | A | B | C | B | B |

The tree based ITOM modulator offers one additional option to tile the unoccupied spectral regions. The ITOM can merge signals of different bandwidth in the available unoccupied, non adjacent, unequal width, frequency band. This that attribute is a unique capability not offered by other multi-channel modulation schemes! We offer a comparison of the attributes and properties of the different multi-channel modulation options we have addressed in this paper plus one we alluded to, conventional OFDM. This comparison is shown compactly as a grade distribution in table 1. The attributes we addressed are ISI, adjacent channel interference (ACI), Bandwidth efficiency, implementation complexity, and mixed bandwidth capability. We assess that ITOM has the highest grade!

## 5. CONCLUSIONS

We have introduced a new modulation format we call Interpolated Tree Orthogonal Multiplexing (ITOM) which inherits some of the properties of the Digital Wavelet Transform (DWPT) as well as exhibits some of the properties of the Shaped Orthogonal Frequency Division Multiplexing (S-OFDM) and a variant known as Offset-OFDM (O-OFDM). We believe the ITOM holds great promise for certain communication system applications and we are actively working towards deployment and demonstration of a prototype system. Between the two authors, we have implemented or are currently developing all the processing blocks required to demonstrate an operating system. The processing blocks include system synchronization, timing and carrier, equalization, and Automatic Gain Control (AGC).

## 6. REFERENCES


[1] Erik Kjeldsen and Alan Lindsey, "Enabling Time-Frequency Agility: Wavelet Packet Modulation in Practice", Proc. of SPIE, Vol. 5207, Wavelets: Applications in Signal and Image Processing, Nov. 2003, pp. 280-291.

[2] Carlos Cordeiro, Kiran Challapali, Dagnachew Birru, and Sai Shankar, "IEEE 802.22 The First Worldwide Wireless Standard based on Cognitive Radios", First International Symposium on New Frontiers in Dynamic Spectrum Access Networks, 2005.

[3] fred harris, "Multirate Signal Processing for Communication Systems", Prentice-Hall, Upper saddle River, NJ, 2004.

[4] Helmut Bölsckei, Pierre Duhamel, and Rima Hleiss, "Design of Pulse Shaping OFDM/OQAM Systems for High Data Rate Transmission over Wireless Channels", Proc. of IEEE Int. Conf. on Communications (ICC)., Vancouver B.C., Canada, pp 559-564, June 1999.