OFDM Modulation Using Square-Root Nyquist Time Domain Kernels to Obtain Reduced Peak-to-Average Power Ratio

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ABSTRACT
The waveform of choice for OFDM signaling is the sinusoid with an integer number of cycles per interval and with an appended cyclic prefix to obtain circular convolution with the channel. This combination makes the channel inversion particularly simple; performed as a ratio between the DFT of the received signal and the DFT of the channel. In fact, this relationship is valid for any periodic function formed as a sum of the basis sinusoids of the DFT. One particularly simple example of this class of signals is the Dirichlet kernel (the periodically extended sinc function). This kernel is used in single carrier OFDM [1]. An advantage of this kernel relative to the complex sinusoid kernel is a 3.4 dB reduction in peak to average power ratio (PAPR). We show here that a windowed version of this kernel exhibits a significantly lower, in fact up to a 10.0 dB reduction in PAPR. The cost to obtain the reduced PAPR is excess bandwidth but that may be a fair trade to obtain higher average transmitted power for a given peak power limited amplifier.

1. INTRODUCTION

The standard kernels of an OFDM modulator are the sampled complex sinusoids with integer number of cycles per modulation symbol interval. These kernels correspond to the basis sequences of the discrete Fourier transform (DFT) which is the reason the OFDM modulator and demodulator are implemented by the fast Fourier Transform (FFT) algorithm.

One attraction of this OFDM signal set is the periodicity of its separate components. This periodicity enables us to append a cyclic prefix to each symbol which, when passed through the signaling channel, converts the linear convolution to a circular convolution. The circular convolution permits us to perform the channel equalization as a simple spectral ratio.

The amplitude of the real and imaginary time series, formed as the sum of a large number of independently weighted sinusoids, tends to be Gaussian distributed with a Rayleigh distributed envelope. The Rayleigh envelope exhibits a large ratio of peak to average value with peak values exceeding 4-times the average value with probability 0.00035. To preserve the fidelity of the OFDM time signal and to avoid spectral artifacts due to power amplifier clipping we often operate the final amplifier stage with the average signal level at one-fourth or less of maximum output capability.

With average amplitude 1/4-th of peak amplitude, average power is 1/16-th of peak power. Power amplifiers are DC to AC converters and power pulled from the DC power supply, which is approximately constant, not delivered to the load is dissipated in the power amplifier. Amplifiers are very inefficient in their transduction process of turning DC power to signal power when they operate at small fractions of their peak power level. Typical efficiencies for amplifier operating with an IEEE 802.11a signal are on the order of 18 % [1]. Thus an amplifier required to supply 1 watt would have a peak power capability of 16 watts and would be pulling 5.5 watts from the power supply while squander 4.5 watts, raising the temperature of its heat sinks, while delivering 1 watt to its external load. There is high motivation to reduce the peak to average power levels of the transmitted signal. Reductions in PAPR result in significant reduction in power supply draw and dissipated waste heat required to deliver a specified average power. Reduced power supply draw offers the desirable benefit of increased operating time between batter charge cycles.

2. AN ALTERNATE MODULATION WAVEFORM

An alternate periodic waveform available for orthogonal signaling in an OFDM system is the Dirichlet kernel [1]. This kernel is the periodic extension of the standard sinc function. The sampled data version of this function is defined in (1) for a sequence of length 256 with two sided bandwidth of 32 bins (± 16.5 bins). This time series is over sampled by 8 so that there are 8 samples per symbol. A closed form of this expression is shown in (2).

$$h(n) = \frac{1}{32}\sum_{k=-16}^{16} a_k \exp(j\frac{2\pi}{256}nk),$$
$$a_k = \begin{cases} 1, & |k| = 0,1,...,15 \\ 0.5, & |k| = 16 \end{cases}, \quad n = 0,1,...,255 \qquad (1)$$

$$h(n) = \frac{\sin(\frac{2\pi}{512}31\cdot n) + \sin(\frac{2\pi}{512}33\cdot n)}{64\cdot\sin(\frac{2\pi}{512}n)}, \; n = 0,1,...,255 \qquad (2)$$

The spectrum and time series corresponding to this periodic time series is shown in figure 1. Note, in this figure and in (1), that the spectral sample value at the band edge is half the value of pass band samples. This is necessary for the translated kernels to be orthogonal. This boundary value is replaced by sqrt(0.5) when the spectrum is partitioned into modulator shaping and demodulator matched filters.

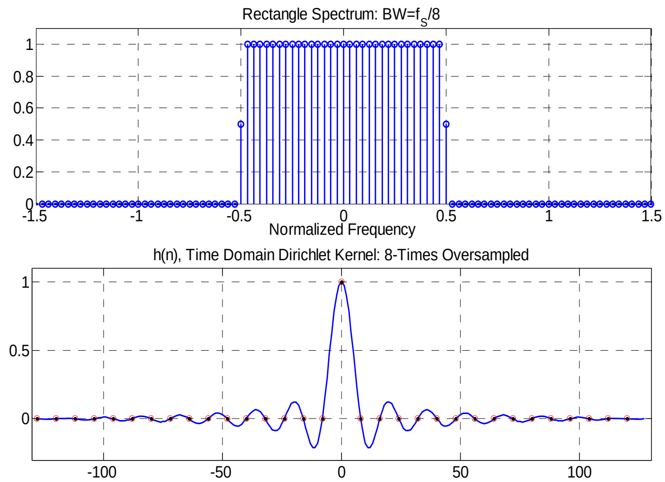

Figure 1. Spectrum and Scaled Time Series of 1-to-8 Over Sampled Dirichlet Kernel

Note that a single period of this kernel resides on a finite support and does not require a multiplicative window applied in the time domain to obtain a finite support as required by the sinc function in the continuous domain. Lacking the applied time window, the spectrum does not exhibit excess bandwidth due to the windowing operation. As with the standard Nyquist function, time translates by multiples of the symbol clock interval of this sequence are mutually orthogonal. Here the time translates are circular shifts rather than linear shifts. In this form of OFDM the data is carried by a vector of circularly shifted (SQRT) Nyquist like Dirichlet wave shapes that span the OFDM symbol interval. Since successive translates reside near the zero crossings of the other component waveforms there is less tendency for the sum to experience large peak values. The time translates interact primarily through their side lobes.

Figure 2 shows the sample histograms for a standard OFDM modulator and for the Dirichlet Kernel OFDM modulator, both modulated with 32 bins of QPSK constellations. Figure 3 shows the complementary cumulative density functions for the same modulation sets. Here we see a reduction of peak amplitude, relative to average amplitude, from 3.50 to 2.21 which represents a drop of 3.9 dB in peak power level.

3. AN IMPROVED ALTERNATE WAVEFORM

The SQRT Dirichlet shaping waveform presented in the previous section exhibits side lobes similar to those of the conventional SQRT Nyquist function. An alternate periodic waveform can be formed by using a window or an equivalent process to reduce these side lobe levels as well as the number of significant side lobes [2]. Our experience and intuition suggests this would reduce the interaction between translated time kernels in the Dirichlet based OFDM signal. The time domain windowing operation would of course be responsible for bandwidth expansion though the equivalent spectral circular convolution.

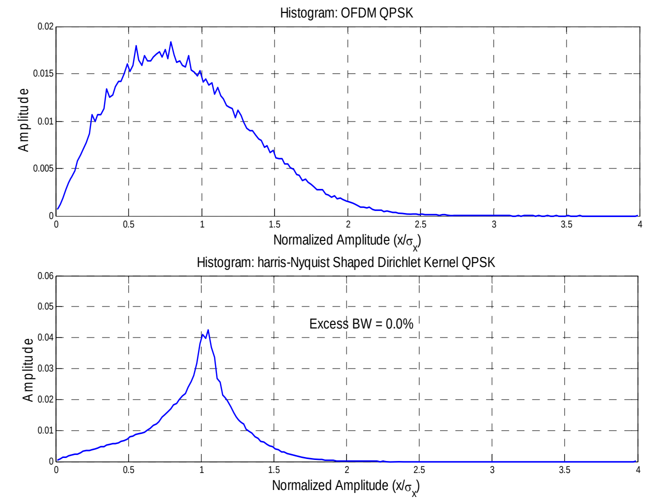

Figure 2. Sample Histograms for Standard and Dirichlet Kernel QPSK Modulated OFDM

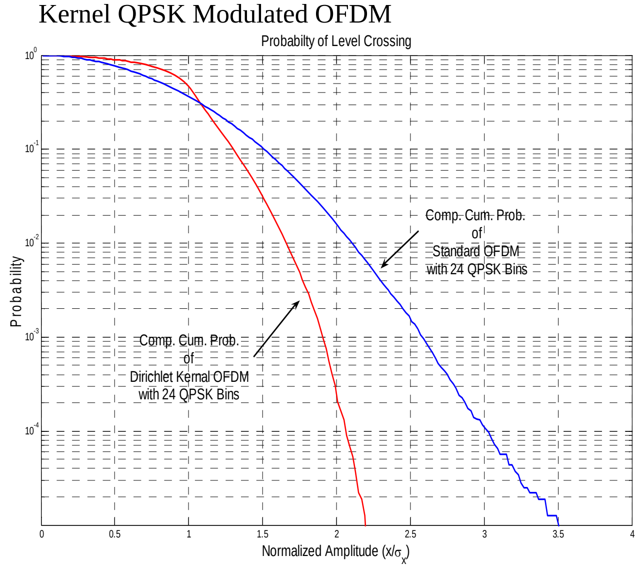

Figure 3. Complementary Cumulative Density Functions for Standard OFDM and Dirichlet Kernel OFDM.

Rather than use a window to reduce the side lobe levels, we can use any filter design routine that directly controls the spectral expansion that would be associated with the window operation. One option we should not use is the ubiquitous cosine tapered sqrt-Nyquist filter available, for instance, from the MATLAB *rcosine* script file. In an earlier paper [3] we noted that the cosine tapered sqrt-Nyquist filter was a poor choice to use for spectral shaping since it involved two windowing operations; one for the taper, and one for the temporal truncation. The second window, not needed here, causes large values of in-band and out of band spectral ripple. The in-band ripple is responsible for residual ISI while the out-of-band ripple makes it difficult to meet spectral mask requirements.

We elected to use a Remez algorithm based design, the harris-Nyquist square root filter [4], which by design exhibits improved ripple levels and controlled bandwidth expansion. The MATLAB script file, nyq_2, is available from the authors. The spectrum and time series corresponding to the nyq_2 time series with nearly 100 % excess bandwidth is shown in figure 4. We see in this figure the spectral expansion required to affect the time-domain side lobes and note too the significant reduction in time domain side lobe levels and in effective signal support. Compare this figure to figure 1. Note too that the sqrt filter has the expected gain of 0.707 at the edge of the symbol band width.

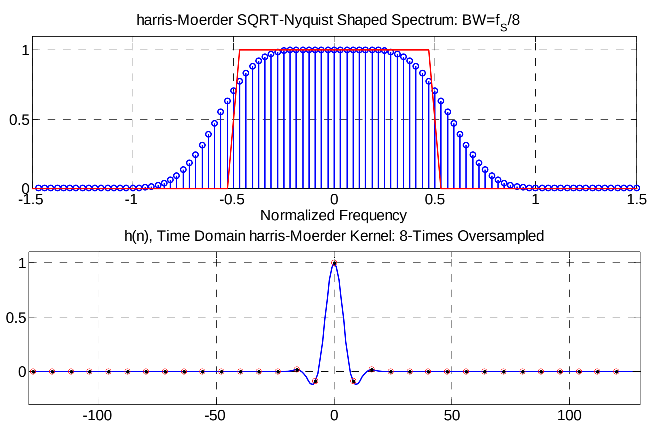

Figure 4. Spectrum and Scaled Time Series of 1-to-8 Over Sampled harris-Nyquist Weighted Dirichlet Filter

Figure 5 shows the sample histograms for a standard OFDM modulator and for the nyq_2 kernel OFDM modulator, both modulated with QPSK constellations. Note the set of distinct peaks in the nyq_2 histogram. This is due to the small amount of coupling between offset symbols and is typically seen in the eye diagrams of a shaped filter with large excess bandwidths. Figure 6 shows the complementary cumulative density functions for the same modulation sets. Here we see a reduction of peak amplitude, relative to average amplitude, from 3.50 to 1.09 which represents a drop of 10.1 dB in peak power level. We did incur a cost to achieve this PAPR reduction; it was the bandwidth increase shown in figure 4.

Figure 7 shows the envelope of 1000 nyq_2 shaped OFDM symbols. These symbols contained a tapered cyclic prefix (CP) as well as a tapered cyclic suffix to control spectral side lobes which had been significantly improved by the shaped kernels. This spectrum is shown in figure 8. The modulation wave shape we just examined achieved its significant reduction in PAPR by allocating nearly 100% excess bandwidth to the signal spectrum. In the next section we show comparable PAPR reductions for reduced amounts of spectral expansion and examine how quickly PAPR can be exchanged for excess bandwidth. We also examine options which trade excess bandwidth against reduced levels of PAPR to obtain higher average transmitted power.

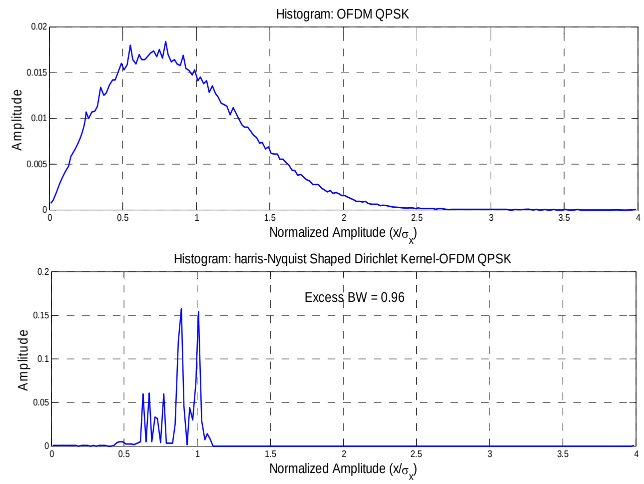

Figure 5. Sample Histograms for Standard and harris-Nyquist Weighted Dirichlet Kernel QPSK Modulated OFDM

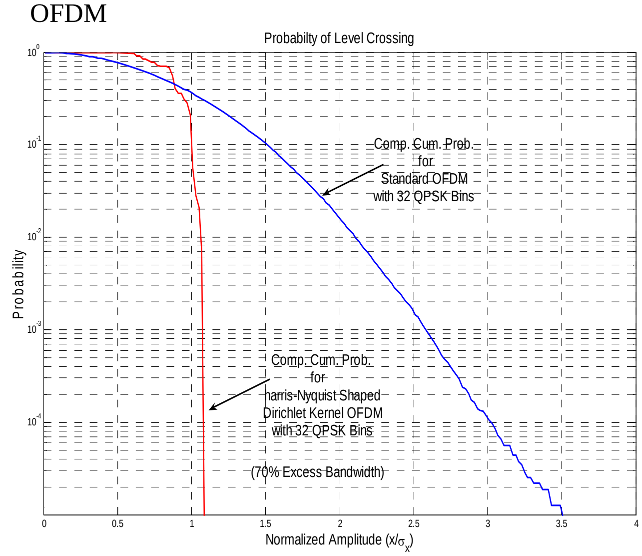

Figure 6. Complementary Cumulative Density Functions for Standard ODM and harris-Nyquist Shaped Dirichlet Kernel QPSK Modulated OFDM.

4. TRADING EXCESS BW FOR PAPR REDUCTION

We examined a range of excess bandwidth carried by the shaping filters to determine the rate at which the trade between excess bandwidth and reduced PAPR occurred. Figure 9 shows the impulse response of a set of filters with successively smaller excess bandwidth which we can think of as transition bandwidth. The first thing we observe is that the filters with larger excess bandwidth have smaller effective time support. This is not surprising since we know that the length of a filter is proportional to the ratio of sample

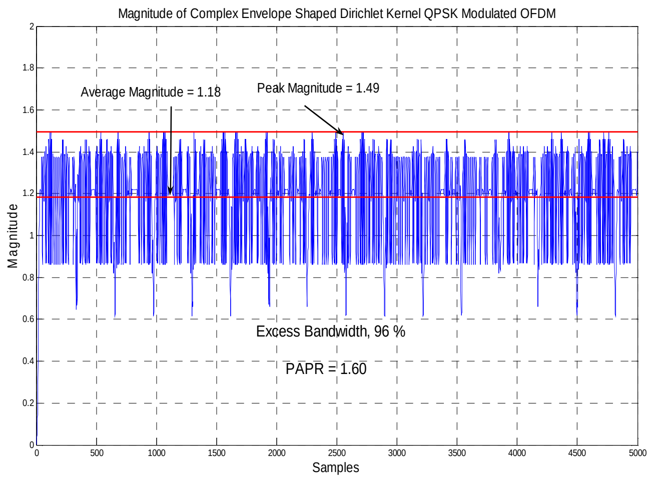

Figure 7. Magnitude of Complex Envelope for harris-Nyquist Shaped Dirichlet Kernel QPSK Modulated OFDM

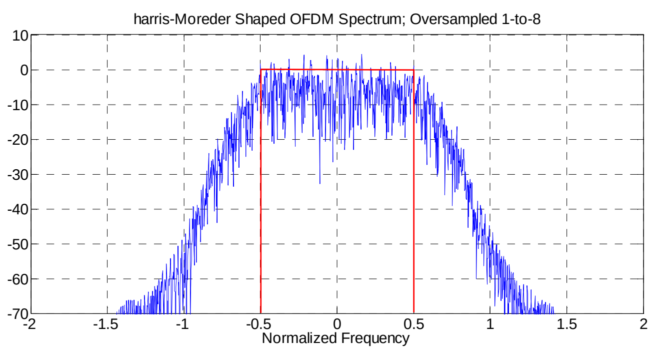

Figure 8. Spectrum of nyq_2 100% Excess Bandwidth Kernel Modulated OFDM Symbols with Tapered Cyclic Prefix and Cyclic Suffix

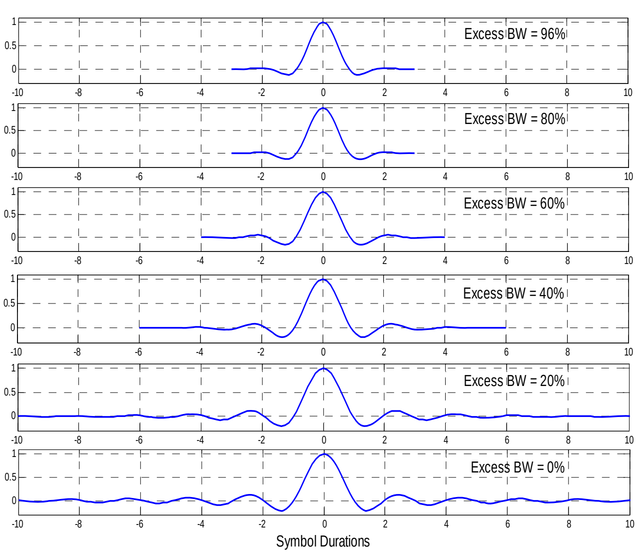

Figure 9. Impulse Response of Shaping Filters with Successively Smaller Excess Bandwidth.

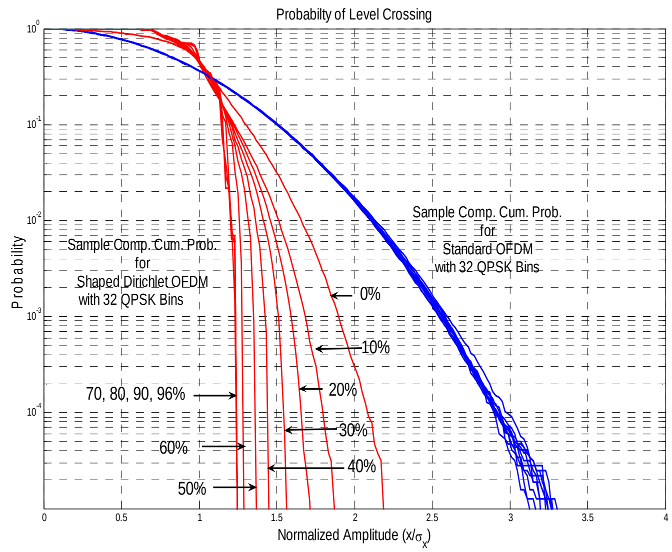

Figure 10. Complementary Cumulative Density Functions for Standard ODM and harris-Nyquist Shaped Dirichlet Kernels, for a Range of Excess Bandwidth, QPSK Modulated OFDM.

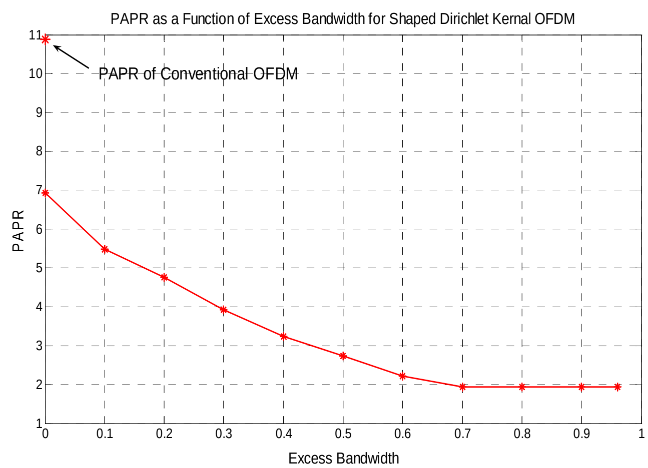

Figure 11. PAPR as Function of Excess BW for Shaped Dirichlet Kernel OFDM

rate to transition bandwidth. To first order, it is this reduction in effective length that is responsible for the improved PAPR. Figure 10 presents the complementary cumulative density functions for the shaped Dirichlet kernels for excess bandwidths of 0% to nearly 100% in 10% steps. We first notice the reduction in PAPR as the excess bandwidth increases from 0% to 60%. A surprise to us was that there was no benefit to increasing the excess bandwidth beyond 70%. We traced this result to the fact that the filter lengths for filters with excess bandwidth exceeding 70% were all the same length. The filter design routine always defaulted to an integer number of symbols and the selected length for these was 6 symbols. Filters with smaller excess bandwidth spanned a larger number of symbol durations and we have concluded that the longer span is the primary cause of increased PAPR. Figure 10 presents a graphical relationship between PAPR and excess bandwidth for the signal set under discussion.

5. EXAMPLE

Consider the following problem. We have a 40-MHz bandwidth available which we want to use for an OFDM modulator with symbols matching the IEEE.802.11a time structure of 3.2 μsec modulation symbol with 0.8 μsec cyclic prefix. Let us examine a number of ways we can apply the material developed here to this task.

Option 1. Standard OFDM, Full Bandwidth: The 3.2 μsec symbol duration tells us the spectral sinc function is (1/3.2) MHz wide and the number of these that fit in the 40 MHz 128. We thus modulate 128 DFT bins of a 256 point FFT to obtain the 128 bins per 3.2 μsec which converts to 128 bins per 4.0 μsec when we include the cyclic prefix (CP). Thus the bin rate is 128 bins per 4 μsec or 32 bins per μsec or 32 M-bins/sec. If each bin carries 2-bits, for QPSK signaling, we have 64 Mbits/sec but with a PAPR of 10.0 dB.

Option 2. Unshaped Dirichlet OFDM, Full Bandwidth: The 40 MHz symbol bandwidth tells us the time sinc function is (1/40) μsec wide and the number of these that fit in the 3.2 μsec interval is 128. We thus modulate 128 time domain Dirichlet kernels up-sampled 1-to-2 in a 256 point FFT to obtain the 256 samples per 3.2 μsec which again converts to 128 kernels per 4.0 μsec when we include the cyclic prefix (CP). Thus the kernel rate is 128 kernels per 4 μsec or 32 kernels per μsec or 32 M-kernels/sec. If each kernel carries 2-bits, for QPSK signaling, we have 64 Mbits/sec but now with a reduced PAPR of 7.0 dB. The drop of PAPR from 11-dB to 7-dB is 5 dB. We have an interesting choice here: we can increase the transmitted power by the same 5-dB using the same output power amplifier to obtain greater range or margin against fade, or we can convert the extra 5-dB into a denser constellation, replacing the 2-bit QPSK with 3-bit PSK to obtain 96 Mbits/sec. To have a complete trade we would require a 5.3 dB increase in power to maintain the same bit error rate. But we are close!

Option 3. Shaped Dirichlet OFDM, 25 MHz Symbol Rate: The 25 MHz symbol bandwidth tells us the time sinc function is (1/25) μsec wide and the number of these that fit in the 3.2 μsec interval is 80. We thus modulate 80 time domain Dirichlet kernels up-sampled 5-to-16 in a and spectrally shaped and extended to 40 MHz bandwidth in a 256 point FFT to obtain the 256 samples per 3.2 μsec which again converts to 80 kernels per 4.0 μsec when we include the cyclic prefix (CP). Thus the kernel rate is 80 kernels per 4 μsec or 20 kernels per μsec or 40 M-kernels/sec. If each kernel carries 2-bits, for QPSK signaling, we have 64 Mbits/sec but now with a further reduced PAPR of 2.2 dB. The drop of PAPR from 11-dB to 2.2-dB is 8.8 dB. We have an interesting choice here: we can increase the transmitted power by the same 8.8-dB using the same output power amplifier to obtain greater range or margin against fade, or we can convert the extra 8.8-dB into a denser constellation and increased range. We can replace the 2-bit QPSK with 3-bit PSK to obtain 60 Mbits/sec and use 5.3 dB of the extra power to maintain the same bit error rate and then use the additional 3.5 dB margin to obtain increased range. Not a bad trade for the increase of bandwidth from the 25 MHz symbol rate to the available 40 MHz bandwidth with the 60% excess bandwidth shaping filter.
Figure 12. Shows the spectra for the 3-options described in this section.

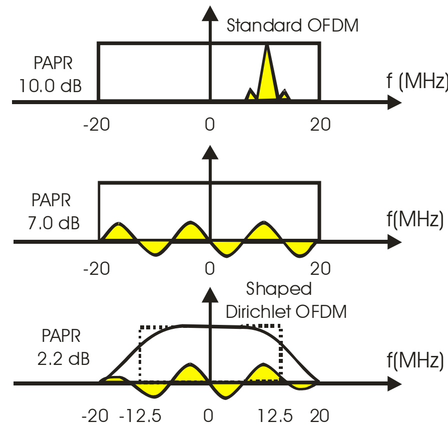

Figure 12. Spectra for 3-OFDM Configurations of Section 5

6. CONCLUSIONS

We have described a variant of OFDM, related to single carrier FDMA that achieves remarkably good PAPR for QPSK and for PSK modulation. The significant reduction in PAPR due to spectral shaping and bandwidth expansion offers the system designer a number of desirable options including smaller power amplifiers and heat sinks, longer intervals between battery charge cycles, or greater range and capacity from existing final amplifier stages. The material presented and reported here concentrated on 32 Dirichlet kernels in the OFDM modulator. While we don't believe the use of more kernels changes the statistics as it does in ordinary OFDM, we recommend that this be verified. This area is rich in research opportunities.

6. REFERENCES

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[4] fred harris, "Multirate Signal Processing for Communication Systems", Prentice-Hall, 2004.