GAME MODELS FOR COGNITIVE RADIO ALGORITHM ANALYSIS

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ABSTRACT

Game theory is a promising approach for analyzing the interactions of adaptive and cognitive radios. This paper describes how the components of the cognition cycle map into normal form game model and describes standard game theory techniques for investigating four important issues that game theory should address: steady state existence, steady state identification, convergence and steady state optimality. This paper then describes three game models that can aid the analyst in addressing these issues and concludes with a discussion of additional ways in which the use of game models aids the analysis and development of cognitive and adaptive radios.

1. INTRODUCTION

Cognitive radio is frequently extolled as a platform for implementing dynamic distributed radio resource management algorithms. In the envisioned scenarios, radios will react to measurements of the network state and change their operation according to some goal driven algorithm. However, when the adaptations of the radios also change the network state, an interactive decision process is realized.

In light of this interactive decision process, before fielding any distributed algorithm, it would be valuable to determine the following: steady state existence and characterization, steady state efficiency, and algorithm convergence properties. These properties can be established through extensive simulation and field testing, or they could be established analytically with game theory.

Several authors have previously commented on the suitability of game theory for analyzing networks with interactive decision processes, [1][2]. Game theory has been used to establish the existence of steady states [3] [4], characterize the steady-states [5], predict steady-state efficiency [6], and establish convergence properties [7]. However, except for a handful of papers, e.g., [7] and [8], these results have been established on an ad-hoc basis so the results of their game theoretic analysis cannot be readily extended to different networks and algorithms. Thus, analysis must start fresh each time, significantly increasing the amount of time and effort required to establish new results, diminishing many of the advantages that analysis provides with respect to simulation.

Instead of effectively reinventing the wheel for each new network and algorithm, this paper proposes the application of game models to the analysis of cognitive radio algorithms. By adopting a model-based approach, analytical effort can be more efficiently spent on establishing results for the game models and game model identification criteria. This paper identifies several attractive game models, notably potential games, supermodular games, and repeated games. Properties related to steady-state existence, characterization, efficiency, and convergence are described and methods for model identification are given. Example applications of each model are cited and drawn from various aspects of radio resource management. As part of this discussion, this paper identifies and describes broader game theoretic concepts applicable to these models that will be important to establishing the suitability of distributed algorithms.

2. COGNITIVE RADIO AND GAME THEORY

This section provides a brief review of cognitive radio, game theory and the application of game theory to cognitive radio.

2.1 Cognitive radio

Cognitive radios are adaptive radios that are aware of their capabilities, aware of their environment, aware of their intended use, and able to learn from experience new waveforms, new models, and new operational scenarios. The operation of a cognitive radio is frequently envisioned as being defined by the cognition cycle shown in Figure 1.

In the cognition cycle, a radio receives information about its operating environment (**Outside world**) through direct observation or through signaling. This information

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is then evaluated (**Orient**) to determine its importance. Based on this valuation, the radio determines its alternatives (**Plan**) and chooses an alternative (**Decide**) in a way that presumably would improve the valuation. Assuming a waveform change was deemed necessary, the radio then implements the alternative (**Act**) by adjusting its resources and performing the appropriate signaling. These changes are then reflected in the interference profile presented by the cognitive radio in the **Outside world**. Throughout the process, the radio is using these observations and decisions to improve the operation of the radio (**Learn**), perhaps by creating new modeling states, generating new alternatives, or creating new valuations. To a large extent, the primary difference between a cognitive radio and an adaptive radio is the cognitive radio's ability to learn.

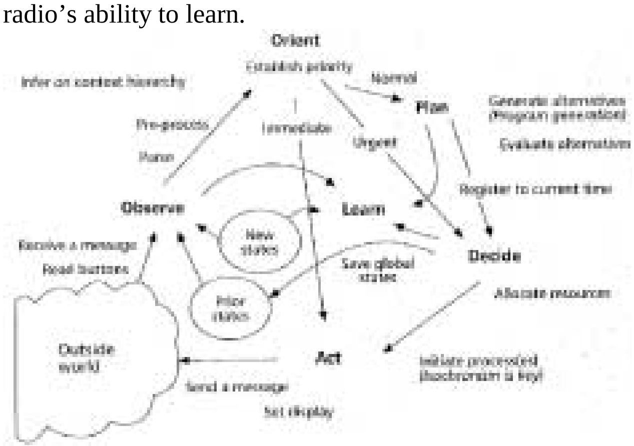

Figure 1 Cognition Cycle [9]

While much research is required to implement a cognitive radio, this paper is particularly concerned with the interaction of decisions in the outside world. Loosely, whenever one cognitive radio changes its interference profile, the remaining cognitive radios in the network may be prompted to change their interference profiles as well. This interactive decision problem is one that game theory handles well.

2.2 Game theory

Game theory is a set of mathematical tools used to analyze interactive decision processes. The fundamental component of game theory is the notion of a game. When expressed in *normal form*, a game, $G = \langle N, A, \{u_i\} \rangle$, has the following three primary components.

1. A finite set of players (decision makers) typically denoted $N = \{1, 2, \ldots, n\}$.
2. An action space, A, formed from the Cartesian product of each player's action set, $A = A_1 \times A_2 \times \cdots \times A_n$.
3. A set of utility functions, $\{u_i\} = \{u_1, u_2, \ldots, u_n\}$, that quantify the players' preferences over the game's possible outcomes. Outcomes are determined by the particular action chosen by player i, a_i, and the particular actions chosen by all of the other players in the game, a_{-i}.

In the game, players are assumed to act in their own self-interest, that is to say, each player chooses its actions in such a way that increases the number returned from its utility function. Other games may include additional components, such as the information available to each player and communication mechanisms.

2.3 Applying game theory to cognitive radio

Examining again the cognition cycle shown in Figure 1, it is readily seen how the interactions of a network of cognitive (or adaptive) radios maps into a game.

Each node in the network that implements the decision step (making it a decision maker) of the cognition cycle is a player in the game. The various alternatives available to a node forms the node's action set, and the action space is formed from the Cartesian product of the radios' alternatives. A cognitive radio's observation and orientation steps combine to form a player's utility function. Loosely, the observation step provides the player with the arguments to evaluate the utility function, and the orientation step determines the valuation of the utility function.

Note that we have ignored the learning step of the cognition cycle. This is not an oversight nor indicative of a limitation of game theory. Rather, it is a limitation of the normal form game model. While the normal form game model is appropriate for any adaptive radio algorithm or for any cognitive radio adaptations that do not require learning, it is not appropriate for analyzing algorithms that learn. In this case, more advanced game models that incorporate learning processes, such as *Bayesian games* should be used. It should also be noted that game theory is not well suited to games where actions and objectives are well defined as may be the case when cognitive radios learn over time.

3. ANALYZING COGNITIVE RADIO

There are four questions that game theory should answer when analyzing an adaptive algorithm:

1. Does the algorithm have a steady state?
2. What are those steady states?
3. Is the steady state(s) desirable?
4. What restrictions need to be placed on the decision update algorithm to ensure convergence?

Most game theory application analyses focus on the first three questions and rarely address the fourth question. However, all four questions should be answered before implementing any cognitive radio algorithm.

3.1 Demonstrating that an algorithm has a steady state

For most game models, the game theoretic equivalent of a distributed algorithm's steady state is a *Nash equilibrium* (NE). An action vector (or alternative vector) a is said to be a NE iff (1) is satisfied.

$$u_i(a) \geq u_i(b_i, a_{-i}) \forall i \in N, b_i \in A_i \quad (1)$$

Restated, a NE is an action vector from which no player can improve its payoff when acting by itself. Without applying more complex game models, a game can be shown to have a NE by applying relevant fixed point theorems. While these fixed point theorems may appear to be quite complex to an unexperienced analyst, the most common application boils down to demonstrating that the following conditions are satisfied.

1. The player set is finite.
2. The action sets are closed, bounded, and convex. Note that intervals and Cartesian products of intervals satisfy this condition.
3. The utility functions are continuous in the action space and quasi-concave. Note that demonstrating that the utility functions are concave, perhaps through a second derivative test, is sufficient to establish quasi-concavity.

In reality, a very large number of algorithms satisfy these conditions, so demonstrating NE existence is not very insightful as there's almost a default assumption that there will be a steady state for a cognitive radio algorithm and there may be numerous NE in a single game. However, not all games and not all algorithms will satisfy these conditions so there remains some value in showing that the algorithm will have a steady-state.

If the radios are permitted to mix their strategies, i.e., if a radio is permitted randomly alternate between playing actions a_i and b_i, then condition 3 is completely removed, and the convexity requirement of condition 2 can be relaxed. Note that a finite action space would satisfy this relaxed condition. These relaxed conditions are known as Nash's fixed point theorem.

3.2 Identifying steady states

In and of itself, demonstrating that a game has a steady-state is not that useful as it provides no insight into the expected behavior of the algorithm. This is why steady-states need to be identified. However, without introducing a more advanced game model, such as the *potential game model*, the normal game model does not provide any tools for identifying NE.

Indeed to identify that action vector, a^*, is a NE, an analyst has to apply (1) and verify that all possible unilateral deviations from a^* does not improve the deviating player's payoff – a polynomial time problem. Then to identify all possible steady-states in a game, this process must be repeated over all possible action vectors tuples in the game, making the problem NP-complete [10].

Indeed when attempting to identify all NE in a game, analysts are forced to turn to simulations – the very step we're intent on minimizing. For example, to show that the modeled GPRS network employing joint rate-power adaptations had four NE, [5] relied on an exhaustive simulation that took days to complete even though the modeled system included only 7 players.

3.3 Determining steady-state desirability

While there are many different ways of identifying whether or not an action vector is a "good" steady-state, the most typically encountered technique is to demonstrate that the action vector is *Pareto optimal* as was done in [11] [2][13]. An action vector, a^*, is said to be Pareto optimal if there exists no other action vector, $a \in A$ such that $u_i(a) \geq u_i(a^*) \forall i \in N$ with at least one player strictly greater. While demonstrating that a steady-state is Pareto optimal seems like a good result, in reality, Pareto optimality is a very weak concept and tells the analyst very little about the desirability of the steady state. This point will be illustrated through two brief analyses, one looking at a distributed power control example and the other examining a call admission problem.

3.3.1 SINR maximizing power control

Consider a single cluster DS-SS network with a centralized receiver where all nodes other than the centralized receiver are adjusting their transmitted power levels in an attempt to maximize their signal-to-interference-plus-noise ratio (SINR) as measured at the receiver. Here our set of players are the nodes in the cluster (other than the centralized receiver); the action sets are the available power levels (presumably a finite number of power levels); and all players' utility functions are given by (2) where p_i is the transmitted power of node i, K is the statistical estimate of the spreading factor, h_i is the gain (presumably less than 1) from a node to the receiver, and σ is the noise at the receiver.

$$u_i(\mathbf{p}) = h_i p_i / \left((1/K) \sum_{k \in N \setminus i} h_k p_k + \sigma \right) \quad (2)$$

As would be indicated by intuition, the unique Nash equilibrium for this game is the power vector where all nodes transmit at maximum power. Clearly this is an undesirable outcome as (1) capacity is greatly diminished due to near-far problems (unless the nodes are all at the same radius from the receiver), (2) the resulting SINRs are unfairly distributed (the closest node will have a far superior SINR to the furthest node), and (3) battery life would be greatly shortened. However, this outcome is Pareto optimal as any more equitable power allocation will reduce the utility of the closest node, and any less equitable allocation will reduce the utility of the disadvantaged nodes. In this scenario Pareto optimality actually misleads the analyst with respect to the desirability of the outcome.

3.3.2 Call admission

Now suppose a number of nodes are requesting data bandwidth from a network with the network allocating bandwidth on a first-come first-serve basis. Here our set of players is the bandwidth requesting nodes; the actions are the amount of bandwidth that each node can request; and we'll assume that the utility function is some monotonic function of received data bandwidth (more bandwidth is always better). We'll introduce a little extra structure to the game in that not all requests are made at the same time. Without going into details, this time dependent scenario is best modeled with an *extensive form game model* though an understanding of the intricacies of this model is not important for understanding the Pareto optimality implications of the result.

In the steady-state, each early arriving node will receive as much bandwidth as it can handle and, assuming a reasonable cap on available bandwidth, late arriving nodes will be blocked from the network. Generally, blocking a potentially large number nodes is not considered to be a good result. However, it is Pareto optimal as any other allocation of bandwidth will decrease the utilities of the early arriving nodes. Further, the traditional call admission scheme by which a small amount of bandwidth is reserved to queue blocked calls is not Pareto optimal as utility functions are expressed solely in terms of received data bandwidth so reassigning this queuing bandwidth to data bandwidth will increase some players' utilities without decreasing any players' utilities.

A far better technique for demonstrating steady-state desirability is to evaluate how the identified steady states perform with respect to some network objective function as was done in [5]. For example the SINR scenario in Section 3.3.1 could be better evaluated through an objective function that measured capacity or total system throughput perhaps augmented by a measure of expected battery life. Rather than evaluating Pareto optimality, the steady state from the call admission scenario in Section 3.3.2 would be better evaluated in terms of Erlang B or Erlang C capacity.

Pareto optimality is a weak concept because it provides little insight into whether or not a steady state is desirable and virtually no insight into whether or not the network designer's objective is being maximized. It is preferable to evaluate any identified steady states using a network objective function that reflects the desires of the algorithm designer.

3.4 Establishing conditions for convergence

Every bit as important as identifying steady states and establishing steady state desirability is establishing under what conditions the algorithm will actually reach the steady state. Consider the game illustrated by the game table shown in Figure 2 that models an abstract interaction of two cognitive radios.

	A	B	C
a	1,-1	-1,1	0,2
b	-1,1	1,-1	1,2
c	2,0	2,1	2,2

Figure 2 Game Table for a Game with Weak FIP

This game table models a network with two cognitive radios where one cognitive radio has alternatives a, b, and c, and the other has alternatives A, B, and C. The implementation of choices by each radio yields different realizations of the outside world, e.g., (A,c) or (B,a), which the radios are capable of observing and valuating. The value that the first radio assigns to an outcome is given by the first entry in each cell of the table, and the value that the second radio assigns to an outcome is given by the second entry.

Now notice that this game has a unique NE, (c,C), which is Pareto optimal and in all likelihood desirable from the perspective of the network planner's objective function (we'll assume a sum of all radio's utilities). However, if the radios adapt their decisions by individually taking the smallest possible steps that improve the adapting radio's payoff, then play can proceed in a cycle. However, if at any point, a radio is permitted to take the largest step, then play will converge to the unique NE. This particular game has what is known as weak FIP (finite improvement path), a property of *supermodular games*. Without implementing the requirement that the radios must at some point take the larger step, the network will not converge and the steady state information will be meaningless.

Unfortunately, the normal form game model provides no insights into convergence criteria so convergence analysis, if performed (for example convergence is not considered in [11] or [12]), would have to be performed separately (as in [13]) or through simulation (as in [5]). Fortunately, more powerful game models exist for establishing convergence and some of these are discussed in the following section.

4. RELEVANT GAME MODELS

This section reviews the repeated game model, the supermodular game model, and the potential game model and examines how each of these game models address questions 1,2, and 4 considered in Section 3. For all models, it is preferable address the third question by substituting the predicted network steady state(s) into a network objective function.

4.1 Repeated games

A repeated game is sequence of "stage games" where each stage game is the same normal form game. Based on their knowledge of the game – past actions, future expectations, and current observations - players choose strategies – a choice of actions at each stage. These strategies can be fixed, contingent on the actions of other players, or adaptive. Further, these strategies can be designed to punish players who deviate from agreed upon behavior. When punishment occurs, players choose their actions to minimize the payoff of the offending player

<u>NE Existence:</u> In general, a repeated game is guaranteed to have a NE only if the stage game has a NE. However, if the players are permitted to "punish" each other, then convergence to virtually any action vector can be assured with the properly designed "punishment" regimen [14].

<u>NE Identification:</u> If the game does permit punishment, then NE identification is solely dependent on the properties of the stage game. However, assuming that the game permits punishment, then the game can be designed to have the desired NE.

<u>Convergence:</u> Assuming a punishment strategy is properly designed, convergence is guaranteed.

<u>Examples:</u> Repeated games are applied to the problem of distributed power control in [1] and to the problem of resource sharing in [12].

4.2 Potential game model

A potential game is a special normal form game where there is a function, $V : A \to \mathbb{R}$, such that when a unilateral deviation occurs, the change in *V*, ΔV, is reflected in the change in value seen by the unilaterally deviating player, Δu_i. If for all unilateral deviations, $\Delta V = \Delta u_i$ the game is called an exact potential game; likewise if $\operatorname{sgn}(\Delta V) = \operatorname{sgn}(\Delta u_i)$ the game is an ordinal potential game.

<u>Model Identification</u>: A game can be shown to be an exact potential game if the action space is compact and the utility functions satisfy (3).

$$\frac{\partial^2 u_i(a)}{\partial a_i \partial a_j} = \frac{\partial^2 u_j(a)}{\partial a_j \partial a_i} \forall i, j \in N, a \in A \qquad (3)$$

Other than applying the definition, there is no well-defined condition for verifying that a game is an ordinal potential game. However, [14] shows that if a sequence of ordinal (monotonic) transformations of the utility functions result in an exact potential game, then the original game is an ordinal potential game.

<u>NE Existence</u> Potential games always have at least one NE [15].

<u>NE Identification:</u> All maximizers of *V* (local and global) are NE [15]. Note this need not be all of the NE in the game, but the only stable NE in the game are maximizers of *V* [16].

<u>Convergence:</u> Potential games have the finite improvement path (FIP) property, so when nodes act in a selfish manner play converges to a NE.

<u>Examples:</u> In [17] potential games are applied to the analysis of adaptive interference avoidance problems. In [8] and [14] potential games are applied to distributed power control.

4.3 Supermodular game model

A game is termed supermodular if the action space forms a *lattice* and the utility functions are *supermodular*. A partially ordered set, *X*, is termed a lattice if for all $a, b \in X$, $a \wedge b \in X$ and $a \vee b \in X$ where $a \vee b = \sup\{a, b\}$ and $a \wedge b = \inf\{a, b\}$. A function, $f : X \to \mathbb{R}$ where *X* is a lattice, is termed supermodular if for all $a, b \in X$,

$$f(a) + f(b) \le f(a \wedge b) + f(a \vee b)$$

<u>Model Identification</u>: While the definition may seem complicated, a game can be identified as a supermodular game if all players' utility functions satisfy the relationship given in (4) and the action space is compact.

$$\frac{\partial^2 u_i(a)}{\partial a_i \partial a_j} \ge 0 \forall j \ne i \in N \qquad (4)$$

<u>NE Existence:</u> By Topkis's fixed point theorem [18], all supermodular games have at least one NE.

<u>NE Identification:</u> By [18], all NE for a game form a lattice. While this does not particularly aid in the process of initially identifying NE, from every pair of identified NE, e.g., a^* and b^*, additional NE can be found by evaluating $a^* \wedge b^*$ and $a^* \vee b^*$.

<u>Convergence:</u> By [20], supermodular games have weak FIP, i.e., from any initial action vector, there exists a sequence of selfish adaptations that lead to a NE. Specifically for supermodular games, a sequence of best responses will converge to a NE [20]. Further, if the radios make a limited number of errors or if the radios are instead playing a best response to a weighted average of observations from the recent past, play will converge [19][20]. These same convergence results also hold for potential games as FIP implies weak FIP.

<u>Examples</u>: Altman [7] demonstrated that the distributed power control scenarios considered by Yates [21] can be modeled using a supermodular game. Other supermodular power control algorithms are discussed in [14].

5. CONCLUSIONS

We have shown how the cognition cycle maps into a normal form game model and identified four issues that any application of game theory to cognitive or adaptive radio should address: steady state existence, steady state identification, steady-state optimality, and convergence. We have made the case that demonstrating Pareto optimality is insufficient for demonstrating the desirability of a steady-state and that evaluation of a network objective function is preferable for determining steady-state desirability. We then described three game models that can be used to address the remaining three issues.

However, the value of using game models extends beyond this limited discussion. In analysis, potential games appear to be less susceptible to the introduction of noise [16] thus steady state stability is implied.

Game models can provide insight into the design and implementation of cognitive radios. For instance, suppose a designer wishes to implement an algorithm that maximizes a network function, *f*. Then the utility functions of the players (the observation and orientation steps of the cognition cycle), can be implemented as $f + d_i(a_{-i})$ where $d_i(a_{-i})$ is a "dummy" function that only depends on the actions of other radios. Alternatively, if an existing algorithm can be shown to be a potential game and a different network steady state is desired, then this can be accomplished by introducing an additive cost function as described in [8]. Game models can also be used to estimate an algorithm's complexity. As discussed in [14], the game model that best models a cognitive radio algorithm can be used as an indicator of algorithm complexity.

When implementing a cognitive radio, a key research task is the development of an ontology for representing the information the radio needs about itself, its waveform, and its network. An ontology that includes mechanisms for describing game models will provide a compact way of representing information about the expected behavior of the network and improve a cognitive radio's ability to predict and plan its performance.

Because of these numerous benefits, adopting an analytic approach that emphasizes the use of game models over a more ad-hoc approach is preferable for analyzing the algorithms of adaptive and cognitive radios.

6. REFERENCES

[1] A. MacKenzie and S. Wicker, "Game Theory in Communications: Motivation, Explanation, and Application to Power Control," *Globecom2001*, pp. 821-825.

[2] C. Sung and W. Wong, "A Noncooperative Power Control Game for Multirate CDMA Data Networks," *IEEE Transactions on Wireless Communications*, Vol. 2, No 1. Jan. 2003, pp. 186-19.

[3] A. MacKenzie and S.Wicker, "Selfish users in Aloha: A Game Theoretic Approach," *VTC Fall 2001*, Vol.3, pp 1354-1357.

[4] C Saraydar, N. Mandayam, D. Goodman, "Pricing and Power Control in a Multicell CDMA Data System," *IEEE JSAC* vol 19, No 10, Oct. 2001.

[5] S. Ginde, R. M. Buehrer, and J. Neel, "A Game Theoretic Analysis of the GPRS Adaptive Modulation Schemes" *Fall VTC 2003*.

[6] C. Saraydar, N. Mandayam, D. Goodman, "Pareto Efficiency of Pricing-based Power Control in Wireless Data Networks," Wireless Communications and Networking Conference, 1999, pp 231-235.

[7] E. Altman and Z. Altman. "S-Modular Games and Power Control in Wireless Networks" *IEEE Transactions on Automatic Control*, Vol. 48, May 2003, 839-842.

[8] J. Neel, J. Reed, R. Gilles, "Game Theoretic Analysis of a Network of Software Radios," *SDR Forum 2002*.

[9] J. Mitola, III. "Cognitive Radio for Flexible Multimedia Communications", *MoMuC '99*, pp. 3 –10, 1999.

[10] S. Sahni, "Computationally Related Problems," *SIAM Journal of Computing*, **3**:262-279, 1974.

[11] D. Krishnaswamy, "Game Theoretic Formulations for Network-assisted Resource Management in Wireless Networks", *VTC Fall* 2002, pp. 1312-1316.

[12] L. DaSilva and V. Srivastava, "Node Participation in Ad-hoc and Peer-to-peer Networks: A Game-theoretic Formulation," *Workshop on Games and Emergent Behavior in Distributed Computing Environments*, Sep. 18, 2004, Birmingham, U.K.

[13] M. Hayajneh, C. Abdallah, "Distributed Joint Rate and Power Control Game-Theoretic Algorithms for Wireless Data," *IEEE Communications Letters*, vol 8, No 8, Aug. 2004.

[14] J. Neel, J. Reed, R. Gilles "Convergence of Cognitive Radio Networks," *WCNC*, March 2004.

[15] M. Voorneveld, "Potential Games and Interactive Decisions with Multiple Criteria," PhD Dissertation, Tilburg University, Netherlands, 1996.

[16] J. Hicks, A. MacKenzie, J. Neel, J. Reed, "A Game Theory Perspective on Interference Avoidance" *Globecom2004*, Nov. 2004.

[17] R. Menon, A. B. MacKenzie, R. M. Buehrer, J. H. Reed, "Game Theory and Interference Avoidance in Decentralized Networks", *SDR Forum 2004 Technical Conference*, Nov. 2004.

[18] Topkis, Donald M. *Supermodularity and Complementarity*, Princeton University Press, Princeton, New Jersey, 1998.

[19] Milgrom, Paul and John Roberts, "Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities," *Econometrica*, Vol. 58, Issue 6 (Nov 1990), pp. 1255-1277.

[20] Friedman, James W. and Claudi Mezzetti, "Learning in Games by Random Sampling," *Journal of Economic Theory* vol. 98, pp. 55-84, 2001.

[21] R. Yates, "A Framework for Uplink Power Control in Cellular Radio Systems," *IEEE JSAC.*, Vol. 13, No 7, Sep. 1995, pp. 1341-1347.