A versatile Filter Structure to Generate and Compress Binary and Polyphase Complementary Spreading Codes

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Abstract

This paper reviews the structure and properties of binary, and polyphase complementary spread spectrum codes, synthesis techniques to implement their code generators and code compressors, as well as a number of applications to modern communication systems.

1. INTRODUCTION

Complementary codes (CC) due to Golay are pairs of orthogonal binary phase sequences with interesting properties. The primary one is that their separate correlation sequences have equal amplitude and opposing polarity side lobes. Consequently, the sum of their correlation sequences exhibit zero value side lobes. Thus while there are no phase codes whose linear correlation sequences are side lobe free, we can synthesize such correlation sequences as the sum of the complementary correlation sequences. The family of CC codes is easily extended from binary CC to non binary CC. The CC, first applied to radar systems, are now enjoying high interest in spread spectrum communication systems as can be seen in IEEE 802.11b as complementary code keying (CCK) and as preambles for random access channels (RACH) 3-G cellular systems.

A simple recursion process to form complementary codes of length 2^p proceeds as follows. From the code pair A(n) and B(n) we form the next code pair A(n+1) and B(n+1) by appending and complementing as shown in (1).

$$\begin{aligned} A(n+1) &= [A(n)\, B(n)] \\ B(n+1) &= [A(n)\, \overline{B}(n)] \end{aligned} \tag{1}$$

The initial code pair is denoted the kernel, and while it can be any arbitrary sequence pair, it is commonly selected as A(0) = B(0) = 1. A sequence of complementary codes formed by this process is shown in (2). For ease of notation we use 1 and 0 as field elements which when converted to waveform levels are mapped to +1 and -1 respectively.

n	A(n)	B(n)
0	[1]	[1]
1	[1 1]	[1 0]
2	[1 1 1 0]	[1 1 0 1]
3	[1 1 1 0 1 1 0 1]	[1 1 1 0 0 0 1 0]

(2)

The coding process can be modified at each step in the iteration process by i) time reverse A, ii) time reverse B, iii) time reverse A and B, iv) complement even bits of A and B, v) complement odd bits of A and B, vi) complement A, vii) complement B, viii), complement A and B, (ix) interchange A and B. A final code generation option, often employed in the radar community, is interleaving of successive elements of A and B and of A and $\overline{B}$.

There exist a set of simple, efficient filter structures that generate the CC as their impulse responses and, more importantly, implement the matched filters that de-spread or compress the CC. The structure shown in figure 1 is a CC spreading filter. Figure 2 presents the impulse response of this filter at successive stages in its cascade. The structure shown in figure 3 is the compressing filter corresponding to the filter shown if figure 1. Figure 4 presents the compression response at successive stages in its cascade.

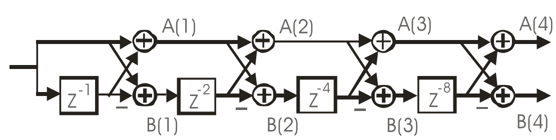

Figure 1. CC Generating Filter

The responses illustrated in figure 4 correspond to the matched filter being time aligned with the received time series and constructing the peak of the correlation series. The full correlation series of the two codes exhibit the side lobe structure the code pair was designed to suppress. If the pair of CC codes are transmitted simultaneously as I and Q components of a complex carrier there is a possibility that there will be residual phase shift between the complex signals at the transmitter and

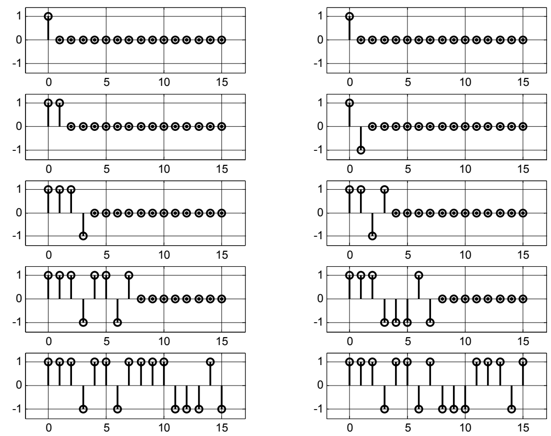

Figure 2. Impulse Response: CC Generating Filter

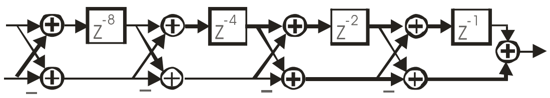

Figure 3. CC Compression (Matched) Filter

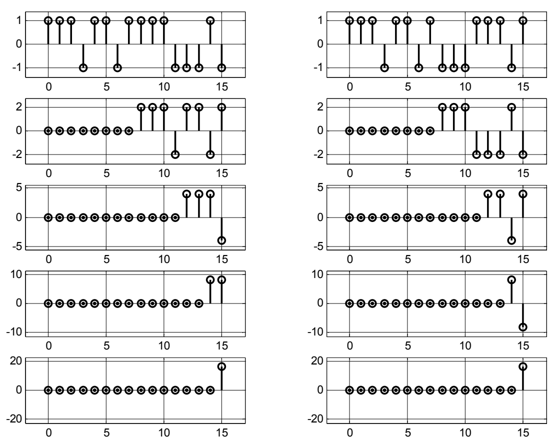

Figure 4 Compression Response: CC Matched Filter

receiver. This phase shift permits coupling between the A and B codes in their matched filters which is observed at the filter output as cross correlation of the two codes. The A and B codes do exhibit significant cross correlation. Figure 5 presents the full response of the matched filters along with their side lobe free sum and the cross correlation of the two codes A and B. The amount of this cross correlation leaked into the desired correlation output is 2 sin(θ), where θ is the phase angle offset of the receiver IQ pair.

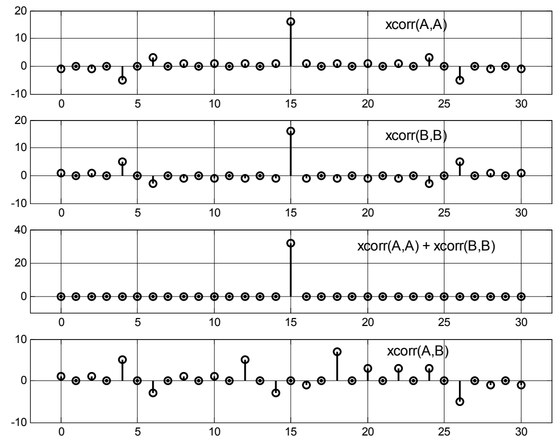

Figure 5. Full Replica and Cross Correlation of A and B Codes

2. Related Codes

The non-zero cross correlation side lobes between the A and B codes can be cancelled by a second pair of sequences called the *code mates* of the first pair. The mates are also complementary and exhibit the same side lobe cancellation property of the original codes. The cross correlation side lobes of the original code pair and of the mate code pair exhibit equal amplitude and opposing polarity side lobes. Hence the sum of the cross correlation sequence of the complementary pair and of the mate pair similarly sum to zero. The correlation sequence assembled by combining the correlation sequences of the complementary pair and of their mates is characterized by a single correlation peak, the response of an ideal probing signal.

The code mates C and D are related to codes A and B as indicated in (3).

$$\begin{aligned} C &= \text{Time Reverse (B)} \\ D &= \text{-Time Reverse (A)} \end{aligned} \tag{3}$$

The reversal of the time series is accomplished by moving the negative sign on the butterflies in the generating filter from the lower leg to the upper leg or equivalently changing the recursion shown in (1) to become that shown in (4). The sign reversal is accomplished by reversing the signs in the lower leg of the final butterfly of the filter cascade.

$$\begin{aligned} A(n+1) &= [\overline{A}(n)\ B(n)] \\ B(n+1) &= [A(n)\ B(n)] \end{aligned} \tag{4}$$

Figure 6 presents the full response of the matched filters along with their side lobe free sum and the cross correlation of the two code mates C and D. Notice that, as expected the sum of the two correlation sequence is side lobe free and that the cross correlation sequence has side lobes of opposing polarity to those seen in figure 5. Note too, that the matched filter side lobes for the A and B codes are also of opposing polarity of the corresponding matched filter side lobes for the code mates C and D. When we form the sum of the two pairs of correlation sequences, any residual cross correlation term in the first sum is canceled by the corresponding cross correlation terms in the second sum.

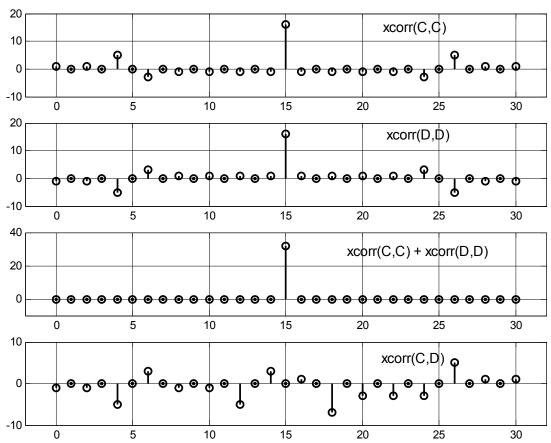

Figure 6. Full Replica and Cross Correlation of C and D Codes

3 Polyphase Codes

We note that the stages of the CC generating filters and of the CC compression filters are formed by delay registers and butterfly structures. The delay registers are all-pass networks, networks that satisfy the relationship shown in (5).

$$|H(\theta)| = 1, \ -\pi \le \theta < \pi \tag{5}$$

All pass networks exhibit unity gain at all frequencies and only contribute phase shift to their transfer functions. It is common to perform frequency transformations of filters by replacing all pass networks in the filter with different all pass networks. The most common such transformation is the low-pass to low-pass transformation shown in (6). Transformations of the form shown here preserve the structure of the filter while inducing a frequency mapping between the original and transformed spectral responses.

$$\frac{1}{z} \Rightarrow \frac{1 - bz}{z - b} \tag{6}$$

The all-pass transformation of interest to us here is the one shown in (7).

$$\frac{1}{z^n} \Rightarrow \frac{1}{z^n} e^{j\varphi(n)} \Rightarrow \frac{1}{z^n} W_n \tag{7}$$

We can modify the CC generating filter by replacing each all-pass delay register segment with another all-pass register segment with an associated arbitrary phase rotator. This transformation modifies the iterative relationship of (1) to the relationship shown in (8) and alters the delayed butterfly structure of the filter to match that shown in figure 7. The relationship shown in (8) first noted by Sivaswamy who generalized the Golay binary CC relationships to include polyphase CC.

$$\begin{aligned} A(n+1) &= [A(n) \ \ W_n B(n)] \\ B(n+1) &= [A(n) \ \ W_n \bar{B}(n)] \end{aligned} \tag{8}$$

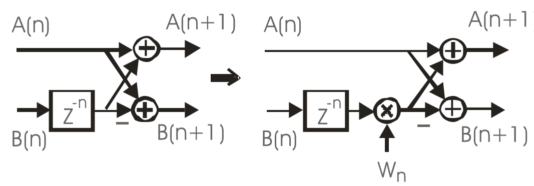

Figure 7. Filter Segment All-pass Frequency Transformation

The butterfly in the modified filter segment has a striking resemblance to the butterfly structure of the fast Fourier transform (FFT). The CC generating filter and compression filter can in fact be cast in the same flow diagram as an FFT.

The polyphase CC maintains all of the properties of the binary CC. These include the complementary canceling side lobes of the pair of cross correlation functions and the cancellation of the cross correlation side lobes with code mates.

A subset of complementary codes of given length N are mutually orthogonal. The number of orthogonal codes of length N is $\log_2(N)$. This corresponds to the number of butterflies in the CC generating filter. At each butterfly stage, the complement (or negation) can be moved from the lower leg to the upper leg, and each distinct sequence of signs in the cascade forms another orthogonal code.

4. Applications of Polyphase Codes

The polyphase complementary codes have found interesting applications in a number of communication tasks. One of them is called complementary code keying (cck), a process now embedded in the IEEE 802.11b standard for 2.4 GHz wireless LAN. In IEEE 802.11 data was originally encoded with a binary phase modulated direct sequence spread spectrum (DS-SS) code. The sequence selected used for the spreading is the 11-bit Barker sequence (1 0 1 1 0 1 1 1 0 0 0), a sequence selected for its good peak to side lobe level ratio of its auto correlation sequence. Modulation uses the Barker sequence as a symbol which can carry 2 bits per symbol by ordinary QPSK modulation. The data rate at 2-bits per symbol using the 11-bit Barker sequence is 2-Mbps.

CCK modulation uses a polyphase sequence of length 8 with the selection of 4-possible cardinal phase angles (0, pi/2, pi, and 3pi/2) at each of three butterfly stages plus one more common phase angle to spin the phase of the entire code sequence. This results in a set of 256 8-chip code words. Since there are 4^8 or 65,536 possible 8-chip sequences, the 256 code words contain an inherent error correcting capability. The encoded sequence can be formed by a 3-stage cc generating filter with two bits assigned to select the phase angle at each stage. Thus each 8-chip sequence can deliver 6-bits of data per symbol. The structure of the polyphase generator filter is shown in figure 8. The impulse response of the filter is shown in (9).

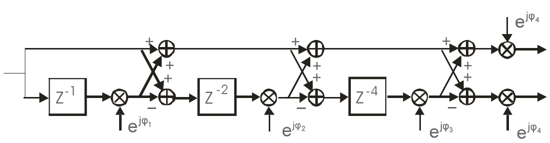

Figure 8. Polyphase CCK Generator Filter

The last phase rotator through angle ϕ_4 does not change the orthogonality of the set of sequences but rather rotates the set so the, zero-phase, first sample moves to one of the four corners of the signal phase space. The phases $\theta(n)$ of the 8-chips is more commonly presented by the mapping shown in (10). Here the phases of the rotators are shown explicitly as $\phi(n)$ and the phase of the sign reversal in the filter is shown as π.

The receiver for the polyphase CC is simply a set of 256 matched filters, one for every one of the possible transmitted sequences. The engine that performs this feat is simply a fast Walsh Transform. The reader is directed to articles on CCK coding for this aspect of CCK decoding.

$$
\begin{aligned}
s_{1-t} &: 1+e^{j\phi_1}z^{-1} \\
s_{1-b} &: 1-e^{j\phi_1}z^{-1} \\
s_{2-t} &: 1+e^{j\phi_1}z^{-1}+e^{j\phi_2}z^{-2}-e^{j(\phi_1+\phi_2)}z^{-3} \qquad (9a) \\
s_{2-b} &: 1+e^{j\phi_1}z^{-1}-e^{j\phi_2}z^{-2}+e^{j(\phi_1+\phi_2)}z^{-3} \\
s_{3-t} &: 1+e^{j\phi_1}z^{-1}+e^{j\phi_2}z^{-2}-e^{j(\phi_1+\phi_2)}z^{-3}+e^{j\phi_3}z^{-4} \\
&\quad +e^{j(\phi_1+\phi_3)}z^{-5}-e^{j(\phi_2+\phi_3)}z^{-6}+e^{j(\phi_1+\phi_2+\phi_3)}z^{-7} \\
s_{3-b} &: 1+e^{j\phi_1}z^{-1}+e^{j\phi_2}z^{-2}-e^{j(\phi_1+\phi_2)}z^{-3}-e^{j\phi_3}z^{-4} \\
&\quad -e^{j(\phi_1+\phi_3)}z^{-5}+e^{j(\phi_2+\phi_3)}z^{-6}-e^{j(\phi_1+\phi_2+\phi_3)}z^{-7}
\end{aligned}
$$

$$
\begin{aligned}
s_{4-t} &: s_{3-t}\ e^{j\phi_4} \\
s_{4-b} &: s_{3-b}\ e^{j\phi_4}
\end{aligned} \qquad (9b)
$$

$$
\begin{bmatrix}\theta_0\\ \theta_1\\ \theta_2\\ \theta_3\\ \theta_4\\ \theta_5\\ \theta_6\\ \theta_7\end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 1 & 0\\
1 & 0 & 0 & 1 & 0\\
0 & 1 & 0 & 1 & 0\\
1 & 1 & 0 & 1 & 1\\
0 & 0 & 1 & 1 & 0\\
1 & 0 & 1 & 1 & 0\\
0 & 1 & 1 & 1 & 1\\
1 & 1 & 1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\\ \pi\end{bmatrix} \qquad (10)
$$

A second application of the polyphase CC is to control peak to average power ratio (PAPR) in Orthogonal Frequency Division Multiplex (OFDM) modulators. As described in the introduction, complementary codes A and B satisfy the correlation relationships shown in (11). The power spectral description of this relationship is shown in (12) and the relationship shown in (13) follows from (12). We also note that since the sample values of A(n) are equal to unity, (14) is also true.

$$A_N(n)*A_N(-n)+B_N(n)*B_N(-n)=2N\delta(n) \qquad (11)$$

$$|A(\omega)|^2+|B(\omega)|^2=2N \qquad (12)$$

$$|A(\omega)|^2\le 2N \qquad (13)$$

$$\text{Average } |A_N(\omega)|^2=N \qquad (14)$$

Examining (13) and (14) we conclude the validity of the relationship shown in (15).

$$\frac{\text{Peak Power}}{\text{Average Power}} \leq \frac{2N}{N} \leq 2 \qquad (15)$$

We now reverse the domains describing the complementary code signal and apply the signal in the frequency domain to access desirable properties in the time domain. We apply the polyphase complementary code as the phases of a set of complex sinusoids of an inverse discrete Fourier transform of an OFDM modulator. With the code applied in the frequency domain, the time signal formed by the sum of the complex sinusoids has the same properties as the power spectrum had when we examined the signal in the time domain. That is, the time signal has a peak mean square value no greater than twice the average mean square value. This is a very desirable property when the time signal must not exhibit peak excursions significantly greater than the average power level. Power amplifiers are often operated at average power levels which are a small fraction of peak power capabilities to avoid operating the amplifier in non linear (saturation) regions. Figure 9 shows the instantaneous power profile of a CC OFDM and of a standard OFDM signal while figure 10 shows the amplitude level probability curves for the two OFDM signals.

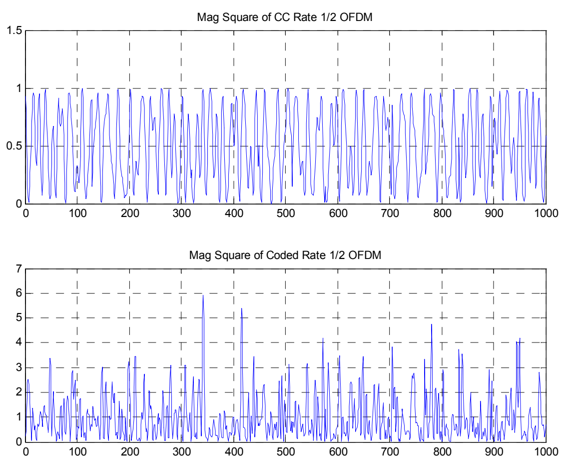

Figure 9. PAPR of CC and Straight OFDM

Concluding Review:

We presented a review of binary phase complementary codes along with a perspective of how to apply digital filters to the task of generating and of compressing the codes. We also examined code mates to suppress cross correlation side lobes in

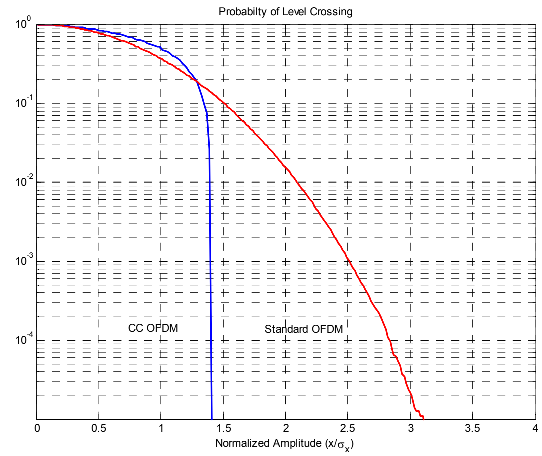

Figure 10. PAPR Amplitude Probabilities

the same manner autocorrelation side lobes are suppressed by the complementary codes. We then extended the filter perspective from binary phase code generators to polyphase code generators. Finally we presented two applications of the polyphase complementary codes to communication tasks.

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