LINE ENHANCER METHODS FOR CARRIER TRACKING IN QAM/PSK DATA SIGNALS

Randall Flint (University of Utah, Salt Lake City, Utah, USA; rkflint@sisna.com);
Behrouz Farhang-Boroujeny (University of Utah, Salt Lake City, Utah, USA; farhang@ece.utah.edu)

ABSTRACT

In mobile communication systems, the Doppler shift is a very common cause of time-varying changes in the carrier-frequency offset. This paper discusses two adaptive line enhancer algorithms which are used to aid in the process of detecting and tracking these time-varying changes. Specifically, it describes the use of an adaptive line enhancer in a quadrature amplitude modulation (QAM), or phase shift keying (PSK), carrier-recovery system. It shows that finite-impulse response (FIR) adaptive line enhancers are not well-suited for tracking changes in the carrier-frequency offset. Then, based on the LMS algorithm, an infinite-impulse response (IIR) adaptive line enhancer algorithm is presented, which is much more effective at tracking the carrier frequency offset. Next, an FIR post-filtering operation is described that is optimal for out-of-band noise rejection, and is used in conjunction with the IIR adaptive line enhancer to further improve its line-enhancement capabilities. Finally, simulation results are presented to compare the effectiveness of the FIR and IIR adaptive line enhancer algorithms at tracking changes in the carrier-frequency offset.

1. INTRODUCTION

In mobile communication systems, the Doppler shift is a very common cause of time-varying changes in the carrier-frequency offset. If these changes are not properly detected, and corrected, the transmitted data will be lost.

This is illustrated by looking at the equation that governs the size of the carrier frequency offset due to the Doppler effect. It is:

$$F_{off} = \frac{F_c}{F_s}\frac{V_r}{c} \qquad (1)$$

where F_c is the carrier frequency, F_s is the sample rate, V_r is the relative velocity between the transmitter and the receiver, and c is the speed of light. In mobile systems, V_r is constantly changing, which results in a time-varying carrier frequency offset. In order to avoid data loss, the receiver must be able to track these time-varying changes and correct for their effects.

One system used for the detection and correction of carrier frequency offsets is shown in Figure 1. This system, which is targeted for use with QAM or PSK data signals, operates by synthesizing a complex sinusoid at the negative of the frequency of the carrier frequency offset, and then by applying that sinusoid to the input signal to shift its spectrum down to baseband.

The left-most block in this system, called the non-linear block, takes the input signal to the M^{th} power in order to convert the QAM or PSK signal into a complex sinusoid buried in a wide-band noise. The frequency of this sinusoid is M times the carrier frequency offset, where the value of M is determined by the modulation type.

The next two blocks in this system prepare the signal for the adaptive line enhancer by whitening the wide-band noise. Then, the adaptive line enhancer removes as much

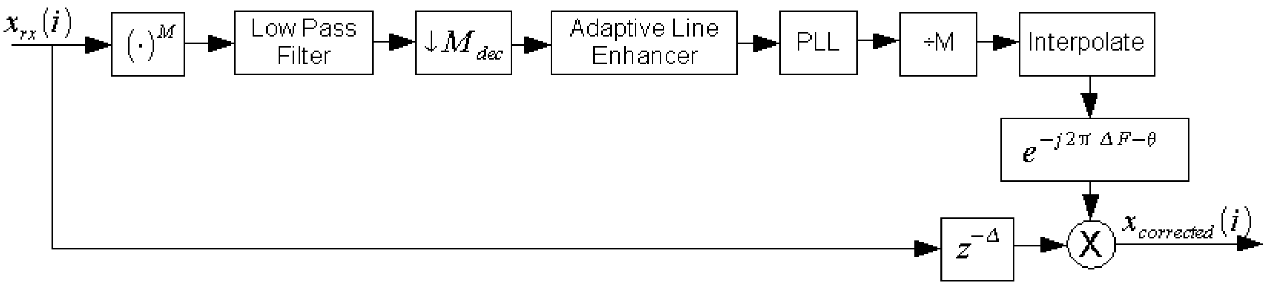

Figure 1: QAM/PSK Carrier Recovery System

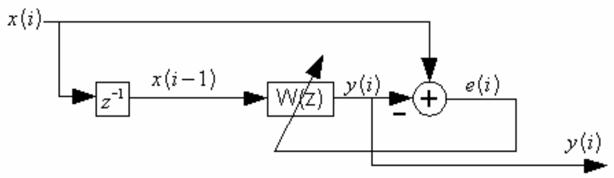

Figure 2: Adaptive line enhancer block diagram

noise as possible from the input sinusoid, to make it possible for the phase locked loop (PLL) to lock onto the frequency of the sinusoid. Finally, the PLL and subsequent blocks synthesize a complex sinusoid, at the negative of the carrier frequency offset, and multiplies the system's input signal by that sinusoid to shift its spectrum down to baseband. A complete explanation of the operation of this system can be found in [1].

In summary, the adaptive line enhancer must be able to track changes in the frequency of a complex sinusoid that is buried in a white noise, and it must remove as much noise as possible from that sinusoid.

Figure 2 shows the block diagram of an adaptive line enhancer. The filter W(z) is called the prediction filter, which is used as a band pass filter in order to minimize the noise in the output sinusoid. The frequency response of the prediction filter must constantly be adapted in order to determine the frequency of the input sinusoid, and to remove as much noise as possible. This means that the optimal frequency response of the prediction filter, for maximum noise rejection, is a band pass filter with a very narrow passband centered at the frequency of the input sinusoid.

This paper will discuss two possible implementations of an adaptive line enhancer algorithm. The first algorithm is implemented using an FIR prediction filter. It will be shown that this method is less effective at noise rejection while tracking changes in the input sinusoid's frequency. The second algorithm is implemented using an IIR prediction filter. It will be shown that this algorithm is much more effective at tracking these changes. Then, a post filtering operation will be discussed that further improves the IIR adaptive line enhancer's noise rejection. Finally, the tracking ability of each adaptive line enhancer algorithm will be shown through simulation.

2. THE FIR ADAPTIVE LINE ENHANCER

It is common practice to implement the adaptive line enhancer using an FIR prediction filter. One advantage of this approach is that the FIR adaptive line-enhancement algorithm is well understood, easy to implement, and its performance can be analyzed mathematically. The main drawback to using this approach is that this algorithm is slow to forget past input frequencies, as it attempts to track changes in the input sinusoid's frequency [2]. This section explains the cause of this drawback, which is referred to as the memory effect.

In order to understand the memory effect, note that the ability of the adaptive line enhancer to reject noise from the input sinusoid depends on the frequency response of the prediction filter. The ideal frequency response would pass only the frequency of the input sinusoid, while rejecting any other frequency content in the signal. The wider the passband of the frequency response, the more noise is passed through to the output signal. Consequently, the effectiveness of the adaptive line enhancer at noise rejection, is determined by the number of coefficients in the prediction filter, or N, and the ability of the adaptive algorithm to determine the optimal filter coefficients.

The ability of the adaptive algorithm to determine the optimal filter coefficients is governed by the fact that the prediction filter's frequency response will converge the fastest in the frequency bands where the input signal contains the most power [3]. This is represented mathematically as the adaptive algorithm's convergence time constant, which is related to the eigenvalues of the input signal’s auto-correlation matrix, according to the equation:

$$\tau_i = \frac{1}{4\mu\lambda_i} \quad (2)$$

where μ is the algorithm step size, and λ_i is an individual eigenvalue of the auto-correlation matrix. For a prediction filter length of N coefficients, the adaptive algorithm will have N convergence time constants, and N eigenvalues. It can be shown that the magnitude of each of these eigenvalues is proportional to the power contained in the input signal in each of the N frequency bands [3]. In other words, the input signal's power-spectral density determines the size of the eigenvalues of the input signal's autocorrelation matrix, and the corresponding convergence time constant in each of the frequency bands.

In this system, the power-spectral density of the input signal has a single peak at the frequency of the input sinusoid, and a constant wide-band component throughout the rest of the frequency spectrum. Consequently, the eigenvalues of the input signal's autocorrelation matrix consist of a single eigenvalue at the frequency of the input sinusoid with a value of $N+\sigma^2$, and N-1 repeated eigenvalues with the value of σ^2, where σ^2 is the variance of the noise in the input signal [3]. From equation 2, it can be seen that the convergence time-constant is shortest at the frequency of the input sinusoid, and longer elsewhere. In other words, the relative sizes of these convergence time constants result in the adaptive line enhancer adapting quickly in the frequency band that contains the input sinusoid's frequency, but at a slower rate throughout the rest of the frequency spectrum.

This fact has a significant impact on the tracking behavior of the FIR adaptive line enhancer. When the frequency of the input sinusoid changes, the peak in the

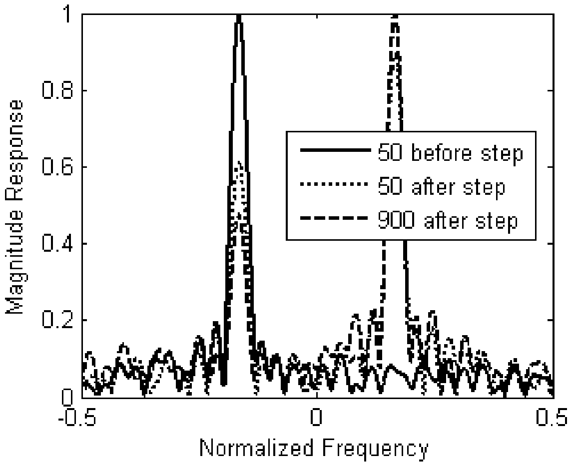

Figure 3: The FIR prediction filter's frequency response before, and after, a step in the input sinusoid's frequency.

power-spectral density moves with it, and so does the frequency band with the largest eigenvalue. The result is that the passband of the prediction filter is quickly adapted to include the new frequency of the input sinusoid. However, the portion of the passband at the past frequency of the input sinusoid is slowly forgotten, due to the smaller amount of power in the input signal at that frequency. This results in a memory of the past frequencies of the input sinusoid, and additional noise being passed through the prediction filter.

In order to illustrate the memory effect, Figure 3 shows the frequency response of the adaptive line enhancer's prediction filter before, and after, a step in the input sinusoid's frequency. Before the step, the passband on the left is centered at the frequency of the sinusoid. After the step, this passband disappears slowly, while the passband on the right, which is at the new frequency of the input sinusoid, is quickly developed. The result of this is that unnecessary noise is passed through the adaptive line enhancer prediction filter at the past frequency of the input sinusoid.

3. IIR ADAPTIVE LINE ENHANCER

A more effective approach to designing an adaptive line enhancer is to use an IIR prediction filter with the following transfer function:

$$W(z) = \frac{(1-\alpha)e^{j2\pi\varphi}}{1-\alpha e^{j2\pi\varphi}z^{-1}} \ . \qquad (3)$$

This transfer function is a single-pole band pass filter. The frequency response of this filter is controlled by two coefficients, φ and α. The coefficient φ determines the center frequency of the filter's passband by rotating the pole location in the z-plane. The coefficient α determines the width of the passband by determining the radius of the pole in the unit circle.

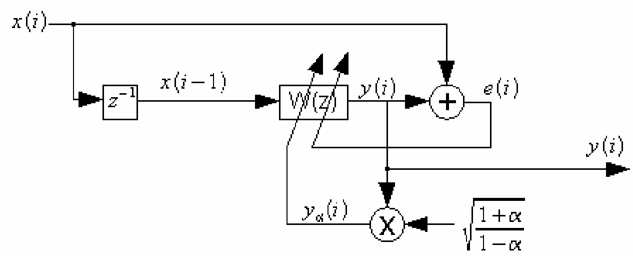

Figure 4: IIR adaptive line enhancer block diagram

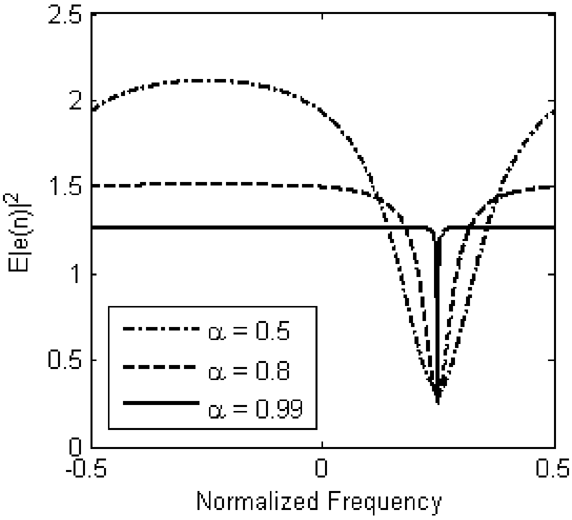

Figure 5: IIR φ cost function

By adapting these coefficients in such a way that the center frequency of the passband is placed at the frequency of the input sinusoid, and the width of the passband is minimized, this filter is very effective at rejecting noise from the input sinusoid.

Figure 4 shows the block diagram of the adaptive line enhancer algorithm used to simultaneously adapt the IIR prediction filter's coefficients. The φ coefficient is adapted by minimizing the cost function $E|e(i)|^2$, and the α coefficient is adapted by maximizing the cost function $E|y_\alpha(i)|^2$.

The LMS algorithm is one method of adapting these coefficients in order to maximize, or minimize, the value of the cost function. This is done by approximating the value of the cost function based on the current filter coefficients, the input signal, and the error signal. Then, based on the slope of the cost function at that point, the filter coefficients are adjusted in the direction of the slope, which moves the value of the cost function either towards its minimum, or its maximum.

A plot of the cost function used to adapt φ is shown in Figure 5. Clearly, the minimum value of the cost function corresponds to the input sinusoid's frequency. By minimizing the value of this cost function, the current approximation of φ will approach the input sinusoid’s

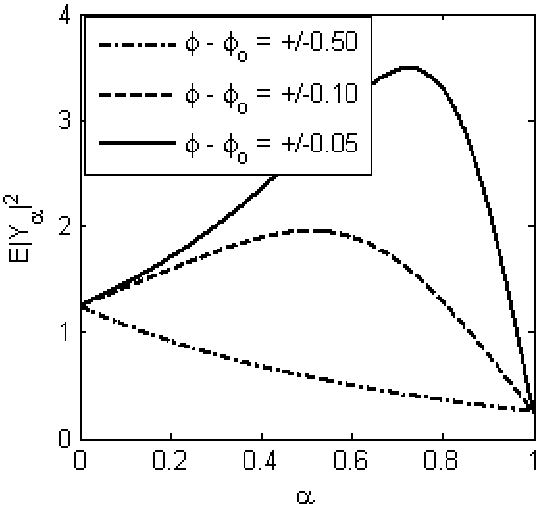

Figure 6: IIR α cost function

Table 1: IIR adaptive line enhancer algorithm

$$\mu_{\varphi} = \mu_0 (1-\alpha(i))^3 \tag{4}$$

$$m(i) = (1-\alpha(i))x(i) + \alpha(i)y(i-1) \tag{5}$$

$$y(i) = m(i)e^{j2\pi\varphi(i)} \tag{6}$$

$$e(i) = x(i+1) - y(i) \tag{7}$$

$$\frac{\delta y^*(i)}{\delta\varphi(i)} = e^{-j2\pi\varphi(i)}[\alpha(i)\frac{\delta y^*(i-1)}{\delta\varphi(i-1)} - j2\pi m^*(i)] \tag{8}$$

$$\nabla_{\varphi}\hat{\zeta} = -2\,\mathrm{Re}[e(i)\frac{\delta y^*(i)}{\delta\varphi(i)}] \tag{9}$$

$$\varphi(i+1) = \varphi(i) - \mu_{\varphi}\nabla_{\varphi}\hat{\zeta} \tag{10}$$

$$\frac{\delta y^*(i)}{\delta\alpha(i)} = e^{-j2\pi\varphi(i)}[\alpha(i)\frac{\delta y^*(i-1)}{\delta\alpha(i-1)} - e^*(i-1)] \tag{11}$$

$$\nabla_{\alpha}\hat{\zeta} = 2\frac{1+\alpha(i)}{1-\alpha(i)}\mathrm{Re}[y(i)\frac{\delta y^*(i)}{\delta\alpha(i)}] + 2\frac{|y(i)|^2}{(1-\alpha(i))^2} \tag{12}$$

$$\alpha(i+1) = \alpha(i)0\mu_{\alpha}\nabla_{\alpha}\hat{\zeta} \tag{13}$$

frequency φ_o, and place the center of the prediction filter's passband at that frequency.

The φ cost function is plotted for several values of α, in order to show that the slope of the cost function is very dependent on α. For α values near one, the slope is very small except near the frequency of the input sinusoid. Consequently, the adaptive line enhancer will converge slowly as for values of α near one, and faster for values of α near zero. This has the unfortunate effect that the filter bandwidth has to be expanded in order to speed convergence of the φ coefficient to its optimal value, and it results in two opposing constraints. First, the value of α must be kept small in order to decrease the φ coefficient's convergence time; and second, the value of α must be kept near one in order to maximize noise rejected from the input sinusoid by the adaptive line enhancer. Consequently, a cost function must be selected that will minimize the value of α, when φ is far from the frequency of the input sinusoid, and maximize its value otherwise. The cost function $E|y_\alpha(i)|^2$ meets this criteria [3][4], where:

$$y_{\alpha}(i) = \sqrt{\frac{1+\alpha}{1-\alpha}}\,y(i)\,. \tag{14}$$

Figure 6 shows a plot of the α cost function for different distances between the current estimate of φ, and its optimal value φ_o. Note that when φ is far from φ_o, the maximum value of the cost function is near 0, and as φ approaches φ_o, this maximum moves toward one. Consequently, by maximizing the value of this cost function, the value of α can be adapted in such a way that the opposing constraints are met. Table 1 shows a listing of the resulting adaptive line enhancer algorithm. Note that the value of α must be explicitly limited between zero and one in order to keep the transfer function's pole within the unit circle.

Finally, Figure 7 shows the frequency response of the IIR prediction filter before, and after, a step in the input sinusoid's frequency. As can be seen, the old frequency of the input sinusoid is completely forgotten shortly after the step. In other words, the IIR adaptive line enhancer does not suffer from the memory effect that limits the tracking ability of the FIR adaptive line enhancer. Also, the figure shows a temporary expansion of the prediction filter's passband. This

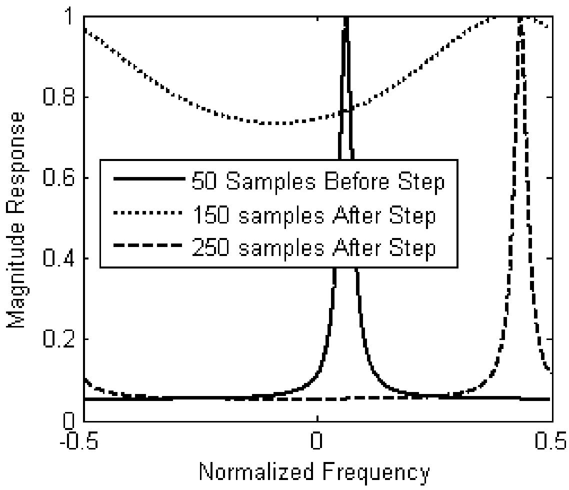

Figure 7: The IIR prediction filter's frequency response before and after a step in the input sinusoid's frequency.

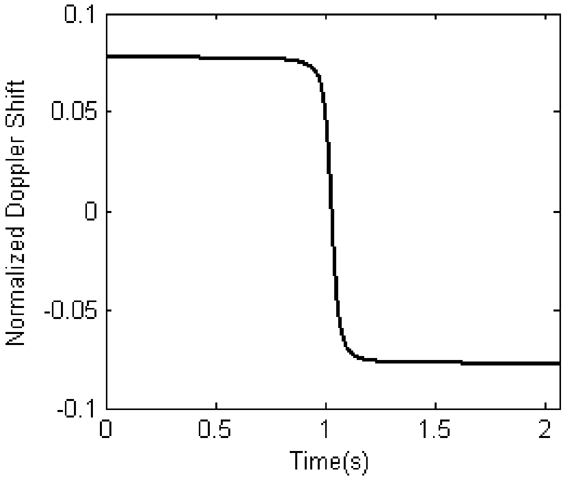

Figure 8: Time-varying carrier frequency offset caused by the Doppler effect in inter-vehicle communication.

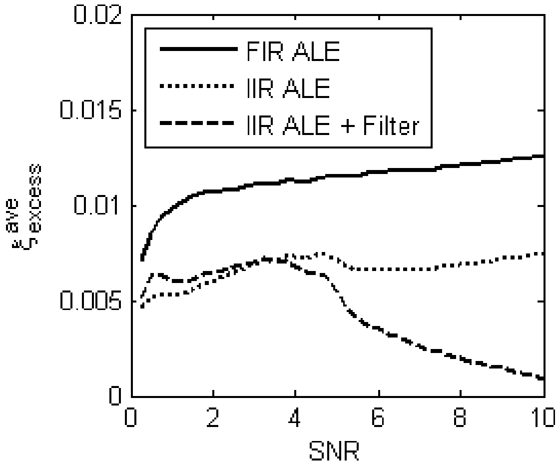

Figure 9: Tracking comparison of the three adaptive line enhancer (ALE) algorithms

is due to the adaptation of α, which is required to minimize the convergence time of the φ coefficient.

4. FIR POST-FILTER

After the convergence of the IIR adaptive line enhancer's φ coefficient, the frequency of the input sinusoid is known by the system. This allows a band pass post-filtering operation to be applied to the adaptive line enhancer's output signal in order to achieve further noise rejection.

This filter is implemented as an FIR band pass filter with the center frequency being determined by the IIR adaptive line enhancer. The transfer function of this filter is:

$$H(z) = \frac{e^{j2\pi\varphi_o}}{N} \sum_{i=0}^{N-1} e^{j2\pi\varphi_o i} z^{-i} \tag{15}$$

where φ is the desired center frequency determined by the IIR adaptive line enhancer. This transfer function represents a filter made up of zeros spaced equally around the unit circle. It can be shown, using the method of Lagrange multipliers, that this transfer function is the optimal FIR filter for use in removing noise from a sinusoid buried in white noise [4]. Although the input signal to the post-filter is a sinusoid buried in noise that has been colored by the adaptive line enhancer, it will be shown that the postfilter is effective at further noise rejection.

Finally, by noting that this transfer function is a geometric series, an equivalent transfer function can be derived that is much more efficient to implement:

$$H(z) = \frac{e^{j2\pi\varphi_o}}{N} \frac{1-\beta^N e^{j2\pi\varphi_o N} z^{-N}}{1-\beta e^{j2\pi\varphi_o} z^{-1}} \tag{16}$$

where β is a constant that is slightly less than one. The purpose for adding β to the transfer function is to keep the pole inside the unit circle, and to ensure stability of the algorithm.

5. SIMULATION RESULTS

A common scenario that necessitates the tracking of the carrier frequency offset due to Doppler shifts is in inter-vehicle communications. Assume two vehicles are communicating wirelessly as they pass each other going opposite directions on the freeway. Due to the Doppler effect, the relative velocity between these vehicles will cause a time-varying shift in the carrier frequency offset that must be tracked by the adaptive line enhancer. Figure 8 shows an example of the Doppler shift as seen by the carrier recovery algorithm.

Based on this scenario, simulations have been done to compare the effectiveness of each of the adaptive line enhancer algorithms in tracking changes in the carrier frequency offset. This is done by comparing a time average of the excess mean-square error, during the tracking phase of the simulation, as a relative measure of each algorithm's effectiveness. Mathematically, this is defined as:

$$\xi_{excess}^{ave} = average(E\,|\,e(i)\,|^2 - \sigma^2)\,. \tag{17}$$

Finally, the convergence rate of the FIR adaptive line enhancer is determined by the signal-to-noise ratio. Consequently, ξ_{excess}^{ave} is measured over a range of signal-to-noise ratios.

Figure 9 shows the results of the simulations. As can be seen, ξ_{excess}^{ave} is much higher for the FIR adaptive line enhancer due to the memory effect. Also, the IIR adaptive line enhancer results in a significant improvement over the FIR implementation, especially for high signal-to-noise ratios. Finally, the postfilter adds an additional advantage to that of the IIR adaptive line enhancer.

6. CONCLUSION

In conclusion, an IIR adaptive line enhancer algorithm and a companion postfiltering operation has been presented that results in a significant improvement in the noise rejection that can be attained over the FIR adaptive line enhancer algorithm. This results from the IIR adaptive line enhancer's lack of a memory of past input frequencies, and the optimal filtering capabilities of the postfilter. Although the exact numbers in this simulation will not match those of certain systems, the effective result will be the same, that of significant noise rejection from the input sinusoid.

10. REFERENCES

[1] B. Farhang-Boroujeny, *Signal Processing Techniques for Software Radio*, Lulu, North Carolina, 2008.

[2] S. D. Hayward, and C. J. Sprigings, "Adaptive line enhancement via subspace tracking," *Signals, Systems and Computers*, Vol. 2, pp. 1872-1877, 2000.

[3] B. Farhang-Boroujeny, *Adaptive Filters*, John Wiley & Sons Ltd, West Sussex, England, 1998.

[4] R. S. Flint, "Tracking Analysis of an Improved Infinite Impulse Response Adaptive Line Enhancer," Master's Thesis, University of Utah, Salt Lake City, Utah, 2008.