ITERATIVE DECODING FOR DIFFERENTIAL MIMO SYSTEMS WITH NEAR-COHERENT PERFORMANCE

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ABSTRACT[#]

In this paper we develop multiple antenna space-time decoding techniques that enable very high capacities in mobile wireless systems. The method can be implemented in small form factor handheld software defined radios. We derive a low-complexity iterative decoding scheme based on diagonal or Cayley differential encoders for multi-input multi-output (MIMO) flat-fading channels. We show that our decoding method guarantees full spatial diversity. More importantly, it bridges the gap between differential and coherent receivers. Simulation results corroborate our theoretical analysis.

1. INTRODUCTION

Multi-antenna wireless communication links provide spatial diversity to combat fading. However, as the number of transmit antennas increases, channel estimation becomes more challenging. Among the difficulty in channel estimation is in the design of optimal training sequences and the associated channel estimation complexity at the receiver as the number of transmitter antennas increase. A natural means of bypassing channel estimation is to utilize differential modulation, which in the MIMO context has been pursued in [1][2][3][4][5].

One of the advantages of the orthogonal differential codes in [4] is their low decoding complexity. However, when the number of antennas is large, orthogonal differential space-time (ST) codes lose rate. Diagonal differential ST codes on the other hand, are relatively simple and flexible to generate [2] [3], and can be decoded fast with the near maximum-likelihood (ML) algorithm of [1]. Cayley codes have also been proposed for a variety of rates and number of transmit antennas [5], and can be decoded efficiently using the near ML algorithm of [6].

Notwithstanding, existing MIMO differential decoders do not outperform non-coherent ML decoders, which are known to be inferior (by about 3 dB) relative to their coherent counterparts. In this paper, we derive iterative decoders to bridge the gap of differential and non-coherent receivers and bring them closer to the performance of coherent MIMO receivers.

2. DIFFERENTIAL DESIGNS

At the transmitter, we adopt the diagonal [2] and Cayley [5] differential designs for a multi-antenna system with N_t transmit- and N_r receive-antennas. Supposing that the N_tN_r channels are independently complex Gaussian distributed with zero mean and unit variance, here we focus on quasi-static flat-fading channels; i.e., the channels are flat in frequency-domain and time-invariant during at least two blocks. At the νth receive antenna, we denote the nth received block as an $N_t \times 1$ vector $\boldsymbol{y}_\nu(n)$ (see [2] and [5] for details). Suppose that $h_{\nu,\mu}$ defines the channel response from the μth transmit-antenna to the νth receive-antenna. The input-output relationship for the nth block is

$$\boldsymbol{y}_\nu(n) = \sqrt{\rho}\boldsymbol{D}_s(n)\boldsymbol{h}_\nu + \boldsymbol{w}_\nu(n), \qquad \nu \in [1, N_r], \tag{1}$$

where $\boldsymbol{h}_\nu = [h_{\nu,1}, \ldots, h_{\nu,N_t}]^T$ is the vector channel response for the νth receive-antenna, $\boldsymbol{w}_\nu$ is the additive white Gaussian noise with zero mean and unit variance, ρ is the expected SNR at each receive antenna, and the unitary matrix $\boldsymbol{D}_s(n)$ is defined as

$$\boldsymbol{D}_s(n) = \begin{cases} \boldsymbol{V}_n\boldsymbol{D}_s(n-1) & n > 0 \\ \boldsymbol{I} & n = 0 \end{cases} \tag{2}$$

with unitary matrix $\boldsymbol{V}_n$ drawn from a constellation $\mathcal{V}$ of unitary-diagonal codes [2] or -Cayley codes [5] of size 2^{RN_t}. R denotes the transmission rate and $\boldsymbol{I}$ is the identity matrix.

Based on the model in (1) and (2), two ML decoders follow readily: the first is the non-coherent one defined as

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$$\hat{\boldsymbol{V}}(n) = \arg\min_{V \in \upsilon} \sum_{\nu=1}^{N_r} \left\| \boldsymbol{y}_\nu(n) - \boldsymbol{V}\boldsymbol{y}_\nu(n-1) \right\|^2; \qquad (3)$$

while the second is the coherent one (which requires channel knowledge at the receiver)

$$\hat{\boldsymbol{D}}_s(n) = \arg\min_{D \in \upsilon} \sum_{\nu=1}^{N_r} \left\| \boldsymbol{y}_\nu(n) - \boldsymbol{D}\boldsymbol{h}_\nu \right\|^2. \qquad (4)$$

$\boldsymbol{V}$ and $\boldsymbol{D}$ are code matrices in the set of all possible candidate constellation codes in (3) an (4), respectively. Apparently, when N_t is large, the size of υ is large, and the decoding complexity for both the non-coherent and the coherent option is as high as $O(2^{N_t R})$. Another issue is that the performance of non-coherent decoder in (3) can be as much as 3 dB worse than the coherent counterpart in (4). To solve this "3 dB" issue we propose iterative decoding for both diagonal and Cayley codes, which reduces the performance gap between the coherent and non-coherent decoders. In addition, to address the issue of complexity, we apply a lattice reduction (LR) algorithm for the iterative decoding of the diagonal codes. We note that sphere decoding can readily be utilized to reduce the complexity of iterative Cayley decoding, but is not presented in this paper.

3. ITERATIVE DECODING

To enable the use of the decoding in (4) requires channel knowledge, and when we differentially encode we do not explicitly send channel training information. We note that differential encoding may be preferable for channels that change sufficiently fast such that periodic training would require excessive training overhead, thus reducing user transmitted bit rate, and in addition excessive channel estimation complexity in the receiver. Thus in this work, we use the first block for differential decoding as a training block and then rely on decision directed channel estimation to subsequently track further channel variation. Specifically, in the first block, we rely on the linear minimum mean square error (LMMSE) channel estimator as:

$$\boldsymbol{h}_\nu = \boldsymbol{y}_\nu(0)/(1+\rho^{-2}), \ \ \nu \in [1, N_r]. \qquad (5)$$

Based on this coarse channel estimate in (5), we can detect the information blocks using any algorithm for coherent detection. Surprisingly, we observe that if the channel is time-invariant, this method (coherent decoding with either a ML or LR algorithm with estimated channel) achieves almost the same performance as that achieved by the non-coherent ML decoder in (3) (shown by simulation later). However, as we expect, the performance gap between the coherent and non-coherent decoders is approximately 3 dB. To bridge the performance gap, we need to further improve the performance of the non-coherent receiver. To this end, we employ a decision-directed approach. Since the channels are quasi-static, using the estimated blocks, we can obtain a

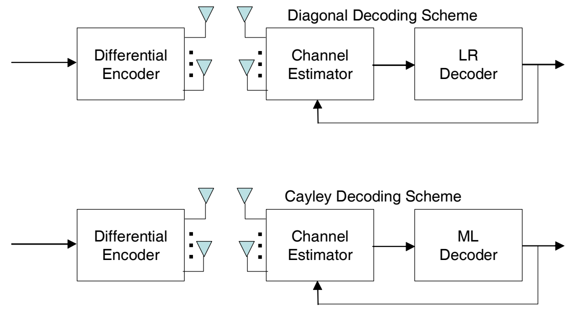

Figure 1: Discrete-time baseband system models for Diagonal and Cayley space-time codes.

refined MMSE channel estimate as:

$$\hat{\boldsymbol{h}}_\nu = \sum_{n=0}^{N_b-1} \left(\hat{\boldsymbol{D}}_s^*(n) \boldsymbol{y}_\nu(n) \right) / (N_b + \rho^{-2}), \ \ \nu \in [1, N_r], \qquad (6)$$

where N_b is the number of blocks, $\hat{\boldsymbol{D}}_s^*(n)$ denotes the estimate of $\boldsymbol{D}_s(n)$, and superscript * stands for conjugation. The number of blocks N_b controls the tradeoff between decoding delay and channel estimation performance.

With a refined channel estimate available, we can more accurately detect the transmitted information blocks. Counting one channel estimation and symbol detection process as a single iteration, we have confirmed by simulations that with a small number (2 or 3) of iterations, the performance of our iterative decoder approaches the performance of the coherent ML decoder. The transceiver design is shown in Figure 1, with the iterative algorithmic process described below.

Iterative Algorithmic Process:

1. *Use the first receive block to estimate channel based on* (5);
2. *Perform decoding algorithm* (*LLL for Diagonal scheme, ML for Cayley scheme*) *as shown in* (4) *with estimated channels*;
3. *After decoding N_b blocks of transmitted symbols, refine estimated channels as in* (6);
4. *If achieves the required number of iterations, then stop; otherwise go back to step* 2 *using updated channel in step* 3.

4. EFFICIENT NEAR ML DECODING

A computationally efficient near-ML solution is possible via a lattice reduction (LR) algorithm. For non-coherent decoding (3) has been proposed in [1]. Inspired by [1], we modify the LLL algorithm in [7] by replacing $\boldsymbol{y}_\nu(n\text{-}1)$ as the channel knowledge $\boldsymbol{h}_\nu$ and deriving a low-complexity LR algorithm approaching ML performance of the coherent decoder in (4). This algorithm is ready by following the steps in [1]. However, here we assume the channel is not known at either transmitter or receiver. Thus, to exploit this low-complexity coherent decoder, we need to get channel information first. Just previously (steps 1 – 4 above), we developed an alternative decoding scheme that provides

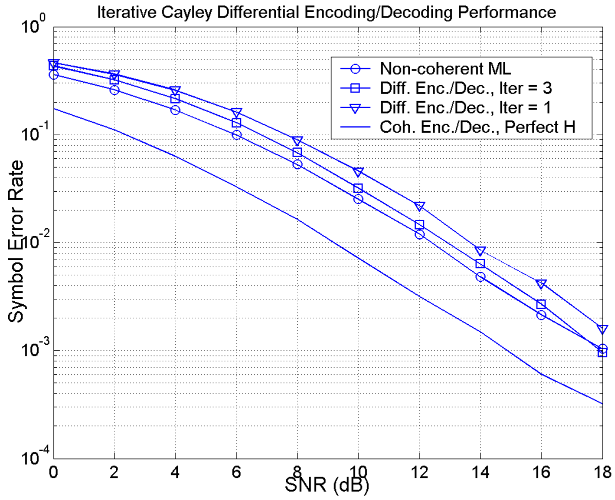

Figure 2: Cayley code iterative differential encoding / decoding performance.

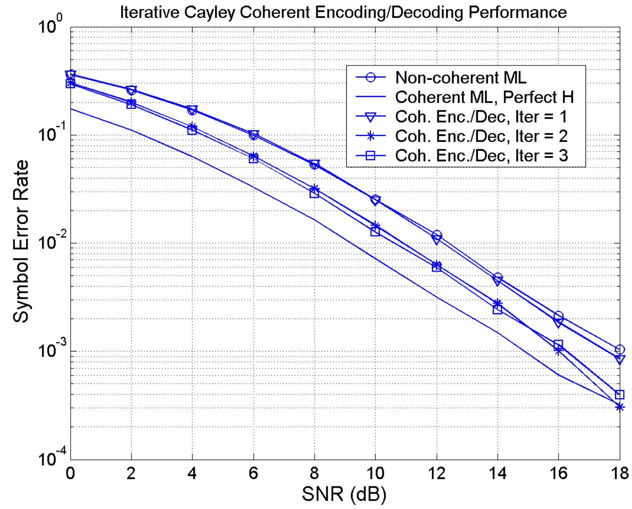

Figure 3: Cayley code iterative coherent encoding / decoding performance.

near-coherent ML performance, without channel knowledge. It can be used with any decoding method (ML-, LLL-, or sphere-decoding).

For diagonal codes we use our coherent LR algorithm to provide near ML coherent performance, but with low complexity.

5. SIMULATION RESULTS

In this work we consider $N_t = 2$ transmit- and $N_r = 1$ receive-antennas, with a transmission rate $R = 1$ bit per channel use. Below we describe each respective scheme used for Cayley and diagonal space-time codes. For both codes, we set the time-invariant length of the channel to be $N_b = 20$ blocks.

5.1. Cayley Codes with Iterative Decoding

The design of the Cayley unitary differential codes can be found in [5]. Specifically, we utilize the code for the $N_t = 2$ transmit antenna case provided in (42) of [5]. For the Cayley codes, the code structure does not belong to a group constellation (in the same sense as the diagonal codes), and thus cannot be coherently detected using the LLL algorithm as with the diagonal codes. Thus we use (2) for differential operation. Typically for coherent operation, training symbols must be sent so that the receiver can provide an estimate of the channel to the coherent ML decoder. We do not use training to enable our coherent operation, however.

Differential modulation, when used in conjunction with a ML non-coherent receiver totally bypasses any need for channel estimates. In the following we provide performance results for both coherent and differential encoding/decoding, but for either case, no training is sent from the transmitter other than a reference block for the first transmission. When performing iterative decoding with Cayley codes, the ML decoding in (4) can be performed either according to a coherent reference of differential reference. In the coherent case, $\boldsymbol{y}_v(n)$ is simply the transmitted code (no differential encoding) passed through the fading channel with noise added at the receiver, and $\boldsymbol{D}$ is simply all possible constellations in the set from (42) in [5] (without differential encoding). Otherwise, for the differentially encoded case, we use (4) as shown.

Figure 2 shows the results of the differentially encoded case using (4) as the decoder. Note, for three iterations, the performance nearly reaches that of the ML non-coherent decoder (which uses no channel estimation whatsoever).

Figure 3 shows results for coherent transmission of the Cayley codes. For one iteration, the coherently encoded transmission is roughly equivalent to the differential transmission using the ML non-coherent decoder. With three iterations, the ML coherent iterative decoder is within nearly 1 dB of a coherent system that has *perfect* channel knowledge (the ideal decoder).

5.2 Diagonal Codes with Iterative Decoding

The design of the unitary diagonal matrix set $\mathcal{V}$ is the same as found in [2]. Each block has length 2. The simulated performance is shown in Figure 4. We observe that: i) with one iteration (the curve with legend "LLL, Iterations = 1"), the proposed decoder achieves the performance of non-coherent ML decoding in (3) with reduced decoding complexity; and ii) as the number of iterations increases, the performance of our scheme approaches the performance of the coherent ML detector in (4). In fact for two iterations, the performance is quickly approaching that of the ideal (assuming we know the channel exactly) decoder in (4).

A comparative analysis of the ML and lattice algorithm decoding times for diagonal codes is now presented based on decoding times found in [1] as measured on a SGI R10000 at 195MHz. For this analysis we assume that the results presented in [1] are applicable to coherent and non-

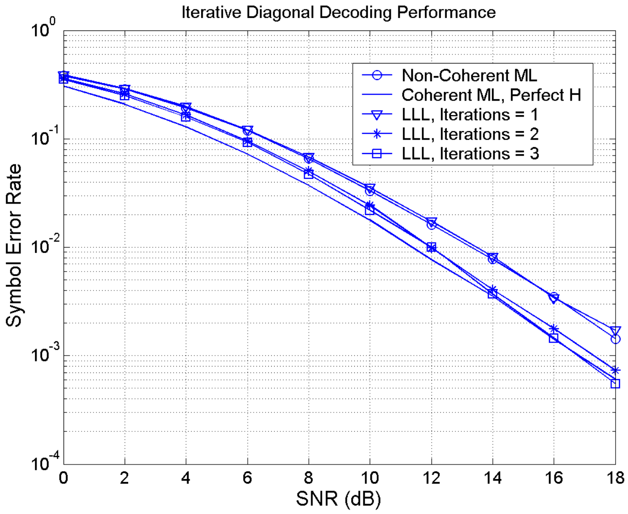

Figure 4: Diagonal code iterative performance using coherent LLL algorithm.

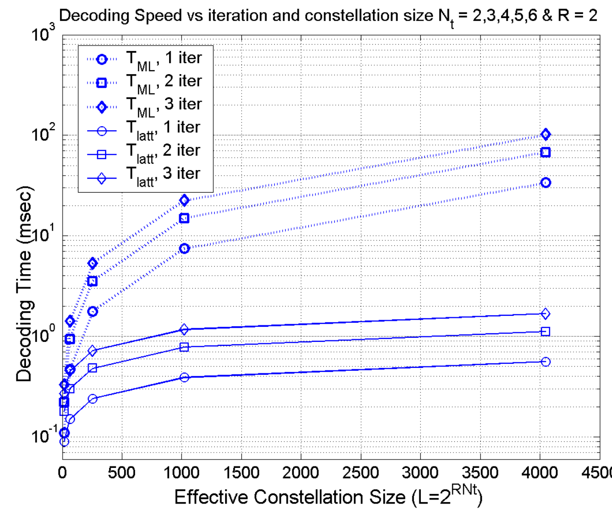

Figure 5: Lattice and ML algorithm decoding time as a function of M (*same as* N_t) and number of decoding iterations for Rate = 2 bits per channel use.

coherent ML decoding. When we apply iterative decoding to reduce the performance gap between differential and coherent systems, we see how the decoding time (or complexity) results compare for multiple iterations of each algorithm.

From our simulations (R = 1), we found near-coherent ML decoding performance is possible with relatively low complexity. However, when the rate is increased, along with an additional number of transmit antennas, we show the importance of a low complexity decoding technique. This data is taken directly from [1], extrapolated, and then plotted to help visualize the decoding times for multiple decoding iterations, as shown in Figure 5. For R = 2 the decoding time is always less complex for the lattice decoder from [1]. The decoding complexity for $N_t = 4$ is an order of magnitude lower in complexity, while for $N_t = 6$ the lattice decoder is two orders of magnitude lower complexity.

6. CONCLUSIONS*

In this paper, we proposed iterative decoding for space-time codes so that transmission of training symbols is not required. Commonly training is used for receiver channel estimation, and with training unavailable we utilized decision feedback to estimate the channel. We evaluated the performance of two different unitary space-time coding structures, one where the Cayley transform is used to generate the code and the other based on group codes producing a diagonal code structure. For both code types, we showed that iterative decoding sufficiently enables the receiver to perform channel estimation, without any training information, such that the performance is nearly the same as the ideal coherent maximum likelihood (ML) receiver. Using the diagonal codes, we showed that a low complexity coherent receiver is plausible using an iterative coherent LLL decoder, thus minimizing complexity for higher rates and additional transmit antennas. This scheme also produces near ideal performance without channel knowledge. Software defined radios are becoming more common for multi-antenna systems which strive to produce robust-high capacity transmission. The method presented in this work enables very high performance mobile MIMO channel performance in software radios that are easily portable, but which have limited signal processing resources.

* The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U.S. Government

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