GAME THEORY AND INTERFERENCE AVOIDANCE IN DECENTRALIZED NETWORKS

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ABSTRACT

In networks with transmitting users having separate un-coordinated receivers, waveform adaptation by greedy interference avoidance (IA) algorithms ([1], [2] and [3]) might not lead to fair network resource allocations. A game theoretic framework for this scenario, based on Potential game theory is presented in this paper. This model provides insight into development of algorithms that are fairer than the greedy IA algorithms and are amenable to distributed implementations.

1. INTRODUCTION

Given the profusion of offered service types, transmission protocols and software radio capabilities available today, networks are becoming less structured and increasingly involve distributed decision making. Nodes need to independently and periodically adapt themselves to changes in the interference environment that result from changes in network configuration (nodes entering or leaving the network), mobility and the nature of the wireless channel. The application of game theory to distributed waveform adaptation/selection techniques aimed at reducing the interference in a network is the focus of this paper. *

In general, transmitting nodes have little or no information about the interference seen at the receiver. One approach to avoid interference is to develop distributed waveform-adaptation algorithms where a minimal amount of feedback is required between receivers and transmitters. A distributed algorithm is proposed in [1] that sequentially updates the signature sequences associated with a particular receiver in a synchronous CDMA system. The sequences are found to decrease the total sum correlation (TSC) of the set. The minimization of the TSC is shown to be equivalent to maximization of the sum capacity [2], which forms a convenient information theoretic social welfare measure. This iterative algorithm (wherein users greedily increase their SINR) converges to a set of orthogonal sequences when the number of users is less than or equal to the processing gain and to a set of Welch Bound Equality sequences otherwise. Reference [2] generalizes this approach to the situation where nodes can adapt their modulation/demodulation methods using a general signal space approach. In [3], game theory is used to model and analyze this interaction of nodes communicating with a common receiver. It is shown in the paper that for two-player games, any combination of metric and receiver types results in a best response potential game that minimizes the total sum correlation function.

In the presence of multiple uncoordinated receivers and asymmetric power constraints for the users, waveform adaptation becomes more difficult as compared to the systems analyzed in [1], [2] and [3], where all nodes transmit to a single common receiver. Greedy IA algorithms based on maximization of sum-capacity might not lead to a fair allocation of the resources in the network. Hence waveform update decisions that improve some system welfare function in addition to reducing the interference at the receiver associated with a particular transmitting node are required. A game theoretic approach to analyzing these systems and constructing solutions that can lead to a fair utilization of network resources is presented in this paper. This framework is based on potential game theory, which makes it easily amenable to a distributed implementation.

The system model for the network scenario under consideration is described in Section 2. Section 3 gives a brief overview of game-theory and potential games. Section 4 presents the game-theoretic framework for fair waveform adaptation in a decentralized network. Section 5 presents an example formulation of a fair waveform adaptation game. Section 6 summarizes the paper and presents directions for future research.

2. SYSTEM MODEL

The network being analyzed is made up of a cluster of nodes. Each transmitting node has a separate node of interest, i.e., a separate receiving node. This leads to the existence of multiple un-coordinated receivers in the network. Figure 1 shows the system model with arrows connecting the transmitting node to its intended receiver node.

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The transmitting nodes are allowed to have different transmit power constraints and the power constraints are assumed to be fixed by a process independent of waveform adaptation. Interference is caused at the receiver nodes by transmissions from user nodes different from the one associated with the particular receiver node. The interference caused is influenced by the power constraints and associated channel characteristics of the user nodes.

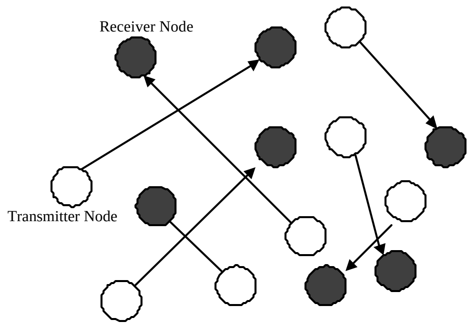

Figure 1: Network with multiple un-coordinated receivers

Let K be the number of transmitting nodes in the network. Let $s_i \in \mathbb{C}^{N\times 1}$ be the signature sequence[1] associated with transmitting node i. The number of available signal dimensions is N. The signature sequences are allowed to have real values (as opposed to bi-polar sequences). Without loss of generality, the signature sequences are assumed to have unit norm $\left(\|s_i\| = 1\right)$. The power received at the j^{th} receiver node from the i^{th} transmitting node is denoted by p_{ij}. This term takes into consideration both the transmit power constraint and the influence of the channel between the transmitting and receiving nodes. The data bit to be transmitted from the i^{th} transmit node is denoted by b_i. The received signal at the receive node is given by

$$r_j = \sum_{i=1}^{N} \sqrt{p_{ij}} b_i s_i + z \tag{1}$$

The vector, $z \in \mathbb{C}^{N\times 1}$, models additive white Gaussian noise with zero mean and unit variance. The channel is assumed to be constant over the time required for waveform adaptation.

3. GAME THEORY AND POTENTIAL GAMES

Consider a normal form game represented [4] as $\Gamma = \left\langle M, \{A_i\}_{i\in M}, \{u_i\}_{i\in M} \right\rangle$, where Γ is a game and $M = \{1, 2, \ldots, |M|\}$, is the set of players of the game. The set of actions available for player i is denoted by A_i and the utility function associated with each player i by u_i. If the set of all available actions for all players is represented by $A = \times_{i\in M} A_i$, then $u_i : A \to \mathbb{R}$. The utility function for each player is thus a function of the actions in the game. Players select actions that maximize their utility functions. A Nash Equilibrium (NE) for a game is an action profile from which no player can increase his utility by unilateral deviations. An action profile $a \in A$ is a NE if and only if

$$u_i(a) \ge u_i(b_i, a_{-i}) \forall i \in M, b_i \in A_i, a_{-i} \in \underset{\substack{j\ne i \\ j\in N}}{\times} A_j \tag{2}$$

Nash equilibria form the steady states of the game.

Suppose that a normal form game is played repeatedly. At each stage of the game, players choose actions that improve their utility functions. The criteria for a particular choice of action gives rise to the best and better response dynamics defined below.

Best Response Dynamic: At each stage, a player i is permitted to deviate from $a_i \in A_i$ to some action $b_i \in A_i$ iff $u_i(b_i, a_{-i}) \ge u_i(c_i, a_{-i}) \forall c_i \in A_i \text{ and } c_i \ne b_i \in A_i$ and $u_i(b_i, a_{-i}) > u_i(a)$.

Better Response Dynamic: At each stage, a player i is permitted to deviate from $a_i \in A_i$ to some action $b_i \in A_i$ iff $u_i(b_i, a_{-i}) > u_i(a)$.

A potential game ([5], [6]) is a normal form game such that any changes in the utility function of any player in the game due to a unilateral deviation by the player is also correspondingly reflected in a potential function.

A function $P : A \to \mathbb{R}$, is called an exact potential function if $u_i(a_i, a_{-i}) - u_i(\hat{a}_i, a_{-i}) = P(a_i, a_{-i}) - P(\hat{a}_i, a_{-i})$, $\forall i$ and $a_i, \hat{a}_i \in A_i$. A game that has an exact potential function is called an exact potential game. A function $P : A \to \mathbb{R}$, is called an ordinal potential function, if $u_i(a_i, a_{-i}) \ge u_i(\hat{a}_i, a_{-i}) \Leftrightarrow P(a_i, a_{-i}) \ge P(\hat{a}_i, a_{-i})$, $\forall i$ and $a_i, \hat{a}_i \in A_i$. A game that has an ordinal potential function is called an ordinal potential game.

NE of potential games are maximizers of their potential functions. All potential games following a best response dynamic converge to a NE. Potential games with finite action spaces have been established to follow a better response dynamic to converge to a NE as well.

[1]The waveforms used by transmitting nodes are assumed to be fully specified by the signature sequence in different signal dimensions [2]. Hence the waveform of a node is analogous to the signature sequence of a node.

4. FORMULATION AS A POTENTIAL GAME

Total sum capacity, defined as the sum of the rates achievable by users in the network, is used as a metric to evaluate the performance of the network in the centralized receiver scenario. All the IA algorithms introduced for the centralized network scenario that greedily improve their own utilities (rate or SINR), are designed to maximize the sum capacity of the network. However, when multiple un-coordinated receivers exist in the network, and when users could have asymmetrical power constraints, the total sum capacity might not be an appropriate metric. Maximization of this metric could lead to allocation of network resources that are biased towards stronger user-receiver pairs, resulting in very poor performance of the weaker user-receiver pairs.

A game-theoretic model based on potential game theory for the fair allocation of resources in decentralized networks is suggested here. Potential games are chosen as these are easy to analyze and give a framework where users can serve the greater good by following their own best interest, i.e., can maximize a global utility by only trying to maximize their own utilities. Hence it can lead to simple game formulations where maximizations of utility of users can lead to improving a global network fairness measure as well.

To incorporate the notion of fairness, the utility function of each user is modified such that in addition to improving the benefit associated with the user's performance at its intended receiver, the function also contributes to the fairness in the network. The utility function is of the form given in Equation 3.

$$\begin{aligned} u_i(s_i,p_i) = f_1(s_i,p_i) - \sum_{j\neq i,\, j=1}^{N} f_2\left(I\left(s_j,s_i\right),p_j,p_i\right) \\ - \sum_{j\neq i,\, j=1}^{N} g_{ij} f_3\left(I\left(s_i,s_j\right),p_i,p_j\right) \end{aligned} \tag{3}$$

Function f_1 quantifies the benefit associated with a particular choice of signature sequence and power. Functions f_2 is the interference measure for user i perceived at its associated receiver due to the other users present in the system. Function I is some function of two signature sequences s_i and s_j. Function f_3 is the interference caused by a particular user at the receivers associated with other users. Coefficient g_{ij} is a weighting factor.

The first two terms of the utility function are intended to reduce the interference at the receiver associated with a particular user. The third term attaches some benefit to being nice to other users (reducing network interference) and is intended to contribute to the fairness in the network.

A possible simple formulation of the potential function for this network is given by Equation 4.

$$Pot(S,P) = \sum_{i=1}^{N} \begin{pmatrix} f_1(s_i,p_i) - a \sum_{j\neq i,\, j=1}^{N} f_2\left(I\left(s_j,s_i\right),p_j,p_i\right) \\ -b \sum_{j\neq i,\, j=1}^{N} g_{ij} f_3\left(I\left(s_i,s_j\right),p_i,p_j\right) \end{pmatrix} \tag{4}$$

This function consists of the sum of the utilities of all users. Coefficients a and b are weighting factors, vector $S=[s_1,s_2,\ldots,s_N]$ and vector $P=[p_1,p_2,\ldots,p_N]$. Separating all the terms involving the i^{th} user,

$$\begin{aligned} Pot(s_i,S_{-i},p_i,P_{-i}) = & f_1(s_i,p_i) - a \sum_{j\neq i,\, j=1}^{K} f_2\left(I\left(s_j,s_i\right),p_j,p_i\right) \\ & - b \sum_{j\neq i,\, j=1}^{K} g_{ij} f_3\left(I\left(s_i,s_j\right),p_i,p_j\right) \\ & - a \sum_{j\neq i,\, j=1}^{K} f_2\left(I\left(s_i,s_j\right),p_i,p_j\right) \\ & - b \sum_{j\neq i,\, j=1}^{K} g_{ji} f_3\left(I\left(s_j,s_i\right),p_j,p_i\right) \\ & \underbrace{- \sum_{k\neq i,\, k=1}^{K} \begin{pmatrix} f_1(s_k,p_k) - \\ a \sum_{j\neq k,\, j\neq i,\, j=1}^{K} f_2\left(I\left(s_j,s_k\right),p_j,p_k\right) \\ -b \sum_{j\neq k,\, j\neq i,\, j=1}^{K} g_{kj} f_3\left(I\left(s_k,s_j\right),p_k,p_j\right) \end{pmatrix}}_{Non-contributing Terms} \end{aligned} \tag{5}$$

Formulation of the utility function and the potential game as an exact potential game requires the following condition (Equation 6) to be satisfied.

$$\begin{aligned} & u_i(s_i',p_i) - u_i(s_i,p_i) = \\ & Pot(s_i',S_{-i},p_i,P_{-i}) - Pot(s_i,S_{-i},p_i,P_{-i}) \end{aligned} \tag{6}$$

This is possible if $a=b=\dfrac{1}{2}$ and under the two scenarios listed below

Case1:

$$\begin{aligned} f_2\left(I\left(s_j,s_i\right),p_j,p_i\right) &= f_2\left(I\left(s_i,s_j\right),p_i,p_j\right), \\ f_3\left(I\left(s_j,s_i\right),p_j,p_i\right) &= f_3\left(I\left(s_i,s_j\right),p_i,p_j\right), \\ g_{ij} &= g_{ji}, \forall i,j \end{aligned} \tag{7}$$

Case2:

$$\begin{aligned} f_2(\bullet) &= f_3(\bullet) \\ g_{ij} &= 1, \forall i,j \end{aligned} \tag{8}$$

In the second case, another formulation of an exact potential function could be as follows (Equation 9).

$$Pot(S,P) = \sum_{i=1}^{K} f_1(s_i, p_i) - \sum_{j\neq i, j=1}^{K} f_3\left(I\left(s_i, s_j\right), p_i, p_j\right) \quad (9)$$

4.1. Ordinal Potential Function

The utility function and the potential function could be formulated as an ordinal potential game if the following condition (Equation 10) is satisfied.

$$u_i\left(s_i', p_i\right) \geq u_i\left(s_i, p_i\right) \Leftrightarrow Pot\left(s_i', S_{-i}, p_i, P_{-i}\right) \geq Pot\left(s_i, S_{-i}, p_i, P_{-i}\right) \quad (10)$$

This is possible when $f_{2i}(\bullet) = f_{3i}(\bullet) = f_{ui}(\bullet)$, where $f_{ui}(\bullet)$ is an ordinal transformation of $f_{pot}(\bullet)$ or when $f_{2i}(\bullet), f_{3i}(\bullet), f_{pot}(\bullet)$ are all ordinal functions where the utility function associated with each user is given by,

$$u_i(s_i, p_i) = f_1(s_i, p_i) - \sum_{j\neq i, j=1}^{K} f_{2i}\left(I\left(s_j, s_i\right), p_j, p_i\right) - \sum_{j\neq i, j=1}^{K} f_{3i}\left(I\left(s_i, s_j\right), p_i, p_j\right) \quad (11)$$

and the potential function is given by,

$$Pot(S,P) = \sum_{i=1}^{K} f_1(s_i, p_i) - \sum_{j\neq i, j=1}^{K} f_{pot}\left(I\left(s_i, s_j\right), p_i, p_j\right) \quad (12)$$

This formulation allows each user to have a different utility function, the only restriction being that the utility functions are ordinal transformations of each other.

5. EXAMPLE FORMULATION OF A POTENTIAL GAME

A possible formulation of a fair waveform adaptation game, based on the game-theoretic framework described in the previous section, is presented here.

Consider the utility function for the i^{th} user defined in Equation 13, modeled on the formulation in Equation 3.

$$u_i(s_i, p_i) = -\sum_{j\neq i} \frac{s_i^H s_j\, s_j^H s_i\, p_{ji}}{p_{ii}} - \sum_{j\neq i} \frac{s_i^H s_j s_j^H s_i\, p_{ij}}{p_{jj}} \quad (13)$$

Here, function f_2 (from Equation 3) is the interference caused by other users in the network at the receiver associated with user i weighted by the receive power of user i. Function f_3 is the interference caused by user i at receivers associated with other users, weighted by the receive power of the other users. As functions f_2 and f_3 are similar in structure ($f_2(\bullet) = f_3(\bullet)$), Equation 9 can be used to form the following exact potential function for this utility.

$$V(S,P) = -\sum_{i=1}^{N} s_i^H \left(\sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{s_j s_j^H p_{ji}}{p_{ii}}\right) s_i \quad (14)$$

The utility function can be rewritten as,

$$u_i(s_i, p_i) = -\frac{s_i^H R_{-ii} s_i}{p_{ii}} - \sum_{j\neq i} \frac{s_i^H s_j\, s_j^H s_i\, p_{ij}}{p_{jj}} \quad (15)$$

where,

$$R_{-ii} = R_i - s_i s_i^H p_{ii} - I = \sum_{\substack{j=1 \\ j\neq i}}^{K} s_j s_j^H p_{ji} \quad (16)$$

and R_i is the cross correlation matrix of the received signal at receiver i associated with user i,

$$R_i = E\left[r_i r_i^H\right] = \sum_{j=1}^{K} s_j s_j^H p_{ji} + E\left[zz^H\right] = \sum_{j=1}^{K} s_j s_j^H p_{ji} + I \quad (17)$$

The first term in the utility function can be identified as is the inverse Signal to Interference Ratio (SIR) at the receiver associated with user i. The second term is a weighted sum of interference caused by the user at other receivers in the system and hence contributes to network fairness. User i, thus tries to maximize its SIR and in addition, also tries to reduce the interference it causes at receivers corresponding to other users.

A similar utility, which increases a social measure, is presented in ([7], [8]). However this is specific to a system where each user talks to multiple receivers and does not take into account fairness in the network.

5.1. Convergence Characteristics

Maximizers of the potential function, V, form the steady states of the network. Exact potential games exhibit best and better response convergence to these steady states. Best response can be implemented by having each user change its signature sequence such that the user derives the best utility for the current state of the network. The utility function of the ith user can be re-written as (Equation 18)

$$u_i(s_i, p_i) = -s_i^H \left(\sum_{j\neq i} \frac{s_j s_j^H p_{ji}}{p_{ii}} + \sum_{j\neq i} \frac{s_j s_j^H p_{ij}}{p_{jj}}\right) s_i \quad (18)$$

Let,

$$R = \left(\sum_{j\neq i} \frac{s_j s_j^H p_{ji}}{p_{ii}} + \sum_{j\neq i} \frac{s_j s_j^H p_{ij}}{p_{jj}}\right) \quad (19)$$

Then the best response of the ith user is the eigenvector corresponding to the minimum eigenvalue of R. To allow the implementation of this best response iteration at the user end, the users are assumed to know the signature sequences of all the other users in the network and the value p_{ij} for all $i, j \in \{1, \ldots, K\}$.

Alternately, better response iterations can also be implemented at the user end. In this approach, the user randomly alters its signature sequence. All the receivers in the network that can hear a particular user send back the change in SINR caused by this signature alteration. If the sum of the inverses of the changes in SINR is negative, the user sticks to its update, else it reverses its signature alteration.

The convergence characteristic for a best response implementation of the algorithm is shown in Figure 2. The simulated network has six users and three signal space dimensions. The users are numbered according to decreasing received powers at their corresponding receiver nodes. It is seen that the game converges in about fifteen iterations. Best response is implemented by allowing each user to update sequentially in a round robin fashion. As each user utility update leads to an increase in the potential function, the algorithm converges even with non-sequential updates.

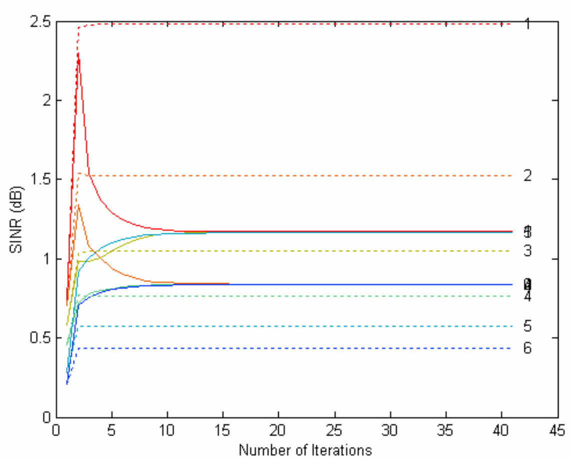

Figure 2: Best-response convergence for a network with multiple un-coordinated receivers (Solid Lines – Fair IA Algorithm, Dashed Lines-Greedy IA Algorithm).

Figure 2 also shows the convergence of a greedy interference algorithm (similar to the algorithms discussed in [1], [2] and [3] for the centralized receiver) extended to the multiple un-coordinated network scenario. In the greedy interference avoidance algorithm, users are concerned only with the maximization of their own SINR. Theil's entropy measure ([9]) an inequality index (defined below), is used to investigate and compare the fairness of the two approaches.

Theil's entropy measure – Inequality Index:
This is a measure of inequality proposed by Theil that derives from the notion of entropy in information theory. The entropy measure, T, is given by:

$$T = \sum_{i=1}^{L} q_i \left(\log q_i - \log\left(\frac{1}{L}\right)\right) \tag{20}$$

where q_i is the share of the i^{th} group, and L is the total number of groups. The index has a potential range from zero to infinity (for very large values of L), with higher values indicating more unequal distribution of resources.

Let r_i be defined as follows.

$$r_i \le \log_2\left(1 + \frac{p_{ii}}{\sum_{\substack{j=1\\ j\ne i}}^{K} s_j^H s_j p_{ji} + I}\right) \tag{21}$$

and

$$q_i = \frac{r_i}{\sum_{j=1}^{K} r_j} \tag{22}$$

The proposed algorithm leads to a fairness measure of *0.0195* as opposed to *0.2495* for the greedy algorithm. Hence the game formulation for waveform adaptation presented in this paper indeed results in fairer allocation of resources than the greedy IA algorithms for waveform adaptation.

In addition to the example exact potential game formulation presented in this paper, several ordinal transformations indicated in [3] could also be used to form fair games for waveform adaptation.

6. SUMMARY

A game theoretic framework based on Potential games, to model waveform adaptation in networks with multiple uncoordinated transmit-receive pairs is presented. This formulation leads to solutions that are fairer than the greedy IA algorithms. An example waveform adaptation algorithm based on this model that leads to a fairer allocation of resources than a greedy IA algorithm is also presented.

However, the implementation of the described fair algorithms requires considerable feedback from the receiver nodes in the network. Hence more efficient feedback mechanisms need to be investigated. Alternately, an investigation of more efficient better response schemes might be of significance, as potential games exhibit better response convergence.

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