# PAPR and the Cubic Metric for Low PA Power consumption

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***Abstract*** **- With the every-increasing complexity of signal waveforms generated by transmitters of modern digital communications systems, there comes a corresponding demand for transmitters to continue to operate efficiently, producing minimum distortion, within specified limits. These days, equipment requirements also include the need to operate in an energy efficient way, minimizing power consumption. In order to operate in the most efficient manner possible, instantaneous control of a Power Amplifier (PA) by measuring appropriate PA input signal characteristics, for example the Peak to Average Power Ratio (PAPR) or Cubic Metric (CM), should result in setting the PA Back-off (sometimes referred to as de-rating) as accurately as possible. PAPR and a CM approaches to determining PA Back-off are compared for OFDM signals, together with a suitable PAPR reduction technique, in terms of their ability to minimize power consumption and ensure efficient PA operation.**

***Index Terms*****—CM, Green Radio, PAPR, Optimum Selective Mapping**

## I. INTRODUCTION

When looking at services which require high user bit rates, wireless communication systems have tended to lag behind wired systems due to problems caused by the less benign radio transmission environment. To the end user service is all important, however, when considering the environment, low power consumption becomes as important. Current perception is that users would like to have the same service offered over a wireless link as they would be able to obtain over a wired one, despite any additional performance requirements. This has led to the design of cellular wireless systems which are capable of working with relatively high data rates, in particular over the wireless link. Recent advances in the third generation (3G) system, currently referred to as Wideband Code Division Multiple Access (WCDMA) – formerly the Universal Mobile Telecommunications System (UMTS) – have resulted in concepts such as High Speed Packet Access (HSPA) where theoretical data rates of 14.4Mbps and 5.76Mbps in downlink and uplink respectively are achievable. The next generation mobile phone system (LTE) aims to increase these data rates further, in a spectrum efficient manner, using "bandwidth on demand" techniques (the allocation of appropriate resource blocks depending on, amongst other items, the amount of data a user requires). The main, driving, forces behind the design of WCDMA and LTE have been the thirst for increased data rates and the requirement for spectrum efficiency (accessible bandwidth being a relatively rare resource). Due to environmental considerations, a third driver, the requirement for power efficiency, needs to be taken into account, aiming ultimately towards the development of Power Efficient Technology (or Green Technology).

## II. GREEN TECHNOLOGIES - GENERAL

Perhaps one of the first, most obvious, areas where high power efficiency has always been a requirement is the transmitter PA.

The increase in data rate, which allows for the delivery of enhanced user services needs to be complimented by the "efficient" delivery of that service, that is to say, resources should not be needlessly wasted and power should not be unnecessarily consumed. Optimisation of such system resources is a necessary task in order to drive down the cost per bit to the end user and associated Operational Energy (OpEn) costs. Multi-media (e.g. internet) service users require relatively high data rates combined with low data access delays, yet internet sessions tend to be bursty in nature, with defined periods of activity and inactivity. To ensure efficient resource usage, users should be assigned only the minimum resources which would allow them to remain connected during these inactive times (i.e. when no data is being sent on the Uplink). In times of no user activity ideally (ignoring transient off/on switching aspects) PA's should be turned off, otherwise their 'back-off' should be kept as small as possible, to ensure efficient operation, whilst also ensuring minimum signal degradation which might otherwise occur (for example through signal clipping). A low back-off value without signal distortion not only ensures increased efficiency but also ensures power is channeled into the desired signal and not "wasted" on spurious signals.

## III. PAPR AND THE CM

Input signals to a PA tend to be very dynamic and this is becoming increasingly true with modern communications systems which involve the deployment of an increased number of channels and associated channel configurations. The way these channels are configured, and their subsequent processing by the PA, is likely to have an impact on any distortion that might be produced by that PA. With second generation, 2G, systems PAPR-based back-off schemes were deemed sufficient for efficient PA operation. However, more recent literature e.g. [2], [3], [4], [5] suggests that PAPR 'back-off' schemes are not sufficiently accurate when working with the, increasingly,

TO BE ADDED

complex signals that occur with enhanced 3G systems (and by implication future, fourth generation, 4G, systems). A measure is required which reflects better a signal's constituent parts, e.g. for OFDM-type signals this may include, for each channel, the number of samples used when implementing the Fast Fourier Transform (FFT) and how many of these samples are "active", the number of samples dedicated to the Cyclic Prefix (CP), the modulation type and the associated bandwidth/number of resource blocks, as well as knowing the way in which the channels themselves are formatted and how they are combined, together with considering other system aspects such as how power control is implemented.

Ref [2] indicates that the (WCDMA) CM has the required accuracy for determining what the PA back-off should be for any given channel format and combination for current, modern, cellular systems when considering WCDMA. Ref[10] provides a similar (CM) approach for LTE.

## IV. CM FOR LTE

The modulator output/input to the PA can be represented as a complex stream of N samples, $\{v_t\}$, as indicated in Figure 1;

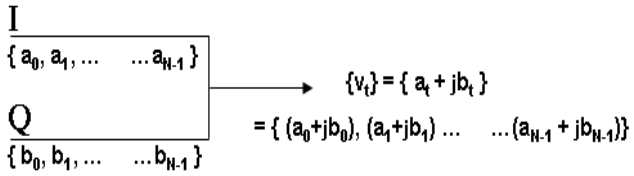


Figure 1 Modulation Output

From the literature, e.g. [9], the CM is given by;

$$CM_{WCDMA} = CEIL([20.\log_{10}((v^3_{norm})_{rms} - 20.\log_{10}(v^3_{norm\mathrm{Re}f})_{rms})]/K) \quad (1)$$

Where,

$v_{norm}$ is the normalized voltage waveform of the input signal.

$v_{norm\mathrm{Re}f}$ is the normalized voltage waveform of the reference signal (12.2kbps AMR Speech) and

$20.\log_{10}(v^3_{norm\mathrm{Re}f})_{rms} = 1.52dB$.

K is a normalizing constant depending on the Spreading Factor (for WCDMA) or the bandwidth/number of resource blocks (for LTE), see ref[2] and ref[10]

The CM can be found using the modulation output samples' cubed power series and the power series itself.

An alternative, more explicit, expression for the CM, ref[1], is thus;

$$c_{rms} = N.\left(\frac{P_{CUBED}}{P^3_{TOTAL}}\right)^{1/2} \quad (2)$$

Where, N is the number of samples considered,

$$P_{CUBED} = (a_0^2 + b_0^2)^3 + (a_1^2 + b_1^2)^3 + \ldots \quad \ldots + (a_{N-1}^2 + b_{N-1}^2)^3$$
$$P_{CUBED} = \sum_{t=0}^{t=N-1}(a_t^2 + b_t^2)^3 \quad (3)$$

And,

$$P_{TOTAL} = (a_0^2 + b_0^2) + (a_1^2 + b_1^2) + \ldots \quad \ldots + (a_{N-1}^2 + b_{N-1}^2)$$
$$P_{TOTAL} = \sum_{t=0}^{t=N-1}(a_t^2 + b_t^2) \quad (4)$$

## V. PAPR & CM COMPARISON

In order to observe the accuracy of PAPR and CM back-off estimations, signals representative of those that would occur in practice might be simulated and the CM calculation performed, making use of equation (2). Comparison of calculated CMs of other system signals is then undertaken. Using the empirical results from ref [2] in the form of a graph the calculated CM values can be mapped onto the required PA power back-off values. A similar procedure can be carried out for back-off as determined by only PAPR measurement. Both sets of results can then be compared to PA back-off measurements determined through experimentation with appropriate PA hardware.

Considering LTE, the standard error for PAPR back-off prediction, is found to be 1.56dB which was also found to be, on average, more than 1.33dB poorer than the CM prediction Ref [10]. Approximated Back-off estimation plots for PAPR estimation and CM estimation can be seen in Figure 2 and Figure 3 respectively.

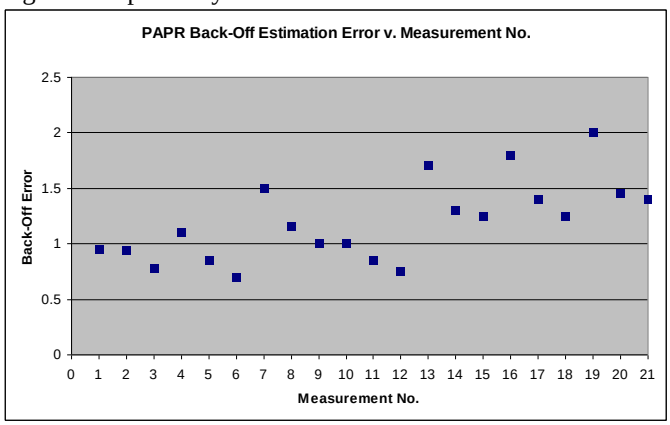


Figure 2 PAPR Back-Off Estimation Error for a number of measurements

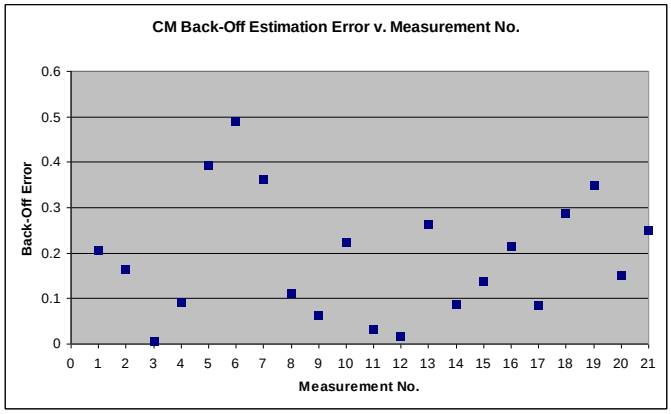


Figure 3 CM Back-Off Estimation Error for a number of measurements

In order to avoid possible signal distortion, the variation in the prediction error of either technique should be taken into account when determining the PA back-off value. As seen from the ideal drawings in Figure 4, this results in a smaller back-off value when using the CM.

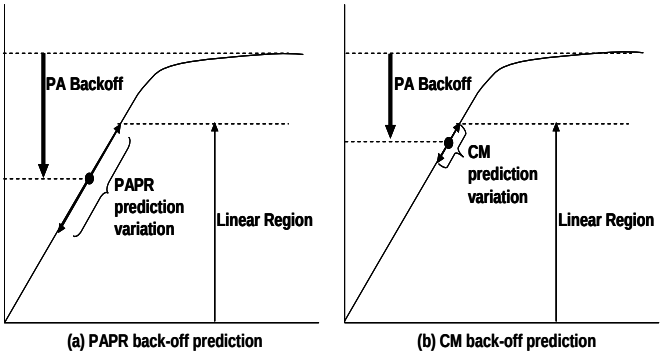


Figure 4 PAPR/CM back-off prediction variation

## VI. PEAK TO AVERAGE POWER RATIO REDUCTION

To further optimize the efficient operation of the PA the PAPR itself might be reduced, thereby reducing the variation of the actual signal and the associated PA backoff prediction error. Various techniques have been proposed to reduce PAPR, among them SeLective Mapping (SLM) ), Ref [11] and as shown in Figure 5, for an OFDMA system. SLM is a non-distortion, stochastic, method that incurs little loss in efficiency.

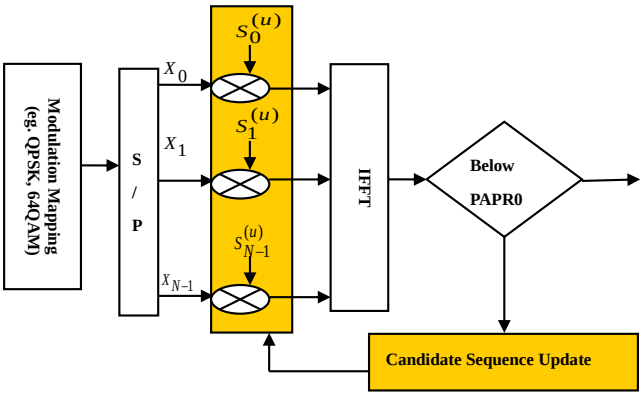


Figure 5 SLM schematic diagram for OFDM transmission

Stochastic approaches often demand more computations due to uncertain properties of the signal statistics, however the approach outlined in this paper, relevant to Figure 5, aims at minimizing the number of computations and associated computation time.

The binary information sequence 1011…is mapped to complex symbols using QPSK, or 64QAM or a similar modulation technique, to produce a parallel data stream denoted by the N-dimensional symbol vector:

$$\mathbf{x} = [x_0 \quad x_1 \quad \ldots \quad x_{N-1}]^T \tag{5}$$

A, phase adjusted, symbol vector is then formed from the component-wise product of **x** with a candidate phase sequence, $\mathbf{s^{(u)}}$, given below:

$$\mathbf{s^{(u)}} = \left[S_0^{(u)} \quad S_1^{(u)} \quad \ldots \quad S_{N-1}^{(u)}\right]^T \quad 1 \le u \le U \tag{6}$$

Applying the Inverse Discrete Fourier Transform (IDFT), with L-times over-sampling, to the phase adjusted symbol vector, produces a sampled version of the corresponding continuous-time multi-carrier signal:

$$\mathbf{x^{(u)}} = \left[x_0^{(u)} \quad x_1^{(u)} \quad \ldots \quad x_{NL-1}^{(u)}\right]^T \tag{7}$$

It is generally assumed, ref [12], that taking L = 4 allows a satisfactory estimate of the PAPR to be calculated from $\mathbf{x^{(u)}}$ as follows:

$$PAPR\left(\mathbf{x^{(u)}}\right) = 10 \times \log_{10}\left(\frac{\max\limits_{0 \le n \le (NL-1)} \left|x_n^{(u)}\right|^2}{\mathrm{E}\left(\left|x_n^{(u)}\right|^2\right)}\right) \text{ dB.} \tag{8}$$

where E(.) denotes expectation. After the PAPR comparisons among the U time signals $x^{(u)}$ for u=1,2…U, the one with the minimum PAPR is selected for transmission.

$$\hat{x} = arg \min_{0 \le u \le U-1} \left\{PAPR\left(\mathbf{x}^{(u)}\right)\right\} \tag{9}$$

Figure 5 shows a serial searching approach by which the search among the candidate phase sequences may terminate when a satisfactory PAPR is achieved. For each search, the complementary cumulative distributed function (CCDF) of PAPR, i.e. the probability that PAPR exceeds a threshold PAPR0, can be estimated by:

$$\Pr\left(\mathrm{PAPR}\left(\mathbf{x^{(u)}}\right) > \mathrm{PAPR0}\right) = 1 - (1 - e^{-\mathrm{PAPR0}})^N \tag{10}$$

This expression assumes that the central limit theorem applies such that the time signal samples are independent complex Gaussian variables with a mean of zero. This is however not accurate when over-sampling is applied. Instead empirical results are often used for the estimation. Nonetheless if U alternative time signals are mutually independent, the PAPR CCDF can be obtained from:

$$\Pr\left(\mathrm{PAPR}_{\mathrm{SLM}} > \mathrm{PAPR0}\right) = \left(\Pr\left(\mathrm{PAPR}\left(\mathbf{x^{(u)}}\right) > \mathrm{PAPR0}\right)\right)^U \tag{11}$$

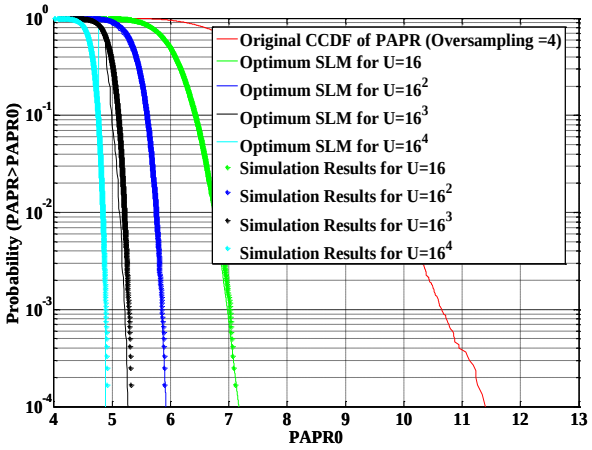


Figure 6 PAPR CCDFs using optimum SLM with incremental phase sequences (with oversampling)

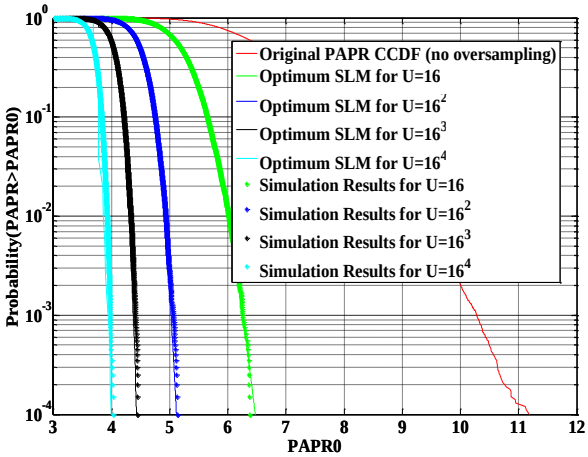


Figure 7 PAPR CCDFs using optimum SLM with incremental phase sequences (without oversampling)

Any two phase sequences with high cross-correlation properties produce PAPR values which are very close to each other hence, in order to keep the search space small, phase sequence design should ensure that candidate sequences have relatively low cross-correlation values. In this way the search time is reduced without compromising the optimum SLM approach, since the number of candidate sequences is thereby reduced.

## VII. PAPR REDUCTION LOW COST SEARCH TECHNIQUE

Referring to the schematic diagram in Figure 5, searching is performed to find a phase sequence, identified by a set of coefficients $\{u_1, u_2, \ldots \ldots, u_w\}$, whose PAPR is no bigger than the specified target threshold, using candidate phase sequences with low cross-correlation properties. Since each coefficient um has M possible values dictated by the order of MPSK modulation, a maximum of MW updates has to be performed by the search. The order of MPSK modulation is restricted on the one hand by the computational cost for the transmitter, and on the other hand by the phase tracking accuracy that can be achieved at the receiver.

At each update, instead of calculating a full NL-point Inverse Fast Fourier Transform (IFFT) for the complete time-domain vector $\boldsymbol{x}$, it is more efficient to compute one sample, $x_n$, at a time. As soon as the value of $x_n$ appears for which it is clear that the PAPR will exceed the threshold, the corresponding phase sequence is dropped, and the search moves on to the next one. The search terminates when a phase sequence is found which gives a PAPR value that is below the threshold specified when all corresponding time-domain samples have been used for that PAPR calculation. This process reduces the search space and thereby minimizes search latency.

Further complexity reduction might be achieved through employing a low complexity search mechanism based on the partial IFFT.

At the receiver, the correct demodulation of all sub-carriers requires a cancellation of the phase adjustments that were applied at the transmitter. The expectation is that a code-book approach would be adopted where the phase sequence used at the transmitter has its code-book number broadcast to the receiver as side information for phase recovery for every OFDM symbol.

## VIII. CONCLUSION

The PAPR or CM relationship with amplifier back-off is dependent on the complexity of modulated signals. Varying the number of component signals and their relative signal strengths affects the PAPR/CM relationship.

For current systems, the PAPR/CM PA back-off estimation can be determined empirically using experimentally obtained data – from appropriate UE Power Amplifiers and through the generation of the appropriate Uplink Modulated Waveforms with their associated power levels.

With a low CM/PAPR, a low PA back-off is needed which leads to a higher PA efficiency and lower power consumption (Greener operation). CM back-off estimation has a lower error variation around the ideal back-off point than PAPR back-off estimation does. To cater for this error the final CM back-off estimation can be lower than the corresponding PAPR back-off estimation.

To further increase the accuracy of the PA back-off estimation, an effective PAPR reduction method, of low complexity, having minimum associated overhead, introducing no distortion, and introducing little latency, such as the method outlined in this paper, is required.

The PAPR technique is also applicable to all types of subcarrier modulation (eg. PSK or QAM) and any number of subcarriers and low complexity is achieved by reducing the statistics search space associated with the optimum SLM, and

employing a low complexity search mechanism based on the partial IFFT.

Finally, usage of PAPR reduction techniques will reduce the errors in PA back-off for both CM and PAPR usage, but the relative gains of CM over PAPR for back-off estimation will still remain, and hence it might be concluded that use of PAPR reduction together with CM estimation for determining PA back-off will result in the "Greenest" methodology.

## IX. REFERENCES


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